Priestley duality for distributive semilattices

G. Hansoul and C. Poussart

Abstract

In this paper, we extend Priestley duality for bounded distributive lattices to all bounded distributive semilattices. We show that we cannot take the prime spectrum as Priestley dual but have to turn to a suitable weakening of the concept of prime ideal.

1991 Mathematics Subject Classification. Primary 06D50, 06D05. Secondary 08B20.

Key words and phrases: Distributive semilattice. Free distributive lattice over a distributive semilattice. Priestley duality.

Introduction

Stone duality for Boolean algebras in 1936 ([10]) and Pontryagin duality for abelian groups ([8]) are major achievements in algebra that opened the door for numerous and fruitful developments: Stone-like dualities exist now in abundance, as nicely shown in Davey’s paper ([4] and [2], see also [3] and [7]). One of the most interesting works in this era is Priestley’s duality for distributive lattices in 1970 ([9]) where a full comprehension of natural dualities was perhaps for the first time made possible.

Another feature of Priestley duality is that it gives a simple alternative to Stone duality for distributive lattices. Now Stone duality very easily extends to distributive semilattices and finds in this larger context its most natural setting, as shown by Grätzer in 1971 ([6]). In this paper we show that rather curiously such an extension is not possible stricto sensu as far as Priestley duality is concerned. We give also an alternative solution in terms of a spectrum that is larger than the prime spectrum and examine some classical dualisations.