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Apsidal Motion in Massive Binaries: Or How to Sound Stellar Interiors Without Asteroseismology

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Abstract

The determination of the apsidal motion in close eccentric binary systems is (one of) the most efficient and reliable observational technique allowing to probe the internal structure of a star. The apsidal motion is the secular precession of the binary orbit's major axis and it is dependent on the tidal interactions occurring between the two stars. The apsidal motion rate is directly related to the internal structure of the stars, in particular their inner stellar density profile. The rate of apsidal motion can be constrained, together with the fundamental parameters of the stars, through state-of-the-art adjustment of combined radial velocity and light curve measurements made over a long timescale. The internal structures of the stars are subsequently constrained through the confrontation of the observationally determined parameters to theoretical models of stellar structure and evolution. This powerful technique has been known for years but has been seldom applied to massive stars. I highlight its interest and discuss recent results concerning several massive binaries, among others one binary that captured our attention for decades notably due to its twin property: HD 152248. While standard 1D stellar evolution models predict stars having a smaller internal stellar structure constant, that is to say, stars having a lower density contrast, than expected from observations, I demonstrate that the addition of mixing inside the models helps to solve, at least partially, this discrepancy. Whether this additional mixing might be fully explained by rotational mixing is under investigation. Ongoing studies with the non-perturbative code MoBiDICT show that the perturbative model assumption is not justified in highly distorted stars. In these cases, the apsidal motion is underestimated, which exacerbates even more the need for enhanced mixing inside the models.

Keywords: stars: early-type, stars: evolution, stars: massive, binaries: eclipsing, binaries: spectroscopic

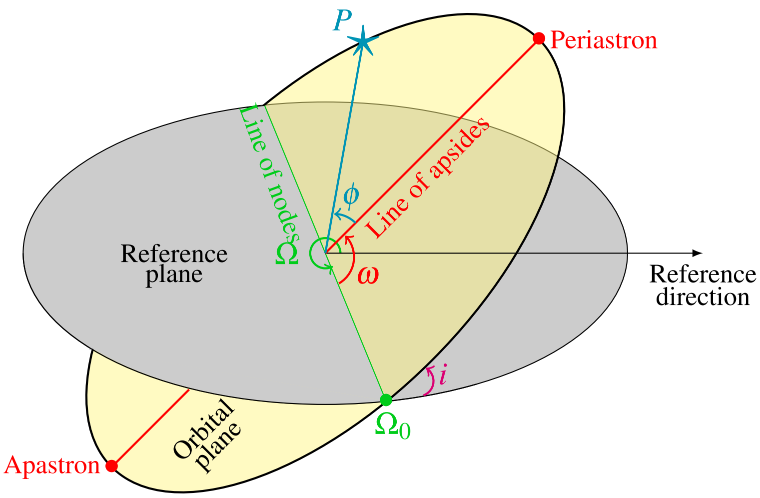

Figure 1: Definition of the orbital elements of a binary system: The plane of the sky (the reference plane, in gray); the orbital plane (in yellow) inclined by an angle i with respect to the reference plane; the intersection between these two planes (the line of nodes, in green); the angle Ω between the line of nodes and the reference direction; the primary star (the cyan star) located in the orbital plane; the line joining the periastron and apastron (the line of apsides, in red). The argument of periastron, ω, is the angle between the line of nodes and the line of apsides, and the true anomaly, ϕ, is the angle between the line of apsides and the position of the primary star; both are measured in the orbital plane.

1. Ode to the Apsidal Motion

There is a very simple test one can do provided an assembly of astrophysicists is given: Ask the audience how to sound the stellar interiors. Chances are high one will very quickly hear the following answer: asteroseismology. If one takes the same audience and asks a slightly more complicated – but also slightly more provocative – question, namely how to sound the stellar interiors without asteroseismology, chances are small one will hear "apsidal motion." Regardless, the apsidal motion in massive binaries is a very powerful technique to probe the interiors of stars, especially for high-mass stars for which asteroseismology is found to be less accurate and reliable. This paper is aiming at showing why and how apsidal motion in massive binaries is a key to sound the stellar interiors.

The Keplerian problem of celestial motion states that in a binary the two stars, assumed to be point-like particles, orbit one another on elliptic orbits which maintain a constant orientation in space over time; only the positions of the stars on their orbit, measured through the true anomaly (see Fig. 1) change with time. This two-body problem, arguably valid for wide eccentric binaries, turns out inappropriate for close eccentric binaries in which the three dimensional extension of the stars cannot be ignored. The tidal forces stars exert on each other are responsible for not only the exchanges of angular momentum in the system, but also the non-gravitational fields of the stars consequent to the stellar deformations. The major observable consequence on the orbital motion is the apsidal motion, that is to say, the precession of the line of apsides – the line joining the periastron and apastron (see Fig. 1) – with time. This motion is usually relatively small, on the order of a few degrees per year. It is of utmost importance to insist upon the fact that this motion is a purely Newtonian motion, or said differently, a non-Keplerian movement, that should not be mistaken with the well-known relativistic apsidal motion. This latter, though contributing to the total rate of apsidal motion in a binary system, is of minor though non-negligible importance for most binaries and surely for those discussed in this paper.

Is this apsidal motion rate, though relatively small, observed and measured in binaries? As illustrated in Fig. 2, the answer is yes. The upper panel reports the apsidal motion period (defined as the time for the orbit to make a complete 360° turn) as a function of the orbital period for 61 systems for which an apsidal motion determination is available in the literature. The last decades have seen a resurgence of interest for the apsidal motion measurement especially in massive binaries thanks notably to the high-precision photometric observations taken with the Transiting Exoplanet Survey Satellite (TESS) and Kepler satellite as well as to the long-term spectroscopic monitoring of promising candidates. Two important studies from the last decade are attributed to Hong et al. (2016) who derived the apsidal motion rate in several binaries belonging to the Milky Way, but also the Large and Small Magellanic Clouds (LMC and SMC), and Zasche and Wolf (2019) and Zasche et al. (2020) who studied 162 eclipsing binaries in the LMC (see bottom panels in Fig. 2), opening up the window to extend the apsidal motion study in other galaxies. The correlation between orbital and apsidal motion periods is evident in these three panels and will find a mathematical justification in the apsidal motion equations (see Sect. 3). The physical explanation is however evident as the closer the stars in the binary, the more important the tidal interactions between the stars at the origin of the apsidal motion.

How the apsidal motion rate is measured in a binary system is explained in Sect. 2 and illustrated by a few well-chosen specific cases. The apsidal motion equations and their link to the internal stellar structure are provided in Sect. 3 as a means to sound the interior of the stars. The confrontation of observationally-determined stellar and orbital parameters with 1D stellar structure and evolution models as well as recently developed 3D non-perturbative models is the topic of Sect. 4. Section 5 reviews the status of the apsidal motion study in massive binaries.

2. Getting the Most (Apsidal Motion Rate) out of Observations

The apsidal motion rate of a binary system can be derived using two different techniques, depending whether the binary system is an eclipsing binary for which photometric observations are available (see Sect. 2.1) or a spectroscopic binary for which radial velocity (RV) measurements are available (see Sect. 2.2). While the former technique is most frequently used because the high-quality photometric data such as TESS allow the accurate determination of the apsidal motion rate using a time span of the observations much shorter than the decades of observations necessary for the RV adjustments, this latter technique turns out to be very powerful and more accurate when sufficient data are available. Indeed, as the apsidal motion is a slow precession,

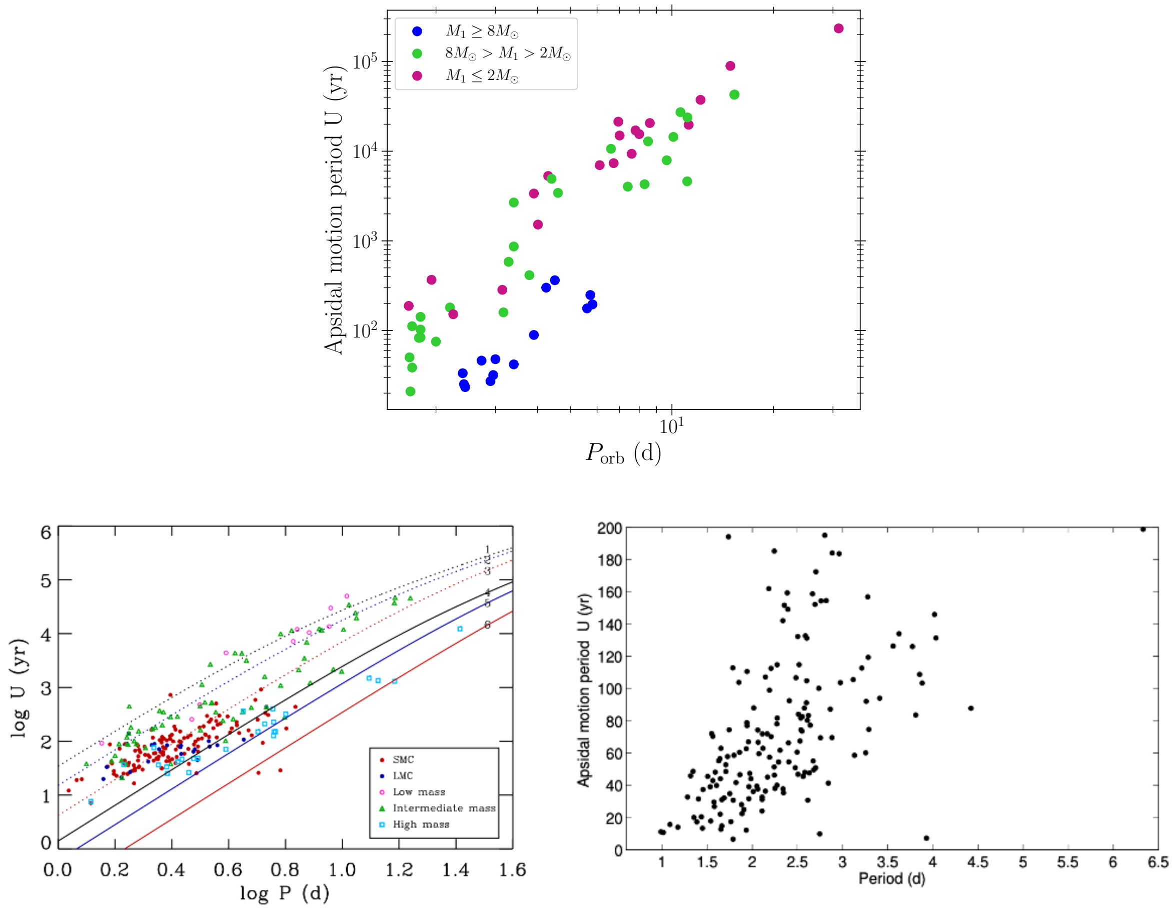

Figure 2: Apsidal motion period as a function of the orbital period. *(Top)* Individual observed systems coming from the literature (Baroch et al., 2021, 2022; Claret et al., 2021; Marcussen and Albrecht, 2022; Rauw et al., 2016; Rosu et al., 2020b, 2022a,b, 2023; Torres et al., 2010; Wolf et al., 2006, 2008, 2010). Only the primaries are plotted, colour-coded by their mass. Figure taken from Rosu et al. (in press). *(Bottom left)* Study of Hong et al. (2016). Credit: Hong et al. (2016, Fig. 6), © The Authors. *(Bottom right)* Studies of Zasche and Wolf (2019) and Zasche et al. (2020). Credit: Zasche et al. (A&A, 640, A33, 2020, Fig. 5), reproduced with permission, © ESO.

observations spanning several years, or even decades, are usually necessary to get an accurate measure of the apsidal motion rate of a system.

We can already forecast that the most promising systems are close short-period eccentric, eclipsing, double-line spectroscopic systems for which photometric, spectroscopic, and RV data can be analysed to constrain the fundamental stellar parameters and the apsidal motion rate of the system. These massive binaries are indeed intrinsically invaluable systems, given that they are the only ones allowing us to constrain the fundamental properties of the stars (such as their masses and radii) in a model independent way (three examples are discussed in Sect. 2.3), while their apsidal motion rate is used to probe the internal structure of the stars (see Sect. 3).

2.1. Eclipsing binaries

The apsidal motion rate of an eclipsing binary can be derived based on a set of photometric observations provided that the eclipses of both stars are observed during one orbital cycle, in other words, that the inclination of the system is close enough to 90°. The best way to illustrate the influence of the longitude of periastron on the light curves of a system is to compute theoretical light curves for a twin binary system (i.e., a system hosting two stars having the same mass m, radius R, and effective temperature $T_{\rm eff}$).

Figure 3 shows nine such light curves computed with the `Nightfall` code (developed and maintained by R. Wichmann at Hamburg Observatory, Germany) for nine different values of ω ranging from 0° to 160°, all other parameters identical, and assuming an eccentricity $e = 0.134$ and an inclination $i = 68.6^\circ$ for the system. For the sake of comparison, the primary eclipse is defined at phase 0 for all the light curves. First, we observe that for these values of ω (except for $\omega = 0^\circ$ where both eclipses share the same depth), the primary eclipse is always the deepest because it happens closer to periastron passage while the secondary eclipse happens closer to apastron passage. Second, we observe a clear change in the depths of both eclipses and in their relative phase shifts, entirely attributable to the change in ω. Indeed, the depth and phase of an eclipse depend upon the orbital separation at conjunction phase, the latter itself depends on the orientation of the ellipse with respect to our line of sight, that is to say, ω.

From this simple picture, we can glimpse that the apsidal motion rate can be derived from either the direct fit of the light curves of a binary taken at different epochs with ω as a free parameter or the fit of the times of minimum of the eclipses of the same light curves. While the former case is straightforward using fitting codes such as the `Nightfall` or `PHOEBE` codes (`PHOEBE` is developed and maintained at Villanova University, US), the latter case involves the use of the equations of Giménez and Bastero (1995) to fit the phase differences between primary and secondary minima at different epochs (see also Zasche and Wolf, 2019; Baroch et al., 2021; Rosu et al., 2022a,b).

2.2. Spectroscopic binaries

The apsidal motion rate can also be determined based on a set of primary (P) and secondary (S) RVs provided the binary is a double-line spectroscopic binary and spectroscopic

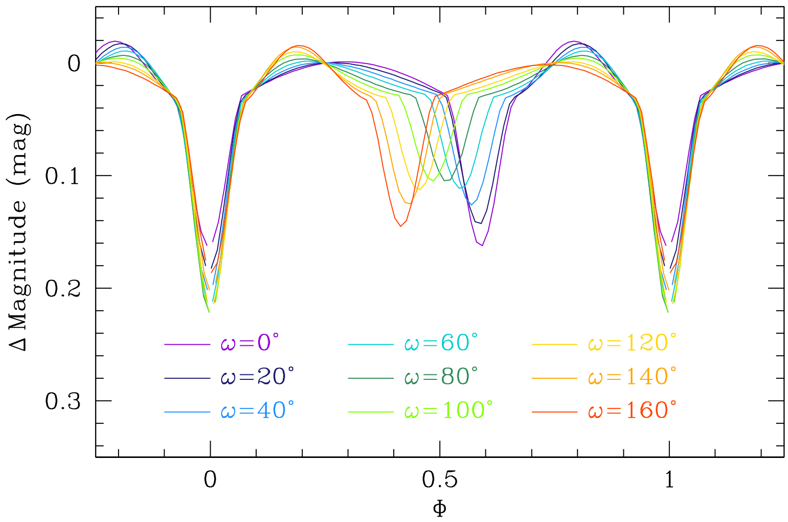

Figure 3: Nine theoretical light curves colour-coded by the assumed value of ω. All other parameters are identical ($e = 0.134$, $i = 68.6°$, and the stars have the same m, R, and $T_{\rm eff}$). Credit: Rosu (2021, Fig. 3) – CC BY 4.0.

observations spanning a relatively long time span are available. These observations are used to disentangle the contributions of the two stars and simultaneously derive the RVs at each time of observation. The derived RVs can be fit according to the following equations

$$RV_{\rm P}(t) = \quad K_{\rm P}\left(\cos(\phi(t)+\omega(t))+e\cos(\omega(t))\right)+\gamma_{\rm P}, \qquad (1)$$

$$RV_{\rm S}(t) = -K_{\rm S}\left(\cos(\phi(t)+\omega(t))+e\cos(\omega(t))\right)+\gamma_{\rm S}, \qquad (2)$$

where K_* and γ_* are the semi-amplitude of the RV curve and the apparent systemic velocity of the corresponding star. Alternatively, the RVs of the secondary star can be converted into equivalent primary RVs using the mass ratio and fitted in a similar way as the primary RVs. The apsidal motion is explicitly taken into account in Eqs. (1) and (2) through the linear variation of ω with time:

$$\omega(t) = \omega_0 + \dot{\omega}(t - T_0), \qquad (3)$$

where $\dot{\omega}$ is the apsidal motion rate and ω_0 is the argument of periastron at the time of reference T_0. Figure 4 illustrates one such resulting RV adjustment for the binary system HD 152248 (see Rauw et al., 2016; Rosu et al., 2020b, 2022a,b, 2023; Barclay et al., 2024, for other examples).

2.3. Three eclipsing and spectroscopic binaries under the light

In the lucky case of a double-line spectroscopic and eclipsing massive binary, the apsidal motion rate determined is very robust and accurate. We illustrate it in the case of three binary systems belonging to the young open cluster NGC 6231.

To the best of my knowledge, HD 152248 is the only binary for which the apsidal motion rate has been derived through the simultaneous adjustment of the RVs and light curves. Rosu et al. (2020b) adjusted all available data at the time of their study using their home-made implementation of the apsidal motion in the PHOEBE code. Figure 5 evidently illustrates that this

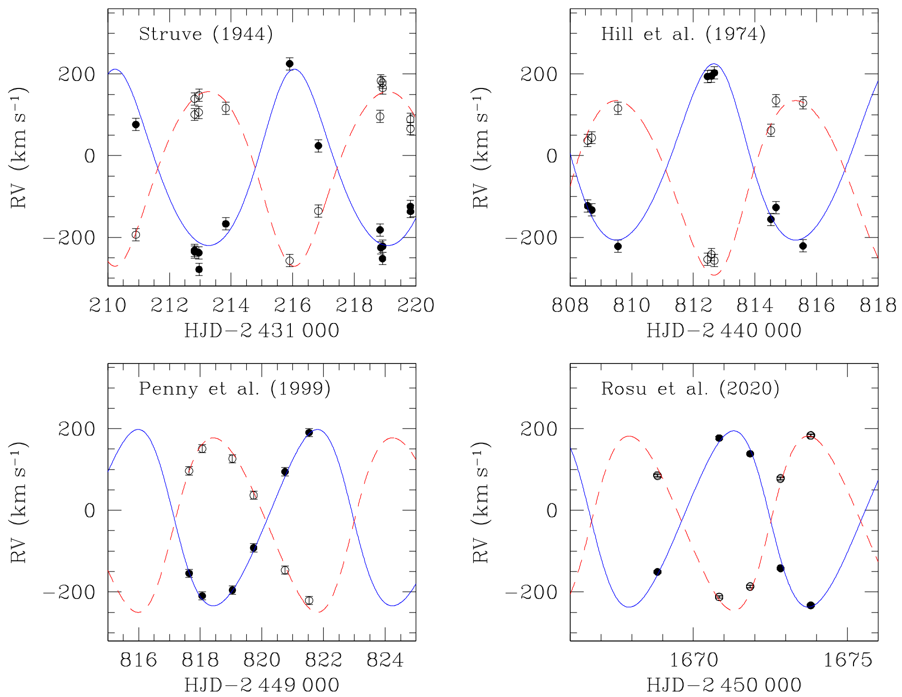

Figure 4: Measured RVs of the primary (filled dots) and secondary (open dots) stars of HD 152248, and best-fit RV curves (blue and red). Data from Struve (1944), Hill et al. (1974), Penny et al. (1999), and Rosu et al. (2020b). Credit: Rosu (2021, Fig. 6) – CC BY 4.0.

simultaneous fit is entirely compatible with the independent fit of the equivalent RVs as well as the individual fits of the light curves at different epochs.

In their in-depth study of the massive binary HD 152219, Rosu et al. (2022a) derived the apsidal motion rate from the RVs and cross-checked their value with the values of ω inferred from the fit of the light curves (see Fig. 6, left). They performed a second cross-check through the fit of the times of minimum of the eclipses and showed these values are consistent with those expected from the RV adjustment (see Fig. 6, right).

The massive binary CPD-41° 7742 suffered high degeneracies in its solution of the RV adjustment due to its very low eccentricity and relatively large apsidal motion rate (Rosu et al., 2022b). Henceforth, the same authors proceeded to the adjustment of the times of minima of the eclipses (see Fig. 7, left) and cross-checked their determination of the apsidal motion rate is consistent with the individual adjustment of the light curves (see Fig. 7, right) and RVs (not shown).

3. Apsidal Motion Rate Equations and Their Link to the Internal Stellar Structure Constants

Claret (2023) in his abstract goes straight to the point by explicitly claiming: “One of the most reliable means of studying the stellar interior is through the apsidal motion in double line

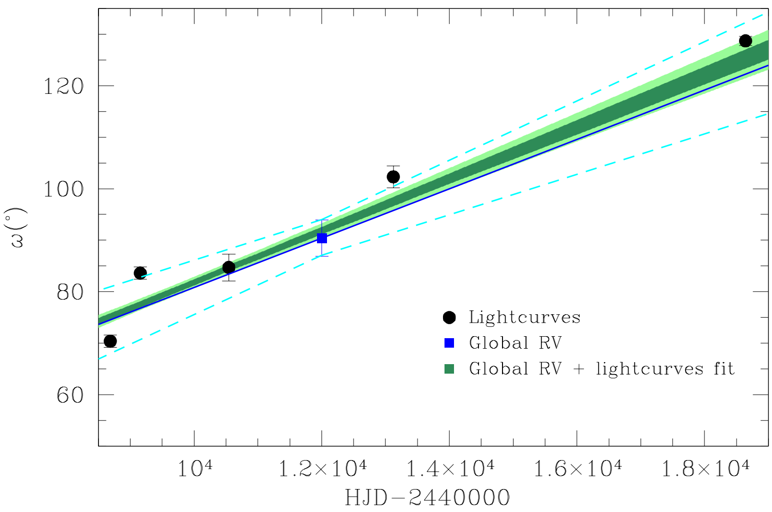

Figure 5: Values of ω as a function of time inferred from the RVs and light curves of HD 152248. The ω_0 value obtained from the global fit of all RVs is indicated by the blue filled square. The best value of $\dot{\omega}$ and its 1σ uncertainties from the equivalent RVs are shown by the solid blue line and the dashed cyan lines. Each individual fit of a light curve is represented by a black dot. The 1σ and 2σ uncertainties on ω from the simultaneous fit of all data with PHOEBE are indicated in dark and light green. Credit: adapted from Rosu et al. (A&A, 635, A145, 2020b), reproduced with permission, © ESO.

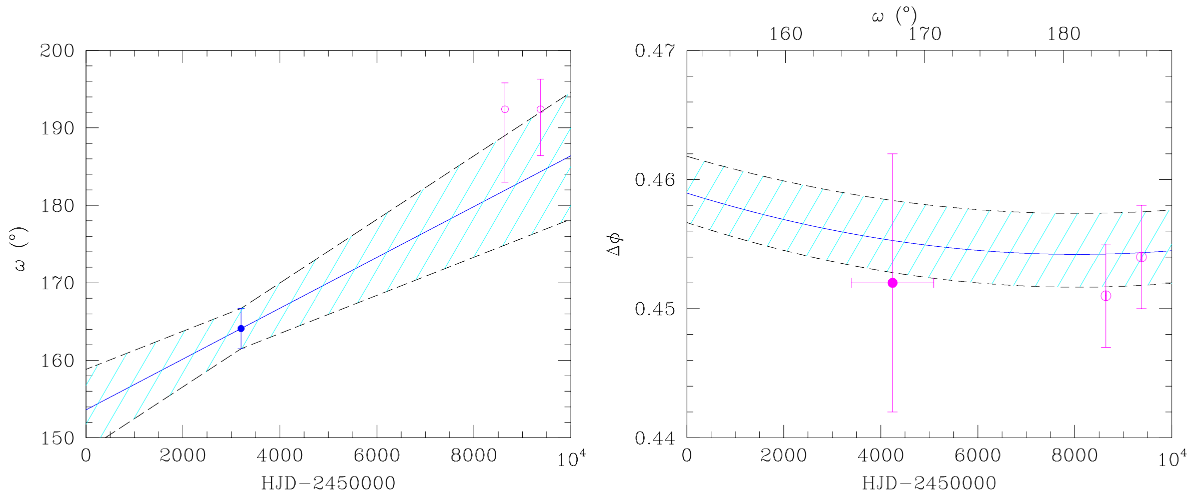

Figure 6: (*Left*) Values of ω as a function of time inferred from the RVs and light curves of HD 152219. The fits of the photometry are symbolised in pink. The ω_0 value obtained from the global fit of all RV data is indicated by the blue dot, while the solid blue line corresponds to the best-fit value of $\dot{\omega}$ inferred from the RVs, and the hatched cyan zone corresponds to the range of values according to the 1σ uncertainties on ω_0 and $\dot{\omega}$. (*Right*) Values of the phase difference $\Delta\phi$ between the primary and secondary eclipses as a function of time and ω inferred from the RVs and light curves. The fits of the photometry are symbolised in pink. The solid blue line corresponds to the best-fit value of $\Delta\phi$ inferred from the RVs, and the hatched cyan zone corresponds to the range of values according to the 1σ uncertainties on e. Credit: Rosu et al. (2022a) – CC BY 4.0.

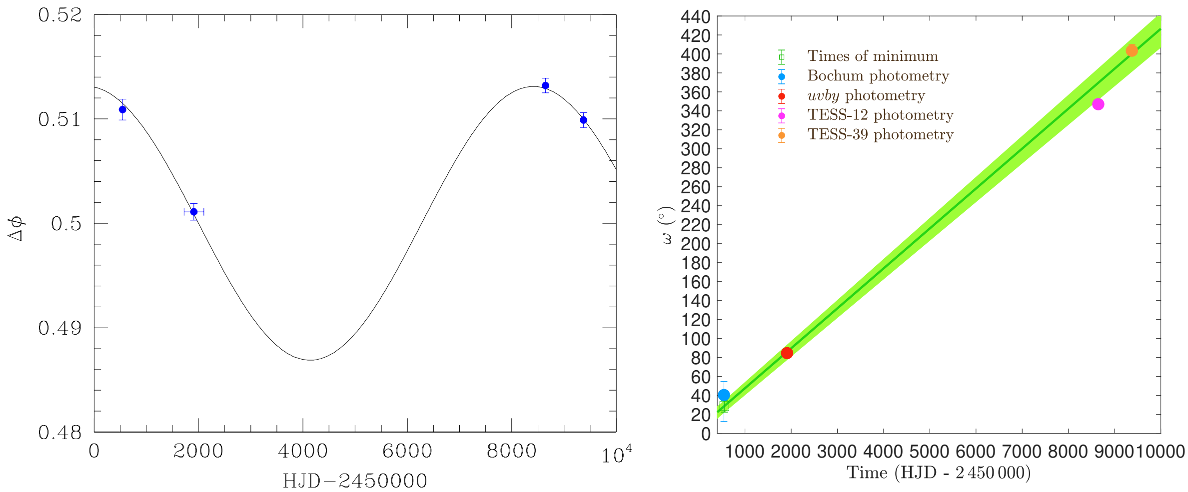

Figure 7: (*Left*) Values of the phase difference $\Delta\phi$ between the primary and secondary eclipses as a function of time inferred from the times of minimum of the eclipses of CPD-41° 7742. The fits of the photometry are symbolised in blue. The solid line corresponds to our best-fit adjustment. (*Right*) Values of ω as a function of time inferred from the times of minimum of the eclipses and the individual adjustments of the light curves. Credit: Rosu et al. (2022b) – CC BY 4.0.

eclipsing binary systems." Let me in this Section put this sentence into context and demonstrate the potency of the apsidal motion as a means to sound the interior of the stars.

The apsidal motion rate of a system is made of, in the perturbative case, the Newtonian contribution $\dot{\omega}_{\rm N}$ and a general relativistic correction $\dot{\omega}_{\rm GR}$. According to Sterne (1939), the expression of $\dot{\omega}_{\rm N}$ is given by

$$\begin{aligned}\dot{\omega}_{\rm N} =& \frac{2\pi}{P_{\rm orb}}\left[15f(e)\left\{k_{2,1}q\left(\frac{R_1}{a}\right)^5+\frac{k_{2,2}}{q}\left(\frac{R_2}{a}\right)^5\right\}\right.\\ &\left.+g(e)\left\{k_{2,1}(1+q)\left(\frac{R_1}{a}\right)^5\left(\frac{P_{\rm orb}}{P_{\rm rot,1}}\right)^2+k_{2,2}\frac{1+q}{q}\left(\frac{R_2}{a}\right)^5\left(\frac{P_{\rm orb}}{P_{\rm rot,2}}\right)^2\right\}\right],\end{aligned} \tag{4}$$

where only the contributions arising from the second-order harmonic distorsions of the gravitational potential are considered. According to Shakura (1985), the expression of $\dot{\omega}_{\rm GR}$ is given by

$$\dot{\omega}_{\rm GR} = \left(\frac{2\pi}{P_{\rm orb}}\right)^{5/3}\frac{3(G(m_1+m_2))^{2/3}}{c^2(1-e^2)}. \tag{5}$$

In these expressions, $q = m_2/m_1$ is the mass ratio, $P_{\rm orb}$ is the orbital period of the system, a is the semi-major axis of the orbit, R_* the radius, $k_{2,*}$ the internal structure constant, and $P_{{\rm rot},*}$ the rotational period of the considered star, $f(e)$ and $g(e)$ are functions of the eccentricity of the orbit which expressions are given in, e.g., Rosu (2021), G is the gravitational constant, and c is the speed of light.

The internal stellar structure constant k_2 is itself an algebraic expression of the η_2 function evaluated at the stellar surface

$$k_2 = \frac{3 - \eta_2(R_*)}{4 + 2\,\eta_2(R_*)}. \tag{6}$$

The η_2 function is the solution of the Clairaut–Radau differential equation (Hejlesen, 1987) that depends upon the density profile inside the star:

$$r\frac{d\eta_2(r)}{dr} + \eta_2^2(r) - \eta_2(r) + 6\frac{\rho(r)}{\bar{\rho}(r)}\,(\eta_2(r) + 1) - 6 = 0, \tag{7}$$

with the boundary condition $\eta_2(0) = 0$, where ρ is the density in a shell at a distance r from the centre, and $\bar{\rho}$ is the mean density inside the sphere of radius r.

Hence, k_2 is a measure of the density stratification between the core and the external layers of the star. An homogenous sphere of constant density has $k_2 = 0.75$, which is the maximum value k_2 can take. On the opposite, massive stars have a dense core and a diluted envelope, and their k_2 can be as low as 10^{-4} (Rosu et al., 2020a). During its evolution, the core of a star contracts while its envelope expands, hence k_2 decreases rendering this quantity a good indicator of stellar evolution. Figure 8 illustrates it through the evolution of k_2 as a function of the hydrogen mass fraction inside the star – the main indicator of stellar evolution – (*left*) as well as as a function of the stellar radius (*right*) obtained with the `GENEC` and `Clés` models with an initial mass of $32.8\,M_\odot$. `GENEC` is developed and maintained at the Geneva Observatory, Switzerland (see, e.g., Eggenberger et al., 2008); `Clés` is developed and maintained at the University of Liège, Belgium (see, e.g., Scuflaire et al., 2008). The impact on the internal structure of models assuming different internal mixing efficiencies is already highlighted on these figures (see Sect. 4 for a thorough discussion).

All terms in Eqs. (4) and (5) are known from observations except for $k_{2,1}$ and $k_{2,2}$. In the special case of a twin system like HD 152248, the k_2 of the stars can be assumed to be identical and an observational value can be inferred for k_2 through Eq. (4). In the general case of two different stars, the situation is slightly more complex but constraints on the primary star can be obtained; we here refer to Rosu (2022) for a thorough mathematical development and discussion, beyond the scope of this paper.

4. When Enhanced Mixing in Stellar Models is the Only Way to Reproduce the Observations… even in 3D

The confrontation of observationally determined apsidal motion rate with theoretical values obtained with one-dimensional stellar structure and evolution models has been attempted in several studies. I here particularly focus on two in-depth studies of the binaries HD 152248 and HD 152219: Rosu et al. (2020a, 2022a). The authors built `Clés` stellar evolution models with different prescriptions for the internal mixing to constrain the internal mixing processes occurring inside the stars. Notably, the overshooting was implemented as a step-function through the overshooting parameter $\alpha_{\rm ov}$, while additional mixing was added through the turbulent diffusion, implemented as a partial mixing process with velocities of chemical elements $V_i = -D_{\rm T}\frac{d\ln X_i}{dr}$,

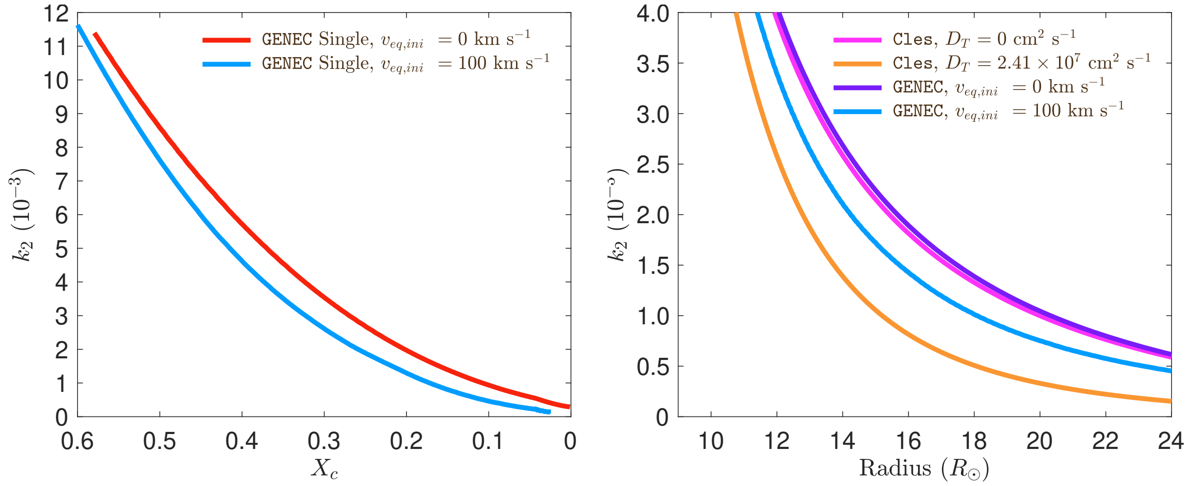

Figure 8: (*Left*) Evolution of k_2 as a function of the hydrogen mass fraction X_c inside the star for two `GENEC` models with an initial mass of $32.8\,M_\odot$, one with initial rotational velocity of $100\,\mathrm{km\,s^{-1}}$ (blue), the other one with zero initial rotational velocity. (*Right*) Evolution of k_2 as a function of the stellar radius for the two `GENEC` models presented in Fig. 8 and two `Clés` models with the same initial mass, without turbulent diffusion (pink) and with a high turbulent diffusion (orange). Credit: adapted from Rosu et al. (2020a, A&A, 642, A221,), reproduced with permission, © ESO.

with the index i indicating the considered element and where D_T is the turbulent diffusion coefficient (measured in $\mathrm{cm^2\,s^{-1}}$). The impact of the turbulent diffusion as implemented is to reduce the abundance gradient of the chemical elements. Overshooting has a similar consequence: it brings additional hydrogen from the external layers of the star to its core, hence fuel for nuclear reactions, therefore increasing the star lifetime during the main-sequence phase. The Levenberg–Marquardt minimisation technique implemented in the `min-Clés` routine was used to search for best-fit models of the stars in terms of observational properties, including the apsidal motion rate. `min-Clés` is developed and maintained by M. Farnir at the University of Liège, Belgium.

Stellar models for the binaries were first built adopting as constraints the stellar mass, radius, and position in the Hertzsprung–Russell (HR) diagram only (see Rosu et al., 2020a,b, 2022a, for further details), and α_ov and D_T were fixed to 0.20 and $0\,\mathrm{cm^2\,s^{-1}}$, respectively. The evolutionary tracks corresponding to the best-fit models in this case are shown in Fig. 9 (purple dotted line) together with the observational values and their error bars (in red), for both stars of HD 152248 (left panel) and the primary star of HD 152219 (right panel). It is clear, especially in the case of HD 152248, that the models are unable to reproduce the physical properties of the stars.

Leaving D_T as a free parameter of the adjustment allowed the authors to find models for which the mass, radius, and position in the HR diagram are perfectly reproduced, as illustrated in Fig. 9 (green dashed lines). The best-fit models are achieved for $D_\mathrm{T} = 1.9\times10^7\,\mathrm{cm^2\,s^{-1}}$

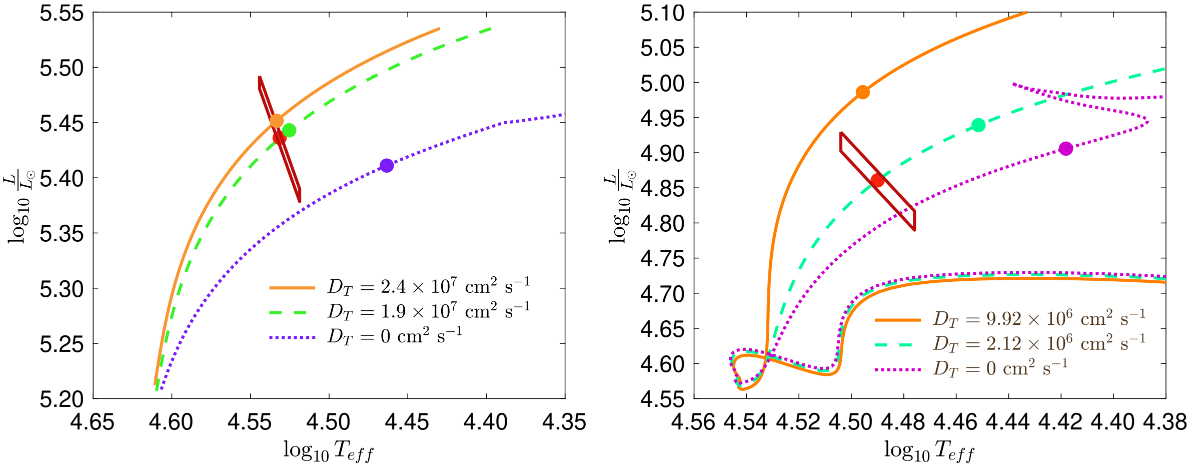

Figure 9: HR diagrams for HD 152248 (*left*) and HD 152219 (*right*): evolutionary tracks of `Clés` models. The observational values and their error bars are represented in red. The dots over-plotted on the tracks correspond to the models that fit the observational k_2. (*Left*) Credit: Rosu et al. (2020a, A&A, 642, A221,), reproduced with permission, © ESO. (*Right*) Credit: adapted from Rosu et al. (2022a) – CC BY 4.0.

and $D_\text{T} = 2.12 \times 10^6\,\text{cm}^2\,\text{s}^{-1}$, respectively, for HD 152248 and HD 152219. If we now look at the models that reproduce the observational k_2, symbolised by the dots over-plotted on the evolutionary tracks, we observe they are located further away on the tracks in comparison to the location of the best-fit models discussed here above. Given that k_2 decreases with time, the best-fit models have too high a k_2-value. In other words, the models have too low a density contrast between the core and external layers of the stars. By enforcing enhanced mixing inside the models through a larger D_T-value, the authors were able to solve, at least partially, this k_2 discrepancy and obtain best-fit models in terms of not only the mass, radius, and position in the HR diagram, but also in terms of the k_2-value and the ensuing apsidal motion rate. These best-fit models have $D_\text{T} = 2.4 \times 10^7\,\text{cm}^2\,\text{s}^{-1}$ and $D_\text{T} = 9.92 \times 10^6\,\text{cm}^2\,\text{s}^{-1}$ for HD 152248 and HD 152219, respectively (plain orange lines in Fig. 9).

While this conclusion – that the standard stellar evolution models predict stars being too homogeneous in terms of density stratification compared to what is expected from observations – is a key result that could not be achieved without the study of the apsidal motion in massive binaries, the physical origin of this turbulent diffusion still needs to be determined. In this avenue, Rosu et al. (2020a) performed a preliminary study and built `GENEC` models accounting for the rotationally-induced mixing inside the models to see whether the turbulent diffusion added in the `Clés` models could be the signature of the stellar rotation. According to their results, the authors conclude that rotation is certainly the key, though further investigations are needed and ongoing.

A question worth asking is whether this need for enhanced mixing is due to the inherent 1D stellar modelling? To answer this question, we used the `MoBiDICT` code, developed and maintained by L. Fellay at the University of Liège, Belgium (Fellay et al., 2024). `MoBiDICT` stands for “Modelling Binaries Deformations Induced by Centrifugal and Tidal forces.” It is a non-perturbative method, as opposed to the perturbative model assumption adopted in the 1D stellar modelling that considers the centrifugal and tidal forces as small perturbations of the spherical symmetry and only accounts for leading terms. `MoBiDICT` computes the entire precise 3D deformed structure of each component including the effects of stellar deformations on the mass redistribution. It calculates the instantaneous non-perturbative tidal acceleration perturbation and its consequence on the apsidal motion (Fellay and Dupret, 2023; Fellay et al., 2024).

The first important conclusion is that the deformations of binaries are significantly underestimated in perturbative models (Fellay and Dupret, 2023). This is illustrated in Fig. 10, top panel, where $\eta_2(R_*)$ is shown as a function of the normalised radius inside the star for four different stars belonging to twin binaries. As expected, stars having significant envelope mass (low-mass main-sequence and red giant branch stars) are subject to higher deformations. The corresponding relative difference in $\eta_2(R_*)$ between perturbative and non-perturbative methods is shown as a function of the semi-major axis of the orbit scaled by the radius of the star in Fig. 10, bottom panel. The authors therefore conclude that the perturbative model assumption is not justified in high distorsion cases, when the orbital separation is comparable to the radii of the stars.

The impact on the theoretical apsidal motion rate follows η_2: the higher the envelope mass compared to the total mass of the stars, the more impacted the stars, as illustrated in Fig. 11, where the relative difference in $\dot{\omega}$ between non-perturbative and perturbative models is shown as a function of the relative separation between the stars for the four systems considered previously and an orbital eccentricity of 0.1 (left panel) and for $20\,M_\odot$ main-sequence stars but different values of the eccentricity (right panel). The most significant discrepancies are reached for low-mass stars and red giants, for which up to 70% and 40% errors are made on the apsidal motion rate determination at low-separation ($a/R_1 \sim 3$) and eccentricity. The discrepancy is even higher for a given orbital separation at higher eccentricity given that the stars get closer during periastron passage, hence the stellar deformations are more pronounced and the ensuing impact on the apsidal motion rate is more important.

Fellay et al. (2024) applied `MoBiDICT` to known twin binary systems and compared the obtained parameters to those from 1D stellar modelling. Overshooting was included in the models as the only mixing mechanism so as to avoid any degeneracies between different mixing mechanisms that would bias a direct comparison between the models. Table 1 summarises the observational parameters of the binaries PV Cas, IM Per, Y Cyg, and HD 152248 together with the fitted overshooting parameter necessary to reproduce the observational properties in both the perturbative and non-perturbative approaches. Except in the most massive binary, HD 152248, the value of $\alpha_{\rm ov}$ is always higher in the non-perturbative than in the perturbative approach. Indeed, given that the deformations are more important in the non-perturbative approach than in the perturbative one, k_2 is smaller and, similarly to the studies presented here above, additional mixing is necessary to reproduce the observed k_2 value. Furthermore, though the non-perturbative approach is a small correction to the perturbative approach, it is noticeable

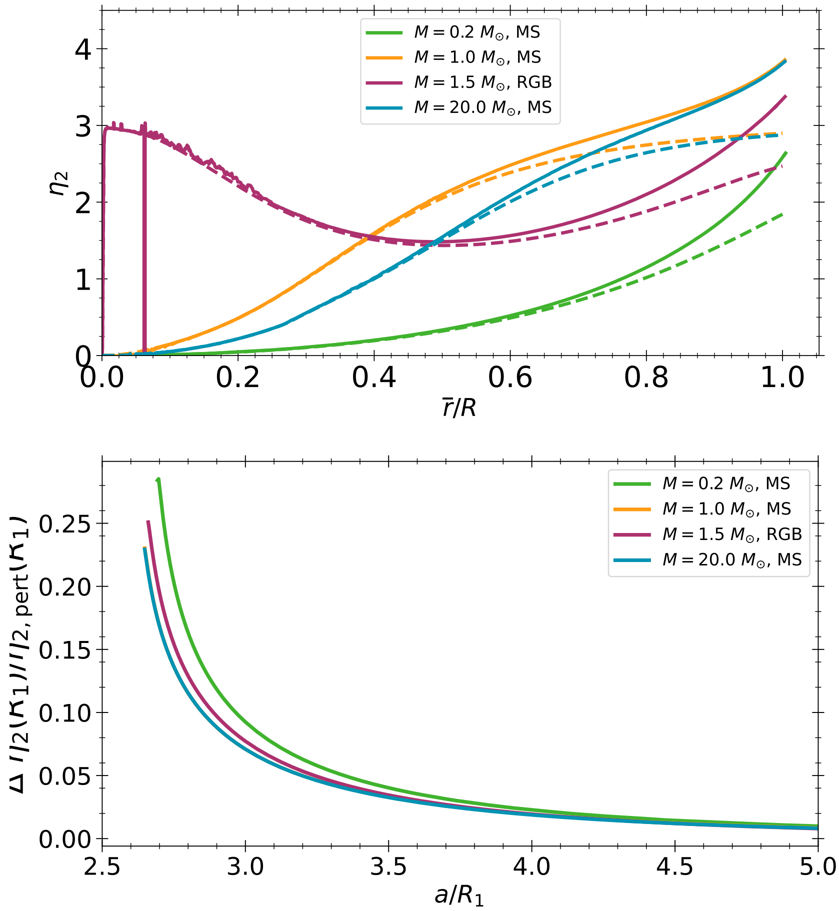

Figure 10: (*Top*) $\eta_2(R_*)$ as a function of the radius inside the star normalised by the total radius for four different stars belonging to twin binaries made of two main-sequence stars of $0.2M_\odot$ (green), $1.0M_\odot$ (orange), or $20.0M_\odot$ (blue) each, or two red giants of $1.5M_\odot$ (purple) each, separated by a distance $a = 2.8R_1$. Solid and dashed lines correspond to non-perturbative and perturbative models, respectively. *Bottom* Relative difference between the $\eta_2(R_*)$ computed with `MoBiDICT` and with the perturbative approach as a function of the semi-major axis of the orbit scaled by the radius of the star for the same systems. Credit: adapted from Fellay and Dupret (2023) – CC BY 4.0.

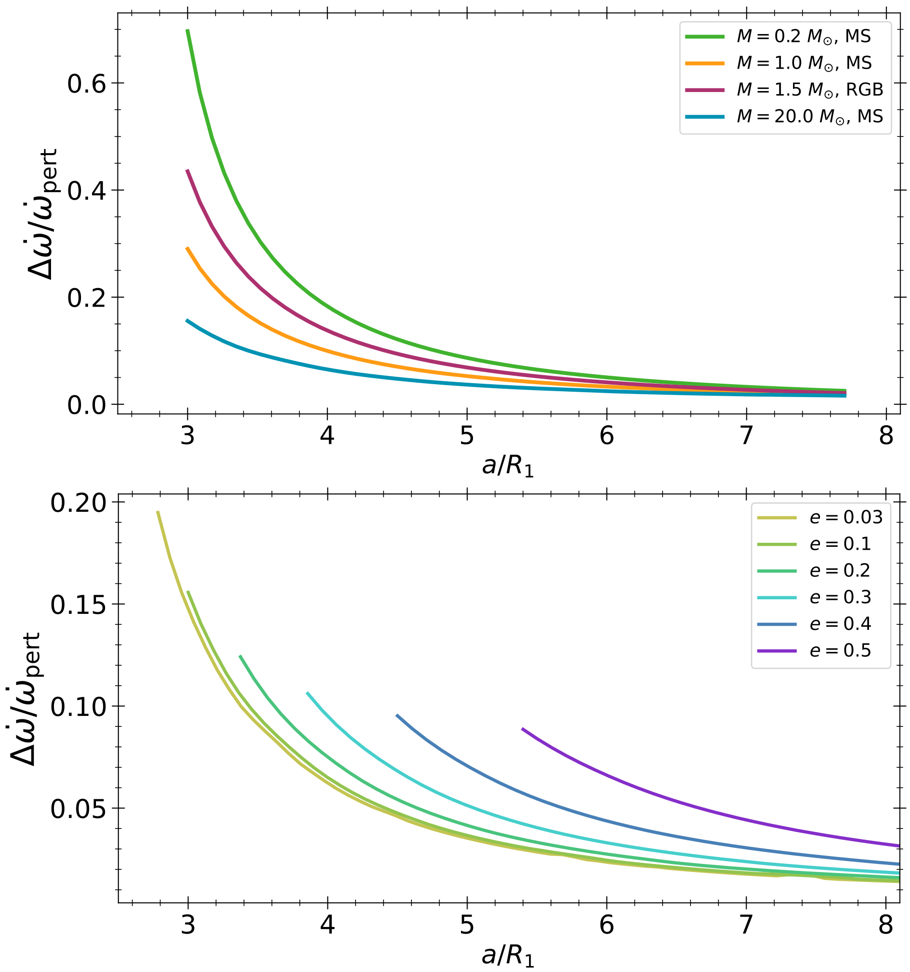

Figure 11: Apsidal motion relative difference as a function of the orbital separation normalised by the stellar radii for the four same twin binaries as in Fig. 10 and an orbital eccentricity of 0.1 (*top*), and for $20M_\odot$ main-sequence stars but different values of the eccentricity (*bottom*). Credit: adapted from Fellay et al. (2024) – CC BY 4.0.

Table 1: Observed and model properties of the systems studied. The first line block corresponds to the observational parameters. The second line block gives the values of the free overshooting parameter obtained performing either a pure perturbative analysis or using `MoBiDICT`. The third line block gives the decomposition of the apsidal motion in the `MoBiDICT` analysis in terms of the general relativistic contribution ($\dot{\omega}_{GR}$), the contribution from the perturbative approach ($\dot{\omega}_{pert.}$), and the correction induced by the non-perturbative approach ($\Delta\dot{\omega}_{non\text{-}pert.}$).

Parameter	**PV Cas** [1,2,3]	**IM Per** [2,4]	**Y Cyg** [5,2,3]	**HD 152248** [6,7]
M ($M_\odot$)	2.78 ± 0.08	1.78 ± 0.01	17.72 ± 0.30	29.5 ± 0.5
P_{orb} (d)	1.75047	2.25422	2.996321	5.816498
a/R_1	4.80	4.60	4.95	3.47
e	0.0325	0.0491	0.145	0.13
$\dot{\omega}_{obs}$ ($^\circ$ yr^{-1})	4.42 ± 0.04	2.37 ± 0.06	7.54 ± 0.04	1.84 ± 0.08
α_{ov} (pert.)	0.923	0.306	1.01	1.29
α_{ov} (`MoBiDICT`)	0.951	0.333	1.05	1.28
$\dot{\omega}_{GR}$ ($^\circ$ yr^{-1})	0.25	0.11	0.35	0.16
$\dot{\omega}_{pert.}$ ($^\circ$ yr^{-1})	4.00	2.12	6.89	1.58
$\Delta\dot{\omega}_{non\text{-}pert.}$ ($^\circ$ yr^{-1})	0.17	0.13	0.30	0.11

[1]Torres et al. (2010); [2]Claret et al. (2021); [3]Marcussen and Albrecht (2022); [4]Lacy et al. (2015); [5]Harmanec et al. (2014); [6]Rosu et al. (2020a); [7]Rosu et al. (2020b).

that it is of the same order of magnitude as the general relativistic correction (see Table 1). Therefore, it confirms that we cannot neglect it anymore in the study of the apsidal motion in close eccentric binaries.

5. The Apsidal Motion as a Means to Sound Stellar Interiors is Still in its Infancy

In massive close eccentric binaries, the longitude of periastron might precess in time, a phenomenon known as apsidal motion. This motion takes its origin from the tidal interactions occurring between the two stars and has been observed and measured in hundreds of systems in the last decades. The measure of the apsidal motion rate in a binary system is a powerful and robust means to sound the interior of the stars. Indeed, in the perturbative case, the apsidal motion rate is directly proportional to the internal stellar structure constants k_2 of the stars, a measure of the inner density profile of the stars. The precious and valuable information the study of apsidal motion in massive binary stars provides us about the interior of the stars makes it (one of) the most efficient and reliable techniques to probe the interior of stars.

Whether our favourite binary is a double-line spectroscopic or an eclipsing binary – or both if we are lucky – we can derive the apsidal motion rate of the system. In a double-line spectroscopic binary, we fit the radial velocities (RVs) of the stars all together leaving the apsidal motion rate as a free parameter of the adjustment. Given that the apsidal motion is a slow motion of only a few degrees per year at most, this technique requires observations collected over a long timespan. In an eclipsing binary for which photometric data show both eclipses during one orbital cycle, we can either adjust the individual light curves or adjust the times of minimum of the eclipses, leaving the longitude of periastron as a free parameter, to infer a value for the apsidal motion rate.

The confrontation of 1D stellar evolution models to the observational constraints allowed us to demonstrate that these standard models predict stars having a too low density contrast between their cores and external layers compared to what is expected from the observations. To reproduce the observational k_2 and the ensuing apsidal motion rate, it was necessary to enhance the mixing inside the models (through the turbulent diffusion in `Clés`). This important result points out that the standard 1D models predict too low an efficiency of the internal mixing and that the convective cores in these massive stars must be more extended than usually considered in stellar evolution calculations. Whether this turbulent diffusion could be entirely attributable to the rotationally-induced mixing has been proposed by Rosu et al. (2020a), though further investigation is needed to confirm this statement.

Whether this enhanced mixing might be inherent to the 1D stellar modelling was also investigated using the non-perturbative `MoBiDICT` code. Fellay et al. (2024) reached the conclusion that the apsidal motion is systematically underestimated in the perturbative case. Consequently, rather than being solved, the enhanced mixing problem is even more enforced. Among the systems presented in Fig. 2, we estimated that about one half of them would be significantly affected, especially massive stars which, despite their lower mass envelope compared to their total mass, are usually found in closer binaries and suffer more important deformations than less massive stars, as illustrated in Fig. 12. These systems would therefore require to be modelled with a proper non-perturbative approach.

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Further Information

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Conflicts of interest

The author declares no conflict of interest.

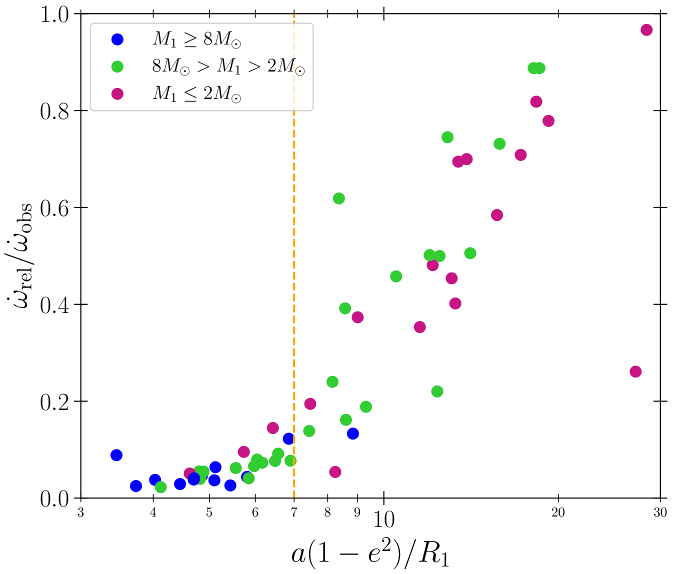

Figure 12: Ratio between relativistic and observational apsidal motion rate as a function of the semi-major axis of the binary orbit scaled by its eccentricity and the radius of the primary star of observed systems coming from the literature (Baroch et al., 2021, 2022; Claret et al., 2021; Marcussen and Albrecht, 2022; Rauw et al., 2016; Rosu et al., 2020b, 2022a,b, 2023; Torres et al., 2010; Wolf et al., 2006, 2008, 2010). The vertical orange line separates the systems which should be significantly affected by the non-perturbative approach (on the left) from the others (on the right).

References

Barclay, K. D. G., Rosu, S., Richardson, N. D., Chené, A.-N., St-Louis, N., Ignace, R., and Moffat, A. F. J. (2024) Using CHIRON spectroscopy to test the hypothesis of a precessing orbit for the WN4 star EZ CMa. *MNRAS*, **527**(2), 2198–2208. https://doi.org/10.1093/mnras/stad3289.

Baroch, D., Giménez, A., Morales, J. C., Ribas, I., Herrero, E., Perdelwitz, V., Jordi, C., Granzer, T., and Allende Prieto, C. (2022) Absolute dimensions and apsidal motion of the eclipsing binaries V889 Aquilae and V402 Lacertae. *A&A*, **665**, A13. https://doi.org/10.1051/0004-6361/202244287.

Baroch, D., Giménez, A., Ribas, I., Morales, J. C., Anglada-Escudé, G., and Claret, A. (2021) Analysis of apsidal motion in eclipsing binaries using TESS data. I. A test of gravitational theories. *A&A*, **649**, A64. https://doi.org/10.1051/0004-6361/202040004.

Claret, A. (2023) Theoretical tidal evolution constants for stellar models from the pre-main sequence to the white dwarf stage. Apsidal motion constants, moment of inertia, and gravitational potential energy. *A&A*, **674**, A67. https://doi.org/10.1051/0004-6361/202346250.

Claret, A., Giménez, A., Baroch, D., Ribas, I., Morales, J. C., and Anglada-Escudé, G. (2021)

Analysis of apsidal motion in eclipsing binaries using TESS data. II. A test of internal stellar structure. *A&A*, **654**, A17. https://doi.org/10.1051/0004-6361/202141484.

Eggenberger, P., Meynet, G., Maeder, A., Hirschi, R., Charbonnel, C., Talon, S., and Ekström, S. (2008) The Geneva stellar evolution code. *Ap&SS*, **316**(1-4), 43–54. https://doi.org/10.1007/s10509-007-9511-y.

Fellay, L. and Dupret, M.-A. (2023) MoBiDICT: New 3D static models of close synchronised binaries in hydrostatic equilibrium. *A&A*, **676**, A22. https://doi.org/10.1051/0004-6361/202346175.

Fellay, L., Dupret, M.-A., and Rosu, S. (2024) Underestimation of the tidal force and apsidal motion in close binary systems by the perturbative approach: Comparisons with non-perturbative models. *A&A*, **683**, A210. https://doi.org/10.1051/0004-6361/202348134.

Giménez, A. and Bastero, M. (1995) A revision of the ephemeris–curve equations for eclipsing binaries with apsidal motion. *Ap&SS*, **226**(1), 99–107. https://doi.org/10.1007/BF00626903.

Harmanec, P., Holmgren, D. E., Wolf, M., Božić, H., Guinan, E. F., Kang, Y. W., Mayer, P., McCook, G. P., Nemravová, J., Yang, S., Šlechta, M., Ruždjak, D., Sudar, D., and Svoboda, P. (2014) Revised physical elements of the astrophysically important O9.5+O9.5V eclipsing binary system Y Cygni. *A&A*, **563**, A120. https://doi.org/10.1051/0004-6361/201323230.

Hejlesen, P. M. (1987) Studies in stellar evolution. III. The internal structure constants. *A&AS*, **69**(2), 251–262. https://ui.adsabs.harvard.edu/abs/1987A&AS...69..251H.

Hill, G., Crawford, D. L., and Barnes, J. V. (1974) Some new spectroscopic binary orbits in NGC 6231 and Sco OB1. *AJ*, **79**, 1271–1279. https://doi.org/10.1086/111672.

Hong, K., Lee, J. W., Kim, S.-L., Koo, J.-R., and Lee, C.-U. (2016) Apsidal motions of 90 eccentric binary systems in the Small Magellanic Cloud. *MNRAS*, **460**(1), 650–663. https://doi.org/10.1093/mnras/stw955.

Lacy, C. H. S., Torres, G., Fekel, F. C., Muterspaugh, M. W., and Southworth, J. (2015) Absolute properties of the eclipsing binary star IM Persei. *AJ*, **149**(1), 34. https://doi.org/10.1088/0004-6256/149/1/34.

Marcussen, M. L. and Albrecht, S. H. (2022) The BANANA Project. VI. Close double stars are well aligned with noticeable exceptions; results from an ensemble study using apsidal motion and Rossiter–McLaughlin measurements. *ApJ*, **933**(2), 227. https://doi.org/10.3847/1538-4357/ac75c2.

Penny, L. R., Gies, D. R., and Bagnuolo, W. G., Jr. (1999) Tomographic separation of composite spectra. VI. The physical properties of the massive close binary HD 152248. *ApJ*, **518**(1), 450–456. https://doi.org/10.1086/307263.

Rauw, G., Rosu, S., Noels, A., Mahy, L., Schmitt, J. H. M. M., Godart, M., Dupret, M.-A., and Gosset, E. (2016) Apsidal motion in the massive binary HD 152218. *A&A*, **594**, A33. https://doi.org/10.1051/0004-6361/201628766.

Rosu, S. (2021) What apsidal motion reveals about the interior of massive binary stars. *BSRSL*, **90**(1), 1–16. https://doi.org/10.25518/0037-9565.10017.

Rosu, S. (2022) *Apsidal Motion in O-Star Binaries: Constraining the Internal Structure of the Stars*. Ph.D. thesis, Université de Liège [Sciences], Liège (BE). https://hdl.handle.net/2268/292652.

Rosu, S., Fellay, L., Rauw, G., and Dupret, M.-A. (in press) Apsidal motion in (massive) binaries. *Central European Astrophysical Bulletin*.

Rosu, S., Noels, A., Dupret, M.-A., Rauw, G., Farnir, M., and Ekström, S. (2020a) Apsidal motion in the massive binary HD 152248: Constraining the internal structure of the stars. *A&A*, **642**, A221. https://doi.org/10.1051/0004-6361/202038380.

Rosu, S., Quintero, E. A., Rauw, G., and Eenens, P. (2023) New insight into the massive eccentric binary HD 165052: self-consistent orbital solution, apsidal motion, and fundamental parameters. *MNRAS*, **521**(2), 2988–3003. https://doi.org/10.1093/mnras/stad780.

Rosu, S., Rauw, G., Conroy, K. E., Gosset, E., Manfroid, J., and Royer, P. (2020b) Apsidal motion in the massive binary HD 152248. *A&A*, **635**, A145. https://doi.org/10.1051/0004-6361/201937285.

Rosu, S., Rauw, G., Farnir, M., Dupret, M.-A., and Noels, A. (2022a) Apsidal motion in massive eccentric binaries in NGC 6231: The case of HD 152219. *A&A*, **660**, A120. https://doi.org/10.1051/0004-6361/202141304.

Rosu, S., Rauw, G., Nazé, Y., Gosset, E., and Sterken, C. (2022b) Apsidal motion in massive eccentric binaries: The case of CPD-41° 7742, and HD 152218 revisited. *A&A*, **664**, A98. https://doi.org/10.1051/0004-6361/202243707.

Scuflaire, R., Théado, S., Montalbán, J., Miglio, A., Bourge, P.-O., Godart, M., Thoul, A., and Noels, A. (2008) CLÉS, Code Liégeois d'Évolution Stellaire. *Ap&SS*, **316**(1-4), 83–91. https://doi.org/10.1007/s10509-007-9650-1.

Shakura, N. I. (1985) On the apsidal motion in binary stars. *SvAL*, **11**, 224–226. https://ui.adsabs.harvard.edu/abs/1985SvAL...11..224S.

Sterne, T. E. (1939) Apsidal motion in binary stars. *MNRAS*, **99**(5), 451–462. https://doi.org/10.1093/mnras/99.5.451.

Struve, O. (1944) Radial velocities of twenty stars of early type in and near the galactic cluster NGC 6231. *ApJ*, **100**, 189–201. https://doi.org/10.1086/144657.

Torres, G., Andersen, J., and Giménez, A. (2010) Accurate masses and radii of normal stars: modern results and applications. *A&AR*, **18**(1-2), 67–126. https://doi.org/10.1007/s00159-009-0025-1.

Wolf, M., Claret, A., Kotková, L., Kučáková, H., Kocián, R., Brát, L., Svoboda, P., and Šmelcer, L. (2010) Relativistic apsidal motion in eccentric eclipsing binaries. *A&A*, **509**, A18. https://doi.org/10.1051/0004-6361/200911671.

Wolf, M., Kučáková, H., Kolasa, M., Štastný, P., Bozkurt, Z., Harmanec, P., Zejda, M., Brát, L., and Hornoch, K. (2006) Apsidal motion in eccentric eclipsing binaries: CW Cephei, V478 Cygni, AG Persei, and IQ Persei. *A&A*, **456**(3), 1077–1083. https://doi.org/10.1051/0004-6361:20065327.

Wolf, M., Zejda, M., and de Villiers, S. N. (2008) Apsidal motion in southern eccentric eclipsing binaries: GL Car, QX Car, NO Pup and V366 Pup. *MNRAS*, **388**(4), 1836–1842. https://doi.org/10.1111/j.1365-2966.2008.13527.x.

Zasche, P. and Wolf, M. (2019) Apsidal motion and absolute parameters of 21 early-type Small Magellanic Cloud eccentric eclipsing binaries. *AJ*, **157**(2), 87. https://doi.org/10.3847/1538-3881/aafc31.

Zasche, P., Wolf, M., Kučáková, H., Kára, J., Merc, J., Zejda, M., Skarka, M., Janík, J., and Kurfürst, P. (2020) First apsidal motion and light curve analysis of 162 eccentric eclipsing binaries from LMC. *A&A*, **640**, A33. https://doi.org/10.1051/0004-6361/202037822.