

A NOTE ON A CLASS OF ANALYTIC FUNCTIONS IN THE UNIT DISK II \*

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ABSTRACT. Let  $A(\alpha)$  be the class of functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the unit disk  $U$  and satisfy

$$|f(z)/z - 1| < \alpha \quad (z \in U)$$

for some  $\alpha$  ( $0 < \alpha \leq 1$ ). The object of the present paper is to show some distortion theorems for the fractional calculus of  $f(z)$  belonging to the class  $A(\alpha)$ .

I. INTRODUCTION

Many essentially equivalent definitions of the fractional calculus (that is, the fractional integrals and the fractional derivatives) have been given in the literature (cf., e.g., [2], [4], [5], [8], [9], and [10]). We find it convenient to recall here the following definitions which were used by Owa [6].

DEFINITION I. The fractional integral of order  $\lambda$  is defined  
by

$$(1.1) \quad D_z^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_0^z \frac{f(\zeta)}{(z - \zeta)^{1-\lambda}} d\zeta,$$

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