DERIVED LIMITS
IN
QUASI-ABELIAN CATEGORIES

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Abstract
In this paper, we study the derived functors of projective limit functors in quasi-abelian categories. First, we show that if \( E \) is a quasi-abelian category with exact products, projective limit functors are right derivable and their derived functors are computable using a generalization of a construction of Roos. Next, we study index restriction and extension functors and link them through the symbolic Hom-functor. If \( J : \mathcal{J} \to \mathcal{I} \) is a functor between small categories and if \( E \) is a projective system indexed by \( \mathcal{I} \), this allows us to give a condition for the derived projective limits of \( E \) and \( E \circ J \) to be isomorphic. Note that this condition holds, if \( \mathcal{I} \) and \( \mathcal{J} \) are filtering and \( \mathcal{J} \) is cofinal. Using the preceding results, we establish that the \( n \)-th left cohomological functor of the derived projective limit of a projective system indexed by \( \mathcal{I} \) vanishes for \( n \geq k \), if the cofinality of \( \mathcal{I} \) is strictly lower than the \( k \)-th infinite cardinal number. Finally, we consider the limits of pro-objects of a quasi-abelian category. From our study, it follows, in particular, that the derived projective limit of a filtering projective system depends only on the associated pro-object.

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