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A GENERALIZATION OF TIETZE'S THEOREM ON LOCAL CONVEXITY FOR OPEN SETS

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Abstract

Let S be a nonempty subset of a real topological linear space L and s a point in cl$S$. A point s of *weak local C-convexity* of S is defined as follows: if there exists a neighbourhood N of s such that $s \in$clC_s, where C_s is a component of $S \cap N$, then $[x, y] \subseteq S$ for each pair of points $x, y \in C_s$, otherwise $[x, y] \subseteq S$ for each pair of points x, y in any component of $S \cap N$. It is proved that an open connected subset S of L whose boundary consists exclusively of C-wlc points of S is convex. This is a version of the Sacksteder-Straus-Valentine generalization of Tietze's local characterization of convexity for open sets.

Key words: weak local C-convexity point, Tietze-type theorem.
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Let S be a nonempty subset of a real topological linear space L. A point s in cl$S$ is said to be a *point of weak local convexity* of S if and only if there is some neighbourhood N of s such that for each pair of points x, y in $S \cap N$, $[x, y] \subseteq S$ [2, Def.4.2]. A point s of *weak local C-convexity* of S is defined as follows: if there exists a neighbourhood N of s such that $s \in$clC_s, where C_s is a component of $S \cap N$, then $[x, y] \subseteq S$ for each pair of points $x, y \in C_s$, otherwise $[x, y] \subseteq S$ for each pair of points x, y in any component of $S \cap N$ (cf. [2, Def.4.5]). Furthermore [1],[2, Def.4.3], a point s in cl$S$ is said to be a *point of strong local convexity (C-convexity)* if and only if $S \cap N$ (each component of $S \cap N$) is convex for some neighbourhood N of s in L. For the sake of brevity, we call points of weak and strong local convexity (C-convexity) of S, respectively, wlc and slc (C-wlc and C-slc) points of S. (xyz) will represent the two-dimensional flat determined by three noncollinear points x, y, z.

Tietze's famous characterization of convexity states that a closed connected subset S of L consisting exclusively of wlc points is convex [2, Th.4.4]. In [1], a generalization was proved that a connected compact subset S of a complete locally convex real topological linear space consisting exclusively of C-slc points is convex. In [3, Cor.2.3], the author proved essentially that an open connected subset S of L whose boundary consists exclu-

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