DERIVED LIMITS IN QUASI-ABELIAN CATEGORIES

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Abstract

In this paper, we study the derived functors of projective limit functors in quasi-abelian categories. First, we show that if $\mathcal{E}$ is a quasi-abelian category with exact products, projective limit functors are right derivable and their derived functors are computable using a generalization of a construction of Roos. Next, we study index restriction and extension functors and link them trough the symbolic Hom-functor. If $J : \mathcal{J} \longrightarrow \mathcal{I}$ is a functor between small categories and if E is a projective system indexed by $\mathcal{I}$, this allows us to give a condition for the derived projective limits of E and $E \circ J$ to be isomorphic. Note that this condition holds, if $\mathcal{I}$ and $\mathcal{J}$ are filtering and J is cofinal. Using the preceding results, we establish that the n-th left cohomological functor of the derived projective limit of a projective system indexed by $\mathcal{I}$ vanishes for $n \geq k$, if the cofinality of $\mathcal{I}$ is strictly lower than the k-th infinite cardinal number. Finally, we consider the limits of pro-objects of a quasi-abelian category. From our study, it follows, in particular, that the derived projective limit of a filtering projective system depends only on the associated pro-object.

Contents

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