Multifractal functions: Recent advances and open problems

Stéphane Jaffard*

Abstract: We raise several questions related to the pointwise regularity of functions and their multifractal analysis.

1 Introduction

The present paper can be seen as a sequence of [25], which was written in 1996 and listed open problems related with the multifractal analysis of functions. The subject at that time was in full bloom, driven by an explosion of applications: All possible kinds of signals were submitted to a “multifractal analysis”, with apparent success. Indeed, as we shall see, the criterium of multifractality for a signal amounts to check that a certain curve, obtained from the data, is not linear, a criterium which is particularly easy to check in practice. Several scientists challenged these results and asked for a scientific interpretation of these numerical computations which would be backed by mathematical theorems. Though multifractal analysis of measures had largely developed at that time, very few mathematical results were available for functions, and the most simple and natural questions were open. The motivation of [25] was to list these questions, organize them as what could be described as a “research program” and hopefully attract students to a very promising area. And, indeed, the subject widely developed, largely due to talented students. It evolved widely since then, and its foundations have been largely renewed. Here are two examples of these changes:

Multifractal analysis is based on the notion of Hölder pointwise regularity; its purpose is to determine the dimensions of the sets of points with a given regularity. All mathematical results concerning this type of regularity assume that the function considered has some uniform Hölder regularity; this prevented the introduction of these tools in image analysis, since images always present discontinuities, as a result of the occlusion phenomenon (some objects are partly hidden behind others). Recently, it was found that a slight weakening of the notion of Hölder pointwise regularity, the $T^p_0$ regularity, allows to derive

*Address: Laboratoire d'Analyse et de Mathématiques Appliquées, Université Paris XII, 61 Avenue du Général de Gaulle, 94010 Créteil Cedex, France. Email: jaffard@univ-paris12.fr The author is supported by the Institut Universitaire de France.