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FACTORIZATION OF FINITE-RANK OPERATORS IN BANACH OPERATOR IDEALS

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Abstract

A condition is given which implies that every finite-rank operator between arbitrary Banach spaces factors through an operator between Banach spaces of finite dimension under control of its norm in a given Banach operator ideal $\mathcal{A}$. This property of $\mathcal{A}$ is an operator-theoretic version of Grothendieck's notion of total accessibility of tensor norms.

1 The result

1.1. Given a quasi-normed operator ideal $(\mathcal{A}, \boldsymbol{A})$ (see [7] and [2] for the definitions and notation) it is interesting to know under which circumstances every $T \in \mathcal{F}(E;F)$ (where E, F are Banach spaces, $\mathcal{F}$ the ideal of finite-rank operators) and $\varepsilon > 0$ there are a closed finite-codimensional subspace $L \subset E$ (notation: $L \in \mathrm{COFIN}(E)$), a finite-dimensional subspace $N \subset F$ (notation: $N \in \mathrm{FIN}(F)$) and an operator $T_0 \in \mathcal{L}(E/L; N)$ such that

$$\begin{array}{ccc} E & \xrightarrow{\quad T \quad} & F \\ {\scriptstyle Q_L^E}\downarrow & \circlearrowleft & \uparrow{\scriptstyle I_M^E} \\ E/L & \xrightarrow{\quad T_0 \quad} & N \end{array} \qquad \text{and } \boldsymbol{A}(T_0) \leq (1+\varepsilon)\boldsymbol{A}(T) \qquad (*)$$

If E' and F have the metric approximation property (m.a.p. for short) this is possible: for a proof use that if F (resp. E') has m.a.p., then for every $T \in \mathcal{F}(E;F)$ and $\varepsilon > 0$ there is an $R \in \mathcal{F}(F;F)$ (resp. $\in \mathcal{F}(E;E)$) with $\|R\| \leq 1 + \varepsilon$ and $R \circ T = T$ (resp. $T \circ R = T$); this *lemma* (see [2, 16.9.] for a proof) will be used various times in this note.

1.2. A quasi-normed operator ideal $(\mathcal{A}, \boldsymbol{A})$ is called *totally accessible*, if $(*)$ holds for all Banach spaces E, F; it is called *right-accessible* if $(*)$ holds for all

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