Weighted composition operators between weighted Bloch type spaces

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Abstract

Let \( D \) be the open unit disk in the complex plane and \( \phi : \mathbb{D} \to \mathbb{D} \) as well as \( \psi : \mathbb{D} \to \mathbb{C} \) be analytic maps. For a holomorphic function \( f \) on \( \mathbb{D} \) the weighted composition operator \( C_{\phi, \psi} \) is defined by \( (C_{\phi, \psi} f)(z) = \psi(z)f(\phi(z)) \) for every \( z \in \mathbb{D} \). We characterize when weighted composition operators acting between weighted Bloch type spaces are bounded resp. compact. Moreover, during these studies we also obtain a characterization of boundedness and compactness of weighted composition operators from weighted Bloch type spaces to weighted Banach spaces of holomorphic functions.

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1 Introduction

Let \( \phi \) be an analytic self-map of the open unit disk \( \mathbb{D} \) in the complex plane \( \mathbb{C} \) and \( \psi \) be an analytic map on \( \mathbb{D} \). Such maps induce the so-called weighted composition operator

\[
C_{\phi, \psi} : H(\mathbb{D}) \to H(\mathbb{D}), \ f \mapsto \psi(f \circ \phi),
\]

where \( H(\mathbb{D}) \) denotes the set of all holomorphic functions on \( \mathbb{D} \). In case that \( \psi(z) = 1 \) for every \( z \in \mathbb{D} \) we simply write \( C_{\phi} \) and obtain the classical composition operator. Such operators have been investigated on various spaces of holomorphic functions and by several authors, see e.g. [4], [5], [6], [9], [10], [13] [16]. In this paper we are interested in operators \( C_{\phi, \psi} \) acting between different weighted Bloch type spaces. For a continuous, strictly positive and bounded function (weight) on \( \mathbb{D} \) we say that the weighted Bloch type space \( B_v \) is the collection of all holomorphic functions \( f \) on \( \mathbb{D} \) such that

\[
\| f \|_{B_v} := \sup_{z \in \mathbb{D}} v(z)|f'(z)| < \infty.
\]

Provided, we identify functions that differ by a constant, \( \| \cdot \|_{B_v} \) becomes a norm and \( B_v \) a Banach space.

In [16] Ohno, Stroethoff and Zhao characterized boundedness and compactness of weighted composition operators in the framework of weighted Bloch type spaces generated by the standard weights, i.e. weights of the form \( v_\alpha(z) = (1 - |z|^2)^\alpha \) with \( \alpha \in \mathbb{R} \) and \( \alpha > 0 \).

Using a completely different approach in this article we characterize these properties of \( C_{\phi, \psi} \) acting between different more general Bloch type spaces.

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