Topology optimization of strut-and-tie models in steel plate shear walls

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Abstract

Since 1970, the steel plate shear wall is regarded as the first choice among latest load resisting systems and employed in several important and modern buildings. In the early and during the 1970s, the steel plate shear walls were used in modern buildings in Japan and United States to improve the seismic behavior of structures. It can be concluded from existing researches that the steel plate shear walls have considerable strength and stiffness in comparison with other systems.

In order to find the optimum size that minimizes structure’s weight under the applied load and also fulfilling the structural requirements and restrictions optimization is required. In this study, tracing the shear wall elements with the lowest strain energy through using the topology optimization based on finite element method and removing them according to particular mechanism is investigated. This procedure gives us the optimized shape of steel plate shear wall. For this purpose, a steel plate shear wall is modeled in Abaqus to obtain its force-displacement curve and subsequently the force-displacement relationship is acquired using virtual work method and MATLAB programming.

Keywords: Topology optimization, Steel plate shear wall, virtual work method, strut-and-tie model

1. Introduction

Although steel plate shear wall has been used for years, and constructors and owners have taken advantage of it, it merits amore meticulous inspection. Particularly, in the last three decades, steel plate shear wall has been rapidly developed and widely used in several countries. In the United States, stiffened and unstiffened steel plate shear walls have been employed in a number of constructions projects. After the attacks performed on September 11'th, researchers have attempted to design cost effective structures which can withstand seismic loads and explosion by combining this system with concrete shear wall system (3-1).

In the studies performed by Sabouri Ghomi and Robert et al, an unstiffened steel plate shear wall was examined under increasing lateral load to reach its failure.
2. Modeling

Herein, the nonlinear static analysis is employed using the general-purpose finite element package. A steel plate, beam, column, and stiffener are modeled whose dimensions and characteristics are presented in table 1. In Abaqus, the model is created using shell elements.

<table>
<thead>
<tr>
<th>Member</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear wall panel</td>
<td>1410</td>
<td>960</td>
<td>2</td>
</tr>
<tr>
<td>Column stiffener</td>
<td>90</td>
<td>140</td>
<td>3</td>
</tr>
<tr>
<td>Vertical stiffener</td>
<td>60</td>
<td>960</td>
<td>4</td>
</tr>
<tr>
<td>Horizontal stiffener</td>
<td>60</td>
<td>1410</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1: Model dimension [13]

Table 2: Shows the dimensions of the beam and columns while table 3 represent the characteristic of the steel plate shear walls.

<table>
<thead>
<tr>
<th>Member</th>
<th>Flange width (mm)</th>
<th>Flange thickness (mm)</th>
<th>Web height (mm)</th>
<th>Web thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>140</td>
<td>20</td>
<td>250</td>
<td>20</td>
</tr>
<tr>
<td>Column</td>
<td>140</td>
<td>15</td>
<td>60</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2: Beam and column dimension [13]

<table>
<thead>
<tr>
<th>Member</th>
<th>Yield stress (Mpa)</th>
<th>Ultimate stress (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>414.9</td>
<td>544</td>
</tr>
<tr>
<td>Column</td>
<td>414.9</td>
<td>544</td>
</tr>
<tr>
<td>Shear wall panel</td>
<td>192.4</td>
<td>288.7</td>
</tr>
<tr>
<td>Stiffeners</td>
<td>240</td>
<td>370</td>
</tr>
</tbody>
</table>

Table 3. Strength capacity of shear wall components [13]

In order to validate the model, the force-displacement curves of two test specimens and the generated model are compared in figure 2.
3. Topology optimization

The topology optimization theory has been fully developed in the last decades by Rezvani et al and Topping [9] [10]. This method has been also used to find the most appropriate strut-and-tie models in reinforced concrete members based on ground structural approach by Kumar and Biondini, in which several continuum concrete structures have been formed from numerous strut-and-tie members and linear programming technique has been used to find the most appropriate topology. Despite the fact that the studies on ground structures have significantly developed the structural topology optimization theory, some incapability of simulating the nature of continuum concrete structures still exists. The evolutionary topology optimization method suggested by Xie and Steven, and Chu et al was designed based on element removal criteria [11] [12] [13]. In addition, a performance index was presented by Liang et al [14] to control the optimization process. In this study, Liang’s performance indices are employed to compare the performance of the structure obtained from different optimization methods.

4. Optimization algorithm

The topology optimization is based on structural finite element method. According to its algorithm, the structure is firstly modeled by finite element method and then analyzed. Afterwards, the elements with the lowest strain energy are found and removed based on particular mechanism. This takes several steps and the additional elements are removed during subsequent cycles similar to the first step. This cycle is repeated until the requirements and restrictions are fulfilled which include displacement and the maximum stress. Figure 3 represents the step-by-step procedure of the optimization algorithm.

It is clear that some elements existing in the finite element model are inefficient. The purpose of optimization is to determine these elements and remove them in a way to reduce structure’s weight while the existing displacements occur within the allowable range. By removing elements systematically from discontinuous steel members, the actual load path is revealed using the remaining elements.
5. Element removal criterion
The purpose of removal criterion is to define a parameter that determines the unimportant elements. In this study, the removal criterion is the level of strain energy. Based on this, the strain energy of all elements is measured at each iteration and the elements with the lowest energy are removed according to the removal procedure described in the next section.

6. Element removal procedure
The number of removed elements in each step depends on the description of problem. Definitely, if the number of removed elements at each iteration is more, the calculation accuracy is higher. However, the lower number of removed elements at each iteration makes the computation time shorter. For removing elements after each cycle, the elements are put in order according to their strain energy. Then, elements with the lowest energy are removed based on the gradually element removal method proposed in earlier studies.

In this method, a particular percentage of remaining elements is removed at each iteration. For instance, if the number of the existing elements is 100 and the defined removing percentage is 5% in an iteration, 5% of remaining elements, which becomes 5 elements, is removed.

Eventually, the shape of the obtained strut-and-tie model for the steel plate shear wall is indicated in figure 4.
Furthermore, the area measured with the help of the program for the two strut-and-tie members can be obtained by the following equation:

\[ A_1 = 0.0024 \text{ m}^2 \quad A_2 = 0.008 \text{ m}^2 \] (1)

7. Force-displacement curve

One of the aims of this study is to present a relationship for drawing force-displacement curves. Figure 5 indicates the obtained strut-and-tie model.

\[
\begin{align*}
E_1 &= 220\text{Gpa} - F_{u1} = 288.7\text{Mpa} - F_{y1} = 240\text{Mpa} \\
E_2 &= 200\text{Gpa} - F_{u2} = 544\text{Mpa} - F_{y2} = 414.9\text{Mpa}
\end{align*}
\]

(2)
Step1: In this step, the applied force is increased to the extent that the members remain in elastic region (figure 6). Since the diagonal member is the critical one, its stress shall not pass 288.7 Mpa.

![Fig. 6: Obtaining the strut-and-tie model using virtual work method in the elastic region](image)

Considering the equilibrium existed in each node of strut-and-tie model, equation 3 yields the force of each member.

\[
F_1 = 1.28p, \quad F_2 = 0.78p \\
F'_1 = 1.28, \quad F'_2 = 0.78
\]  

(3)

By using virtual work equation:

\[
X = \sum \left( \frac{P'F' \cdot l_i}{AE} \right) = \left( \frac{1.28p \times y \times l_1}{AE_1} \right) + \left( \frac{0.78p \times y \times l_2}{AE_2} \right)
\]

Step2: In this step, the diagonal member enters into the elastic region while other member remains in plastic region (figure 7).

![Fig. 7: Obtaining the strut-and-tie model using virtual work method in the plastic region](image)

Similar to the previous section, the equilibrium exists in each node of the strut-and-tie model.

\[
F_1 = 1.28p, \quad F_2 = 0.78p \\
F'_1 = 1.28, \quad F'_2 = 0.78
\]  

(5)

Thus, by using virtual work equation:

\[
X = \sum \left( \frac{P'F' \cdot l_i}{AE} \right) = \left( \frac{1.28p \times y \times l_1}{AE_1} \right) + \left( \frac{0.78p \times y \times l_2}{AE_2} \right)
\]  

(6)
The applying force increases until the stress of diagonal member reaches its ultimate capacity. Eventually, after taking the two mentioned step, the force-displacement curve is obtained and compared with curve acquired from Abaqus as it is illustrated in figure 8.

**Fig. 8**: Comparing the force-displacement curve obtained from the equation with the one resulted from modeling

### 8. Conclusion

It can be concluded from this study that:

1. It is possible to remove some parts of steel plate shear wall without considerably decreasing its load-bearing capacity. Moreover, it is possible to cost-effectively reduce the structure’s weight.
2. This study succeeded to present a relationship for drawing the force-displacement curve of steel plate shear wall without opening.

For the elastic region of diagram, the relationship is as follows:

\[
X = \sum \frac{\text{FE element}}{\text{AB}} = \left( \frac{1.2 \times 10^5 \times p \times I_1}{AE_1} \right) + \left( \frac{0.7 \times 10^5 \times p \times I_2}{AE_2} \right)
\]

And for the plastic region we have

\[
X = \sum \frac{\text{FE element}}{\text{AB}} = \left( \frac{1.2 \times 10^5 \times p \times I_1}{AE_1} \right) + \left( \frac{0.7 \times 10^5 \times p \times I_2}{AE_2} \right)
\]

### References


