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RELATIVE PARACOMPACTNESS AS TAUTNESS CONDITION IN SHEAF THEORY

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RESUME : Nous introduisons la paracompacité relative. Cette notion nous permet d'obtenir un critère de raideur qui unifie et généralise les résultats classiques de [2].

INTRODUCTION

Let X be a topological space, S a subset of X, Φ a family of supports in X and $\mathcal{V}_S$ the set of the open neighborhoods of S in X, ordered by the relation $\supset$. In this paper, we consider only sheaves of abelian groups. We say that S is Φ-taut in X if the canonical morphism

$$(r^{\bullet}_S : \varinjlim_{V\in\mathcal{V}_S} H^{\bullet}_{\Phi\cap V}(V,\mathcal{F}_{|V}) \longrightarrow H^{\bullet}_{\Phi\cap S}(S,\mathcal{F}_{|S}))$$

is an isomorphism whenever $\mathcal{F}$ is a sheaf on X. In [2] G.E. Bredon proves that it is equivalent to say that the canonical morphism

$$(r_{SX} : \Gamma_{\Phi}(X,\mathcal{F}) \longrightarrow \Gamma_{\Phi\cap S}(S,\mathcal{F}_{|S}))$$

is onto and that $\mathcal{F}_{|S}$ is $\Phi\cap S$-acyclic whenever $\mathcal{F}$ is a flabby sheaf on X. The tautness appears in the hypothesis of many important theorems of sheaf theory. So, for pratical use, we need criteria stating that S is Φ-taut in X under more explicit topological assumptions on S and Φ. For example, it is trivial to see that an open subset of X is Φ-taut. In [2] it is proved that S is Φ-taut in X if one of the following conditions is satisfied :

a) Φ is paracompactifying for the pair (X,S)

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