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SPECTRAL MEASURES IN CLASSES OF FRÉCHET SPACES

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Abstract

A detailed investigation is made of the canonical atomic spectral measure defined in such Fréchet spaces as the Köthe echelon sequence spaces and the sequence spaces ℓ^{p+}, as well as the (non-atomic) "natural" spectral measures in such Fréchet spaces of measurable functions as the space of locally p-th power integrable functions on $\mathbb{R}$ and L_{p-} on $[0,1]$. Of particular interest are questions concerned with the range of the spectral measure, whether or not it has finite variation (for certain operator topologies), the Radon-Nikodým property of the underlying spaces involved and, most importantly, does the spectral measure admit *unbounded* integrable functions?

1 Introduction and preliminaries

The theory of Boolean algebras of projections/spectral measures in Banach spaces was initiated by W. Bade, N. Dunford and others, [12], and is by now well understood. In contrast, there is a distinct lack of concrete, non-trivial examples in the non-normable setting, even within the class of *Fréchet* (locally convex) spaces. An attempt to rectify this (to some extent) can be found in [5]. The aim of this paper is two-fold. Firstly, we wish to summarize the main results of [5] and secondly, to expand on these results and elaborate further on some closely related topics. In order to do so, we begin with some general notation and definitions, so that the questions (some answers) and examples can be properly formulated.

Let X be a locally convex Hausdorff space (briefly, lcHs) and $L(X)$ denote the space of all continuous linear operators from X to itself. The space $L(X)$ is denoted by $L_s(X)$ (resp. $L_b(X)$) when it is equipped with the topology τ_s (resp. τ_b) of uniform

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