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Differential Operators on Conic Manifolds: Maximal Regularity and Parabolic Equations

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Dedicated to the memory of Pascal Laubin

ABSTRACT. We study an elliptic differential operator A on a manifold with conic points. Assuming A to be defined on the smooth functions supported away from the singularities, we first address the question of possible closed extensions of A to L_p Sobolev spaces and then explain how additional ellipticity conditions ensure maximal regularity for the operator A. Investigating the Lipschitz continuity of the maps $f(u) = |u|^\alpha$, $\alpha \geq 1$, and $f(u) = u^\alpha$, $\alpha \in \mathbb{N}$, and using a result of Clément and Li, we finally show unique solvability of a quasilinear equation of the form $(\partial_t - a(u)\Delta)u = f(u)$ in suitable spaces.

1. Introduction

Parabolic equations and associated initial value problems or boundary value problems are common models appearing in science and engineering. A well-known example is the mixed initial-boundary value problem for the heat equation

$$\begin{cases} \partial_t u(t,x) - \Delta u(t,x) = g(t,x) & \text{on }]0,T[\times \Omega, \\ u(0,x) = u_0(x) & \text{on } \Omega, \\ u(t,x)|_{\partial\Omega} = u_1(x) & \text{for } t \in]0,T[, \end{cases} \tag{1.1}$$

where Ω is a domain (or manifold) with smooth boundary $\partial\Omega$.

A typical approach to solve (1.1) consists in rewriting it as an abstract evolution equation

$$\begin{cases} \dot{u}(t) + Au(t) = g(t) & \text{on }]0,T[, \\ u(0) = u_0 \end{cases} \tag{1.2}$$

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