Bulletin de la Société Royale des Sciences de Liège, Vol. 70, 4-5-6, 2001, pp. 435-440

THE TENSOR ALGEBRA OF THE SPACE s

DIETMAR VOGT

Dedicated to the memory of Pascal Laubin

In view of recent work on noncommutative geometry involving various Fréchet spaces (resp. algebras), isomorphic to the space s of rapidly decreasing sequences, and their tensor algebras (see [2]) it appears to be of interest to determine the isomorphy type of the tensor algebra $T(s)$ of s. We show that $T(s) \cong s$ and that, moreover, for any complemented subspace E of s with $\dim E \geq 2$ we have $T(E) \cong s$. This, in particular, includes all nuclear power series spaces of infinite type as e.g. the space of entire holomorphic functions in any finite dimension.

It is interesting to compare the situation with the case of the symmetric tensor algebra $S(E)$. The isomorphy type of $S(E)$ for certain power series spaces E has been determined in [1] There we also have $S(s) \cong s$, however $S(E) \not\cong s$ in general, even for power series spaces. Of course, the proof of Theorem 5 owes much to the methods developed in [1].

For any Fréchet space we set

$$E^{\otimes n} := E \otimes \cdots \otimes E$$

the n-fold complete π-tensor product, and by $T(E)$ we denote the completion of

$$\bigoplus_{n=1}^{\infty} E^{\otimes n}$$

with respect to the Fréchet space topology given by the seminorms

$$x = x_1 \oplus \cdots \oplus x_n \mapsto \sum_n p^{\otimes n}(x_n)$$

where p runs through all continuous seminorms on E and $p^{\otimes n}$ denotes the n-fold π-tensorproduct of p. $T(E)$ is called the tensor algebra of E.