# CO<sub>2</sub>/CH<sub>4</sub> Sorption Behavior of Glassy Polymeric Membranes Based on Dual Mode Sorption Model

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### Abstract

Among the various transport models for gas separation via membrane, the best description is done by solution-diffusion model. The main parameters of this model are the sorption and diffusion of the penetrant through the membrane. Studies conducted by various researchers in the field of gas separation indicate that the thermodynamic interaction (sorption effects) in glassy polymers has the major role in the diffusivity, permeability, and selectivity of the membrane, especially in the multicomponent gas mixture. In glassy polymers, dual-mode sorption model is frequently used to describe the equilibrium sorption behavior of a polymer-gas system and, based on this sorption model; the permeation behavior is described by the partial immobilization model. In this study, the difference between the sorption mechanism of CO<sub>2</sub> and CH<sub>4</sub> in glassy polymeric membranes was analyzed by separating the sorption mode and introducing  $P_{50/50}$  parameter (the pressure at which the contribution of both Henry and Langmuir sorption is equal), which was done using the available experimental data in the scientific references for the sorption values of CO2 and CH4. The contribution of the sorbed molecules in Henry and Langmuir sites was investigated and its changes with pressure were evaluated for CO<sub>2</sub> and CH<sub>4</sub> permeation. This study was also attempted to provide a correct definition of F factor, used in references, in the form of the multiplication of mobile concentration ratios ( $C_{Hmobile}/C_{H}$ ) by the diffusivity coefficients in two modes ( $D_{H}/D_{D}$ ).

**Keywords:** Glassy polymer, Permeability coefficient, Dual-mode sorption model, Partial immobilization model

### 1. Introduction

The usage of natural gas as a clean and efficient fuel is constantly growing around the world. In addition, methane as the main component of natural gas is considered a principal feedstock for petrochemical and chemical industries. At present, natural gas purification is one of the most important industrial gas separation processes. In this process, acid gas ( $CO_2$ ,  $H_2S$ , and mercaptans), mainly  $CO_2$ , is removed from natural gas [1-5].

Pressure swing adsorption (PSA), the absorption of acid gases in solvents (such as dimethylamine or diethylamine solutions), and the membrane process are some examples of natural gas purification technologies [4-6].

Recently, membrane separation has received great attention due to its high-energy efficiency, ease of scale-up, and being environmentally friendly. A variety of polymers has been synthesized as membranes and investigated for gas separation applications. Among these materials, glassy

polymers (such as polyimides, polysulfone, and polycarbonate) are among the most attractive and favorable materials as a result of some admirable properties such as high thermal stability, mechanical strength, chemical resistance, and spinnability [1, 4-12].

Gas transport across a polymeric membrane mainly follows the solution-diffusion mechanism, in which permeability is affected by solubility and diffusivity coefficients [13-17]. In this mechanism, a gas molecule is sorbed onto the membrane surface, then diffuses across the polymer medium, and subsequently is desorbed from the other face of the membrane. Therefore, both kinetic factors, like segment mobility and free volume, which are mostly dependent on the penetrant size, and thermodynamic factors, like condensability of the gas and its interaction with polymer segments, can affect the permeation process [17].

The effects of pressure and concentration of penetrant molecules are crucial in gas transport modeling. As can be seen in Figure 1 and Figure 2, the effect of pressure on permeability and solubility can be divided into four conditions. The first one shown in Figure 1.a and Figure 2.a regards sorption and diffusion as independent from the gas pressure, which is proper for ideal conditions. This case is applied for the sorption and permeation of supercritical gases in amorphous polymers and can be defined by Henry's law. The second case shown in Figure 1.b and Figure 2.b is useful for the sorption and permeation of organic compounds in elastic polymers. In this case, due to the fineness and flexibility of these kinds of polymers, sorption and permeation are enhanced by increasing the pressure. The third case shown in Figure 1.c and Figure 2.c is applied for the sorption and permeation of gases with high solubility in glassy polymers without the plasticization behavior. In this case, sorption obeys the dual-mode sorption model. The last case shown in Figure 1.d and Figure 2.d is relevant for the sorption and permeation of highly condensable gases in glassy polymers with the plasticization effect.



**Fig.1:** Effect of pressure on permeability in four conditions a) Ideal b) organic compound in elastic polymers c) High solubility gases in glassy polymers without plasticization behavior d) Highly condensable gases in glassy polymers with plasticization effect



Fig.2: Effect of pressure on solubility in four conditions a)Ideal b) organic compound in elastic polymers c) High solubility gases in glassy polymers without plasticization behavior d) Highly condensable gases in glassy polymers with plasticization effect

Researchers have found that the effect of the thermodynamic factor (sorption) is more crucial than that of the kinetic factor (diffusion) in gas permeability and selectivity. Then, the sorption behavior of gases in glassy polymers can be a key factor to select the polymer for the mixed gas separation [5, 16-28]. In this study, we summarized the sorption behavior of  $CO_2$  and  $CH_4$  in a variety of glassy polymers, as listed in Table 1, and explained the difference between the sorption behaviors of these polymers. For this propose, we defined a pressure that has the same sorption in Henry and Langmuir sites as  $P_{50/50}$  and also calculated the gas sorption in Henry and Langmuir site for each polymer.

| Polymer                   | Condition                    | Designation    | T<br>(°C) | Press.<br>(atm) | Ref.    |  |  |  |  |
|---------------------------|------------------------------|----------------|-----------|-----------------|---------|--|--|--|--|
|                           | Treat @ 180 °C, 18 h         | CPI1-180       | 35        | 7               | [1]     |  |  |  |  |
| 6FDA-DAM/DABA (3/2)       | Treat @ 230 °C, 18 h         | CPI1-230       | 35        | 7               | [1]     |  |  |  |  |
|                           | Treat @ 300 °C, 1 h          | 6F-bisAPAF-300 | 35        | 1               | [5]     |  |  |  |  |
|                           | Treat @ 350 °C, 1 h          | 6F-bisAPAF-350 | 35        | 1               | [5]     |  |  |  |  |
| 6FDA-DISAPAF              | Treat @ 400 °C, 1 h          | 6F-bisAPAF-400 | 35        | 1               | [5]     |  |  |  |  |
|                           | Treat @ 450 °C, 1 h          | 6F-bisAPAF-450 | 35        | 1               | [5]     |  |  |  |  |
| PIM-1                     | Casting In Chloroform        | PIM-1          | 25        | 1               | [16]    |  |  |  |  |
|                           | Original                     | PPO            | 35        | 10              | [18]    |  |  |  |  |
|                           | Carboxylated PPO             | СРРО           | 35        | 10              | [18]    |  |  |  |  |
| Polypnenylene Oxide (PPO) | Methyl esterified CPPO       | MeCPPO         | 35        | 10              | [18]    |  |  |  |  |
|                           | Brominated PPO               | BPPO           | 35        | 10              | [18]    |  |  |  |  |
| Polystyrene               |                              | PS             | 35        | 10              | [19,20] |  |  |  |  |
|                           | Treat @ 200 °C, 2 h          | CPI2-200       | 35        | 1               | [21]    |  |  |  |  |
| 6FDA-Durene/DABA (9/1)    | Treat @ 425 °C, 2 h          | CPI2-425       | 35        | 1               | [21]    |  |  |  |  |
|                           | Grafted y-CD & Treat @ 200 ° | 35             | 1         | [21]            |         |  |  |  |  |

**Table 1:** List of selected glassy polymers

|                                     | 35                             | 1             | [21] |     |         |
|-------------------------------------|--------------------------------|---------------|------|-----|---------|
| 6FDA-6FpDA                          |                                | 6F-6FpDA      | 35   | 10  | [22]    |
| 6FDA-1,5-NDA                        |                                | 6F-1,5-NDA    | 35   | 10  | [23]    |
|                                     | Original                       | PSF           | 30   | 10  | [24]    |
| Delyculfone (DSE)                   | Dimethyl Polysulfone           | DMPSF         | 30   | 10  | [24]    |
| Polysullone (PSF)                   | Brominated Polysulfone         | BPSF          | 30   | 10  | [25]    |
|                                     | Trimethylsilylated Polysulfone | TMSPSF        | 30   | 10  | [25]    |
| Polycarbonate                       | -                              | PC            | 35   | 10  | [26]    |
| Polyhydroxyether                    |                                | PH            | 35   | 10  | [27]    |
| Polyetherimide                      |                                | PEI           | 35   | 10  | [27]    |
| Polyarylate                         | -                              | PA            | 35   | 10  | [27]    |
| Polycarbonate                       |                                | PC            | 35   | 10  | [27]    |
| 6FDA-TADPO polypyrrolone            | -                              | 6F-TADPO      | 35   | 10  | [40]    |
| Bromo Trimethylsilylated polysulfor | ne                             | BTMSPSF       | 30   | 10  | [41]    |
|                                     |                                | 6F-HAB        | 35   | 10  | [42,43] |
|                                     | TR 350 °C, 1 h                 | 6F-HAB-TR-350 | 35   | 10  | [42,43] |
| огранав                             | TR 400 °C, 1 h                 | 6F-HAB-TR-400 | 35   | 10  | [42,43] |
|                                     | TR 450 °C, 0.5 h               | 6F-HAB-TR-450 | 35   | 10  | [42,43] |
| Ultem®1000                          | -                              | Ultem         | 35   | 3.5 | [44,45] |
| Matrimid®5218                       |                                | Matrimid      | 35   | 3.5 | [44,45] |
| Poly (1-trimethylsilyl-1-propyne)   | Casting in THF                 | PTMSP         | 35   | 1   | [46,47] |

### 2. Theory and background

### 2.1 Dual-Mode Sorption Model

Gas transport through glassy polymeric membranes is frequently described by the dual-mode sorption model. The Henry sorption is the main mechanism of transporting sorption to the matrix component and is defined by  $C_D$ , while the Langmuir sorption governs the sorption into the microvoid region and is defined by  $C_H$ . Therefore, total sorption (*C*) is written as:

$$C = C_D + C_H = k_D p + \frac{C'_H b p}{1 + b p}$$

$$\tag{1}$$

where *C* is the concentration of the gas sorbed in the polymer with a unit of gas volume (cm<sup>3</sup>) per polymer volume (cm<sup>3</sup>), *p* (atm) is the pressure of the feed gas in contact with the polymer,  $k_D$  is the Henry's law constant indicating gas dissolution into the equilibrium-defined polymer matrix,  $C'_H$  is the Langmuir capacity of the glassy polymer that is related to the un-relaxed volume, which is a measure of the departure from equilibrium in the glassy state, and *b* is the Langmuir affinity parameter describing the affinity of the gas to a Langmuir site. The Langmuir capacity can be viewed as a measurement of the excess free volume of a polymer [16-18, 29-31]. Solubility coefficient (*S*) of a gas in a glassy polymeric membrane is described as the ratio of the equilibrium gas concentration to the applied gas pressure, as written in Eq. (2).

$$S = \frac{C}{p} = k_D + \frac{C'_H b}{1 + bp}$$
(2)

Gas sorption may display an unusual isotherm if a gas is polar in nature and highly condensable. Since gas sorption isotherm can be experimentally performed, the three dual-mode sorption parameters ( $k_D$ ,  $C'_H$ , and b) can be obtained by the curve fitting using a nonlinear least squares method [32-38].

#### 2.2 Partial Immobilization Model

In the dual-mode sorption theory, the Langmuir mode species is originally considered not to be mobile at all [31]. In contrast, the partial immobilization model is based on the independent dual diffusion of the Henry and Langmuir modes, but assumes that the species in the Langmuir mode can relatively mobilize in glassy polymeric membranes. Since the concentration C of the gas sorption is assumed to obey the dual-mode sorption model as written by Eq. (1), permeation flux (permeation amount per unit time) J and permeability coefficient P through the glassy polymeric membrane can be written by Eq. (3) and Eq. (4), respectively [21, 31-32].

$$J = -D(C)\frac{\partial C}{\partial y} = -D_{D}\frac{\partial C_{D}}{\partial y} - D_{H}\frac{\partial C_{H}}{\partial y} = -D_{D}\left(1 + \frac{FK}{(1+bp)^{2}}\right)\frac{\partial C_{D}}{\partial y} = -D_{D}\left(\frac{1 + \frac{FK}{(1+bp)^{2}}}{1 + \frac{K}{(1+bp)^{2}}}\right)\frac{\partial C}{\partial y}$$
(3)  
$$C = \left(\begin{array}{c}K\\ C_{D}\end{array}\right) - \left(\begin{array}{c}K$$

$$P = S \times D = \frac{C}{p} \times D = k_{D} \left( 1 + \frac{K}{1 + bp} \right) \times D = k_{D} D_{D} + \frac{C_{H}^{*} b D_{H}}{1 + bp} = k_{D} D_{D} \left( 1 + \frac{FK}{1 + bp} \right)$$
(4)

where *C* is the concentration of the penetrant, *y* is the membrane thickness, D(C) is the concentration-dependent diffusion coefficient,  $D_D$  is the diffusion coefficient under the Henry mode,  $D_H$  is the diffusion coefficient under the Langmuir mode, *F* is the ratio of the two diffusion coefficients  $(D_H/D_D)$ , *K* is equal to  $bC'_H/k_D$ , *D* is the average diffusion coefficient, and *P* is the average permeability coefficient. The permeability coefficient, *P*, decreases slowly with an increase in the system pressure and, then, reaches  $k_D D_D$ , irrespective of the *F* value because of the saturation of Langmuir sites. For the binary gas mixture, dual-mode sorption model can be developed. For this purpose, Henry constant for each gas can be assumed independent and uninfluenced by other gases and the pressure of the permeate side for both gases is zero. For the binary gas mixture, total sorption of gasses in polymer and permeability of gas *A* can be written by Eq. (5) and Eq. (6), respectively [1, 24-25, 33].

$$C = k_{D_A} p_A + k_{D_B} p_B + \frac{C'_{H_A} b_A p_A + C'_{H_B} b_B p_B}{1 + b_A p_A + 1 + b_B p_B}$$
(5)

$$P_{A} = k_{D_{A}} D_{D_{A}} \left( 1 + \frac{F_{A} K_{A}}{1 + b_{A} p_{A} + b_{B} p_{B}} \right)$$
(6)

where  $p_A$  and  $p_B$  are partial pressures of gases A and B in the polymer, respectively.

### 3. Results and discussion

### 3.1 Investigation of CO<sub>2</sub> and CH<sub>4</sub> sorption behavior in glassy polymers

As mentioned in the references,  $CO_2$  has smaller molecular size and higher Tc than  $CH_4$  [6, 16]. Also, according to the solution-diffusion model, diffusivity and solubility that are related to the size and condensability of the penetrant molecules, respectively, can affect permeability. It is clear that penetrant molecules with smaller size and higher condensability (Tc) have higher diffusivity and solubility and, then, higher permeability.

Table 2 and Table 3 show dual-mode sorption parameters for  $CO_2$  and  $CH_4$  in different glassy polymers, respectively. As can be seen, all the polymers have higher  $k_D$ ,  $C_H$ , and b for  $CO_2$  than  $CH_4$ , which can be related to higher solubility of  $CO_2$  than  $CH_4$  in these polymers.

| Designation     | k <sub>D</sub> | C <sub>H</sub> | b     | K      | P <sub>50/50</sub> | θ     | lim C <sub>D</sub> /C | lim C <sub>D</sub> /C <sub>H</sub> | lim C <sub>H</sub> /C | S <sup>∞</sup> |
|-----------------|----------------|----------------|-------|--------|--------------------|-------|-----------------------|------------------------------------|-----------------------|----------------|
| CPI1-180        | 2.422          | 44.000         | 0.730 | 13.255 | 16.8               | 0.925 | 0.070                 | 0.075                              | 0.930                 | 34.521         |
| CPI1-230        | 1.976          | 45.900         | 0.648 | 15.065 | 21.7               | 0.934 | 0.062                 | 0.066                              | 0.938                 | 31.741         |
| 6F-bisAPAF-300  | 1.600          | 11.200         | 0.800 | 5.600  | 5.8                | 0.821 | 0.152                 | 0.179                              | 0.848                 | 10.560         |
| 6F-bisAPAF-350  | 1.230          | 30.000         | 0.500 | 12.195 | 22.4               | 0.918 | 0.076                 | 0.082                              | 0.924                 | 16.230         |
| 6F-bisAPAF-400  | 1.600          | 35.000         | 0.550 | 12.031 | 20.1               | 0.917 | 0.077                 | 0.083                              | 0.923                 | 20.850         |
| 6F-bisAPAF-450  | 1.750          | 43.000         | 0.450 | 11.057 | 22.3               | 0.910 | 0.083                 | 0.090                              | 0.917                 | 21.100         |
| PIM-1           | 2.351          | 106.796        | 0.421 | 19.124 | 43.1               | 0.948 | 0.050                 | 0.052                              | 0.950                 | 47.312         |
| РРО             | 0.849          | 25.865         | 0.240 | 7.296  | 26.3               | 0.863 | 0.121                 | 0.137                              | 0.879                 | 7.044          |
| CPPO            | 0.792          | 26.278         | 0.285 | 9.465  | 29.7               | 0.894 | 0.096                 | 0.106                              | 0.904                 | 8.286          |
| MeCPPO          | 0.652          | 20.613         | 0.268 | 8.465  | 27.9               | 0.882 | 0.106                 | 0.118                              | 0.894                 | 6.175          |
| BPPO            | 0.991          | 37.794         | 0.305 | 11.626 | 34.8               | 0.914 | 0.079                 | 0.086                              | 0.921                 | 12.515         |
| PS              | 0.650          | 7.700          | 0.370 | 4.383  | 9.1                | 0.772 | 0.186                 | 0.228                              | 0.814                 | 3.499          |
| CPI2-200        | 1.990          | 41.600         | 0.640 | 13.379 | 19.3               | 0.925 | 0.070                 | 0.075                              | 0.930                 | 28.614         |
| CPI2-425        | 2.310          | 70.600         | 0.630 | 19.255 | 29.0               | 0.948 | 0.049                 | 0.052                              | 0.951                 | 46.788         |
| CPI2-g-γ-CD-200 | 2.330          | 49.700         | 0.560 | 11.945 | 19.5               | 0.916 | 0.077                 | 0.084                              | 0.923                 | 30.162         |
| CPI2-g-γ-CD-425 | 2.650          | 87.700         | 1.100 | 36.404 | 32.2               | 0.973 | 0.027                 | 0.027                              | 0.973                 | 99.120         |
| 6F-6FpDA        | 1.277          | 40.700         | 0.364 | 11.601 | 29.1               | 0.914 | 0.079                 | 0.086                              | 0.921                 | 16.092         |
| 6F-1,5-NDA      | 1.380          | 42.800         | 0.930 | 28.843 | 29.9               | 0.965 | 0.034                 | 0.035                              | 0.966                 | 41.184         |
| PSF             | 0.630          | 16.503         | 0.356 | 9.326  | 23.4               | 0.893 | 0.097                 | 0.107                              | 0.903                 | 6.505          |
| DMPSF           | 0.482          | 12.166         | 0.287 | 7.244  | 21.8               | 0.862 | 0.121                 | 0.138                              | 0.879                 | 3.974          |
| BPSF            | 0.429          | 17.700         | 0.261 | 10.769 | 37.4               | 0.907 | 0.085                 | 0.093                              | 0.915                 | 5.049          |
| TMSPSF          | 0.324          | 20.719         | 0.155 | 9.912  | 57.5               | 0.899 | 0.092                 | 0.101                              | 0.908                 | 3.535          |
| PC              | 0.685          | 18.805         | 0.262 | 7.185  | 23.6               | 0.861 | 0.122                 | 0.139                              | 0.878                 | 5.608          |
| PH              | 0.289          | 10.010         | 0.184 | 6.373  | 29.2               | 0.843 | 0.136                 | 0.157                              | 0.864                 | 2.131          |
| PEI             | 0.758          | 25.020         | 0.366 | 12.081 | 30.3               | 0.917 | 0.076                 | 0.083                              | 0.924                 | 9.915          |
| PA              | 0.685          | 18.810         | 0.262 | 7.194  | 23.6               | 0.861 | 0.122                 | 0.139                              | 0.878                 | 5.613          |
| PC              | 0.631          | 22.690         | 0.215 | 7.731  | 31.3               | 0.871 | 0.115                 | 0.129                              | 0.885                 | 5.509          |
| 6F-TADPO        | 1.526          | 34.084         | 1.023 | 22.849 | 21.4               | 0.956 | 0.042                 | 0.044                              | 0.958                 | 36.394         |
| BTMSPSF         | 0.643          | 15.906         | 0.257 | 6.357  | 20.8               | 0.843 | 0.136                 | 0.157                              | 0.864                 | 4.731          |
| 6F-HAB          | 1.400          | 35.000         | 0.340 | 8.500  | 22.1               | 0.882 | 0.105                 | 0.118                              | 0.895                 | 13.300         |
| 6F-HAB-TR-350   | 1.100          | 51.000         | 0.420 | 19.473 | 44.0               | 0.949 | 0.049                 | 0.051                              | 0.951                 | 22.520         |
| 6F-HAB-TR-400   | 1.500          | 60.000         | 0.580 | 23.200 | 38.3               | 0.957 | 0.041                 | 0.043                              | 0.959                 | 36.300         |
| 6F-HAB-TR-450   | 1.600          | 62.000         | 0.530 | 20.538 | 36.9               | 0.951 | 0.046                 | 0.049                              | 0.954                 | 34.460         |
| Ultem           | 1.040          | 17.300         | 0.355 | 5.905  | 13.8               | 0.831 | 0.145                 | 0.169                              | 0.855                 | 7.182          |
| Matrimid        | 1.440          | 25.500         | 0.367 | 6.499  | 15.0               | 0.846 | 0.133                 | 0.154                              | 0.867                 | 10.799         |
| PTMSP           | 1.050          | 130.000        | 0.040 | 4.952  | 98.8               | 0.798 | 0.168                 | 0.202                              | 0.832                 | 6.250          |

**Table 2:** Dual mode sorption parameters and calculated parameters of different glassy polymers for CO2

The unit of  $k_D$  is (cm<sup>3</sup>/cm<sup>3</sup>.atm),  $C_H$  is (cm<sup>3</sup>/cm<sup>3</sup>polymer), b is (atm<sup>-1</sup>),  $P_{50/50}$  is (atm) and S<sup> $\infty$ </sup> is (cm<sup>3</sup>/cm<sup>3</sup>.atm) and lim  $C_D/C$ , lim  $C_D/C_H$ , and lim  $C_H/C$  are in P=0.

| polymers for eff4 |                |        |       |        |                    |       |                       |                                    |                       |                       |
|-------------------|----------------|--------|-------|--------|--------------------|-------|-----------------------|------------------------------------|-----------------------|-----------------------|
| Designation       | k <sub>D</sub> | С'н    | b     | K      | P <sub>50/50</sub> | θ     | lim C <sub>D</sub> /C | lim C <sub>D</sub> /C <sub>H</sub> | lim C <sub>H</sub> /C | $\mathbf{S}^{\infty}$ |
| CPI1-180          | 0.064          | 27.200 | 0.190 | 80.750 | 419.7              | 0.988 | 0.012                 | 0.012                              | 0.988                 | 5.232                 |
| CPI1-230          | 0.068          | 25.100 | 0.170 | 62.750 | 363.2              | 0.984 | 0.016                 | 0.016                              | 0.984                 | 4.335                 |
| 6F-bisAPAF-300    | 0.700          | 3.100  | 0.700 | 3.100  | 3.0                | 0.677 | 0.244                 | 0.323                              | 0.756                 | 2.870                 |
| 6F-bisAPAF-350    | 0.700          | 11.000 | 0.400 | 6.286  | 13.2               | 0.841 | 0.137                 | 0.159                              | 0.863                 | 5.100                 |
| 6F-bisAPAF-400    | 0.820          | 18.000 | 0.370 | 8.122  | 19.2               | 0.877 | 0.110                 | 0.123                              | 0.890                 | 7.480                 |
| 6F-bisAPAF-450    | 1.180          | 20.000 | 0.240 | 4.068  | 12.8               | 0.754 | 0.197                 | 0.246                              | 0.803                 | 5.980                 |
| PIM-1             | 0.592          | 64.966 | 0.150 | 16.461 | 103.1              | 0.939 | 0.057                 | 0.061                              | 0.943                 | 10.337                |
| PPO               | 0.267          | 19.101 | 0.108 | 7.693  | 62.1               | 0.870 | 0.115                 | 0.130                              | 0.885                 | 2.325                 |
| СРРО              | 0.298          | 15.531 | 0.125 | 6.533  | 44.1               | 0.847 | 0.133                 | 0.153                              | 0.867                 | 2.246                 |
| MeCPPO            | 0.270          | 10.106 | 0.124 | 4.637  | 29.4               | 0.784 | 0.177                 | 0.216                              | 0.823                 | 1.521                 |
| BPPO              | 0.418          | 24.935 | 0.151 | 9.025  | 53.0               | 0.889 | 0.100                 | 0.111                              | 0.900                 | 4.193                 |
| PS                | 0.175          | 2.530  | 0.146 | 2.111  | 7.6                | 0.526 | 0.321                 | 0.474                              | 0.679                 | 0.544                 |
| CPI2-200          | 0.497          | 14.196 | 0.143 | 4.078  | 21.6               | 0.755 | 0.197                 | 0.245                              | 0.803                 | 2.524                 |
| CPI2-425          | 0.641          | 20.148 | 0.181 | 5.679  | 25.9               | 0.824 | 0.150                 | 0.176                              | 0.850                 | 4.284                 |
| CPI2-g-γ-CD-200   | 0.551          | 14.315 | 0.154 | 4.010  | 19.5               | 0.751 | 0.200                 | 0.249                              | 0.800                 | 2.761                 |
| CPI2-g-γ-CD-425   | 0.787          | 40.149 | 0.200 | 10.198 | 46.0               | 0.902 | 0.089                 | 0.098                              | 0.911                 | 8.808                 |
| 6F-6FpDA          | 0.287          | 20.630 | 0.117 | 8.410  | 63.3               | 0.881 | 0.106                 | 0.119                              | 0.894                 | 2.701                 |
| 6F-1,5-NDA        | 0.330          | 27.580 | 0.140 | 11.701 | 76.4               | 0.915 | 0.079                 | 0.085                              | 0.921                 | 4.191                 |
| PSF               | 0.167          | 9.044  | 0.118 | 6.390  | 45.7               | 0.844 | 0.135                 | 0.156                              | 0.865                 | 1.234                 |
| DMPSF             | 0.078          | 7.348  | 0.108 | 10.174 | 84.9               | 0.902 | 0.089                 | 0.098                              | 0.911                 | 0.872                 |
| BPSF              | 0.096          | 8.810  | 0.084 | 7.709  | 79.9               | 0.870 | 0.115                 | 0.130                              | 0.885                 | 0.836                 |
| TMSPSF            | 0.199          | 8.299  | 0.090 | 3.753  | 30.6               | 0.734 | 0.210                 | 0.266                              | 0.790                 | 0.946                 |
| PC                | 0.147          | 8.382  | 0.084 | 4.786  | 45.0               | 0.791 | 0.173                 | 0.209                              | 0.827                 | 0.852                 |
| PH                | 0.051          | 2.700  | 0.067 | 3.547  | 38.0               | 0.718 | 0.220                 | 0.282                              | 0.780                 | 0.232                 |
| PEI               | 0.207          | 7.310  | 0.136 | 4.803  | 28.0               | 0.792 | 0.172                 | 0.208                              | 0.828                 | 1.201                 |
| PA                | 0.147          | 8.380  | 0.084 | 4.789  | 45.1               | 0.791 | 0.173                 | 0.209                              | 0.827                 | 0.851                 |
| PC                | 0.181          | 6.450  | 0.100 | 3.564  | 25.6               | 0.719 | 0.219                 | 0.281                              | 0.781                 | 0.826                 |
| 6F-TADPO          | 0.327          | 22.838 | 0.160 | 11.175 | 63.6               | 0.911 | 0.082                 | 0.089                              | 0.918                 | 3.981                 |
| BTMSPSF           | 0.211          | 9.375  | 0.105 | 4.665  | 34.9               | 0.786 | 0.177                 | 0.214                              | 0.823                 | 1.195                 |
| 6F-HAB            | 0.440          | 11.000 | 0.110 | 2.750  | 15.9               | 0.636 | 0.267                 | 0.364                              | 0.733                 | 1.650                 |
| 6F-HAB-TR-350     | 0.340          | 23.000 | 0.130 | 8.794  | 60.0               | 0.886 | 0.102                 | 0.114                              | 0.898                 | 3.330                 |
| 6F-HAB-TR-400     | 0.460          | 32.000 | 0.180 | 12.522 | 64.0               | 0.920 | 0.074                 | 0.080                              | 0.926                 | 6.220                 |
| 6F-HAB-TR-450     | 0.490          | 36.000 | 0.160 | 11.755 | 67.2               | 0.915 | 0.078                 | 0.085                              | 0.922                 | 6.250                 |
| Ultem             | 0.170          | 5.830  | 0.210 | 7.202  | 29.5               | 0.861 | 0.122                 | 0.139                              | 0.878                 | 1.394                 |
| Matrimid          | 0.136          | 14.30  | 0.105 | 11.040 | 95.6               | 0.909 | 0.083                 | 0.091                              | 0.917                 | 1.638                 |
| PTMSP             | 0.500          | 62.000 | 0.050 | 6.200  | 104.0              | 0.839 | 0.139                 | 0.161                              | 0.861                 | 3.600                 |

 Table 3: Dual mode sorption parameters and calculated parameters of different glassy

 polymers for CH.

The unit of  $k_D$  is (cm<sup>3</sup>/cm<sup>3</sup>.atm),  $C_H$  is (cm<sup>3</sup>/cm<sup>3</sup>polymer), b is (atm<sup>-1</sup>),  $P_{50/50}$  is (atm) and  $S^{\infty}$  is (cm<sup>3</sup>/cm<sup>3</sup>.atm) and lim  $C_D/C$ , lim  $C_D/C_H$ , and lim  $C_H/C$  are in P=0.

As mentioned above,  $CO_2$  has smaller molecular size than  $CH_4$  and can quickly saturate Langmuir sites and then immediately start to occupy Henry sites (higher  $k_D$ ); however,  $CH_4$  needs higher pressure to saturate Langmuir sites. As can be seen in Eq. (7) and Eq. (8), increasing the pressure in the dual-mode sorption leads to a decrease in the Langmuir sorption to the total sorption ratio up to zero and increase the Henry sorption to the total sorption ratio up to one.

$$\lim_{P \to \infty} \frac{C_H}{C} = 0 \tag{7}$$

$$\lim_{P \to \infty} \frac{C_D}{C} = 1 \tag{8}$$

These two equations confirmed that, for  $CO_2$ , by increasing the pressure, Langmuir sites are saturated very fast and, afterwards, the Henry sorption is dominant. However,  $CH_4$  could not easily enter the Langmuir site, which could make the Langmuir sorption significant up to high pressure. According to the sorption curves for the Langmuir and Henry sorption, at the beginning, penetrant molecules are motivated to occupy the Langmuir sites and to fill these pores and, at low pressure for most of the penetrant in the polymers, Langmuir sorption is dominant. After a while, due to the saturation of most of the Langmuir sites, the sorption of the penetrant molecules in the Langmuir site become more difficult, this resulted in an increase of the sorption in Henry sites.

The infinite dilution solubility coefficient ( $S^{\infty}$ ) from the dual-mode sorption model given by Eq. (9) represents the solubility coefficient in the limit of zero concentration (very low pressure) [16]. Table 2 and Table 3 list the values of  $S^{\infty}$  for CO<sub>2</sub> and CH<sub>4</sub>, respectively.

$$S^{\infty} = \lim_{P \to 0} S = \lim_{P \to 0} \left( k_{D} + \frac{C'_{H}b}{1+bp} \right) = k_{D} + C'_{H}b = k_{D} \left( 1 + K \right)$$
(9)

This parameter can be a piece of good evidence for the solubility of penetrant in special polymers. Comparison of this parameter for special penetrant in different polymers shows the polymers with higher solubility for that penetrant. While the gas sorption at low pressures tends to be dominated by the non-equilibrium nature of glassy polymers (Langmuir site) and  $C_H b$  is much higher than  $k_D$  at very low pressures,  $S^{\infty}$  is equal to  $C_H b$ . As pressure increased, the solubility coefficients are reduced, especially for the more condensable gases [16].

### 3.2 P<sub>50/50</sub> parameter

 $P_{50/50}$  is the pressure with an equal sorption amount in Henry and Langmuir sites. This parameter is defined to compare the difference of sorption in various polymers and can be calculated by Eq. (10).

At 
$$P_{50/50}$$
:  $C_D = C_H \Rightarrow k_D P_{50/50} = \frac{C'_H b p_{50/50}}{1 + b p_{50/50}} \Rightarrow P_{50/50} = \frac{C'_H b \cdot k_D}{b k_D} = \frac{K \cdot 1}{b}$  (10)

Table 2 and Table 3 show the calculated amount of this parameter for CO<sub>2</sub> and CH<sub>4</sub>, respectively. As can be seen in Figure 3 and Figure 4, this pressure is lower for CO<sub>2</sub> than it is for CH<sub>4</sub>, showing that at the low pressure, the contribution of the Langmuir sorption to the total sorption is higher. However, for some polymers, this pressure is very high, which cannot be covered by the sorption data and the Langmuir sorption is always dominant. Higher  $P_{50/50}$  means that the Langmuir sorption is more important than the Henry sorption. This parameter can be used to make a decision about the sorption model. For extremely small  $P_{50/50}$ , the sorption model is near the Henry sorption and, for very high  $P_{50/50}$ , the sorption model is close to the Langmuir sorption.



**Fig.3:** Henry, Langmuir and Dual mode sorption isotherms for CO<sub>2</sub> and CH<sub>4</sub> in Matrimid<sup>®</sup>5218 and Ultem<sup>®</sup>1000 @ 35°C



**Fig.4:** Henry, Langmuir and Dual mode sorption isotherms for CO<sub>2</sub> and CH<sub>4</sub> in PIM-1 and CPI2-g- $\gamma$ -CD-425 @ 35°C

3.3 Saturated percentage of Langmuir sites in  $P_{50/50}(\theta)$ 

Saturated percentage of Langmuir sites is crucial due to the effect of pressure on sorption and permeation. This parameter shows the extent to which Langmuir sites are saturated at  $P_{50/50}$ . The higher this parameter, the faster the sorption of penetrant is in Langmuir sites and the more the

Langmuir sorption is sensitive to pressure. Moreover, the sorption also occurs at low pressures. The saturated percentage of Langmuir sites at  $P_{50/50}$  can be calculated by Eq. (11).

$$\theta = \frac{C_{H} @ p : p_{50/50}}{C_{H} @ p : p_{inf.}} = \frac{\frac{C'_{H}bp_{50/50}}{(1+bp_{50/50})}}{\frac{C'_{H}bp_{inf.}}{(1+bp_{inf.})}} = \frac{\frac{C'_{H}b\frac{(C'_{H}b-k_{D})}{bk_{D}}}{bk_{D}}}{C'_{H}} = \frac{C'_{H}b-k_{D}}{C'_{H}b} = 1 - \frac{k_{D}}{C'_{H}b} = 1 - \frac{1}{K}$$
(11)

According to the shape of the sorption curve, it is obvious that the Langmuir mode is more sensitive to pressure than the Henry mode; based on the results of Table 2 and Table 3, most of the Langmuir sites are usually saturated at  $P_{50/50}$ . Therefore, a small amount of difference at  $P_{50/50}$  is crucial. Other important parameters calculated in Table 2 and Table 3 is the Henry sorption to the Langmuir sorption (Eq. (12)) as well as the Henry sorption to the total sorption ratio both at zero pressure (Eq. (13)).

$$\lim_{P \to 0} \frac{C_{D}}{C_{H}} = \lim_{P \to 0} \frac{k_{D}p}{\underline{C'_{H}bp}} = \frac{k_{D}}{C'_{H}b} = \frac{1}{K}$$
(12)

$$\lim_{P \to 0} \frac{C_{D}}{C} = \lim_{P \to 0} \frac{k_{D}p}{k_{D}p + \frac{C'_{H}bp}{1+bp}} = \frac{k_{D}}{k_{D} + C'_{H}b} = \frac{1}{1+K}$$
(13)

As mentioned previously, K is a dimensionless variable that is related to all dual-mode sorption parameters. Figure 3 and Figure 4 show CO<sub>2</sub> and CH<sub>4</sub> sorption as functions of pressure in Henry and Langmuir sites for different polymers, respectively. Moreover, Figure 5 and Figure 6 demonstrate the contributions of Henry and Langmuir sorption as functions of pressure for different polymers, respectively.



**Fig.5:** Contribution of Henry and Langmuir sorption from total sorption for CO<sub>2</sub> and CH<sub>4</sub> in Matrimid<sup>®</sup>5218 and Ultem<sup>®</sup>1000



**Fig.6:** Contribution of Henry and Langmuir sorption from total sorption for CO<sub>2</sub> and CH<sub>4</sub> in PIM-1 and CPI2-g-γ-CD-425

### 3.4 Analysis of partial immobilization model

As mentioned before, the partial immobilization model is based on the independent dual diffusion of the Henry and Langmuir modes. However, it assumes that species in the Langmuir mode can relatively mobilize in glassy polymeric membranes and *F* factor is the portion of mobile sorbed penetrant in Langmuir sites. The amount of *F* factor ranges from 0.01 to 0.2 for different penetrants in various polymers, showing that the portion of the Langmuir sorption is less than 20 percent of the total sorption. Researchers have defined *F* factor as  $D_{H}/D_D$  and assumed this amount to correspond to  $C_{Hmobile}/C_H$ , meaning that the portion of the concentration of mobile molecules in the Langmuir site corresponds to the ratio of diffusivity in the Langmuir site to the Henry site [39-41]. This assumption is correct under particular conditions. In general, these two definitions, two parameters  $F_C$  and  $F_D$  are defined as below.

$$F_{c} = \frac{C_{H_{mobile}}}{C_{H}}$$
(14)

$$F_{D} = \frac{D_{H}}{D_{D}}$$
(15)

Generally, the penetrant concentrations are divided into mobile and immobile concentrations. Mobile concentration and diffusivity coefficient are defined as  $C_m$  and D, respectively, and the immobile concentration is defined as  $C-C_m$ . Thus, the mobile and immobile concentrations can be defined as Eq. (16) and Eq. (17).

Mobile Concentration : 
$$C_m = C_D + F_C C_H = k_D p + F_C \left(\frac{C_H b p}{1 + b p}\right)$$
 (16)

Immobile Concentration : 
$$C - C_m = (1 - F_C)C_H = (1 - F_C)\left(\frac{C_H bp}{1 + bp}\right)$$
 (17)

The permeation flux through the membrane can be written in two conditions. In the first one, two diffusion coefficients are defined for Henry sites  $(D_D)$  and Langmuir sites  $(D_H)$  and the driving force is total concentration (C). In the second condition, the diffusion coefficients of both Langmuir and Henry sites are the same (D), but the driving force is only mobile concentration  $(C_{Hmobile})$ . Eq. (18) and Eq. (19) show the permeation flux in these two conditions. As can be seen in these two equations, if  $D=D_D$  and  $F_C=F_D$ , these two permeation fluxes are equal.

$$J = -D_{D} \frac{\partial C_{D}}{\partial y} - D_{H} \frac{\partial C_{H}}{\partial y} = -D_{D} \frac{\partial C_{D}}{\partial p} \cdot \frac{\partial p}{\partial y} - D_{H} \frac{\partial C_{H}}{\partial p} \cdot \frac{\partial p}{\partial y} = -D_{D} \left( \frac{\partial C_{D}}{\partial p} - F_{D} \frac{\partial C_{H}}{\partial p} \right) \frac{\partial p}{\partial y}$$
(18)

$$J = -D \frac{\partial C_D}{\partial y} - D \frac{\partial C_H_{mobile}}{\partial y} = -D \frac{\partial C_D}{\partial p} \cdot \frac{\partial p}{\partial y} - D \frac{\partial (F_c \cdot C_H)}{\partial p} \cdot \frac{\partial p}{\partial y} = -D \left( \frac{\partial C_D}{\partial p} - F_c \frac{\partial C_H}{\partial p} \right) \frac{\partial p}{\partial y}$$
(19)

To show the difference of these two conditions, the permeation flux and permeability coefficient can be written as shown below:

$$J = -D_D \frac{\partial C_D}{\partial y} - D_H \frac{\partial C_{H_{mobile}}}{\partial y} = -D_D \frac{\partial C_D}{\partial p} \cdot \frac{\partial p}{\partial y} - D_H \frac{\partial (F_C \cdot C_H)}{\partial p} \cdot \frac{\partial p}{\partial y}$$
(20)

$$C_{m} = C_{D} + F_{C}C_{H} = k_{D}p + F_{C}\frac{C_{H}bp}{1+bp}$$

$$\tag{21}$$

$$J = -D_{D} \frac{\partial C_{D}}{\partial p} \cdot \frac{\partial p}{\partial y} - D_{H} \frac{\partial (F_{C} \cdot C_{H})}{\partial p} \cdot \frac{\partial p}{\partial y} = \left(-k_{D} D_{D} - D_{H} F_{C} \frac{C_{H} b p}{\left(1 + b p\right)^{2}}\right) \cdot \frac{\partial p}{\partial y}$$
(22)

$$J = \left(-k_{D}D_{D} - D_{H}F_{C}\frac{C_{H}bp}{\left(1+bp\right)^{2}}\right)\cdot\frac{\partial p}{\partial y} = -k_{D}D_{D}\left(1+\frac{F_{D}F_{C}\frac{C_{H}b}{k_{D}}}{\left(1+bp\right)^{2}}\right)\cdot\frac{\partial p}{\partial y}$$
(23)

$$P = -\frac{\left(\int_{0}^{L} J \cdot dy\right)}{\Delta p} = \frac{\int_{p_{1}}^{p_{2}} k_{D} \cdot D_{D} \left(1 + \frac{F_{D}F_{C}\frac{C_{H}b}{k_{D}}}{\left(1 + bp\right)^{2}}\right) \cdot \partial p}{\Delta p} = k_{D}D_{D} \left(1 + \frac{F_{D}F_{C}K}{\left(1 + bp\right)}\right)$$
(24)

As can be seen, if  $F=F_C \times F_D$ , then Eq. (24) is equal to Eq. (4) ,which has been used in most of the references. An important point that is not analyzed cautiously in the partial immobilized model is the portion of penetrant sorption in the Langmuir site to the total sorption. Most scholars use the first condition mentioned above, in which the diffusivity coefficient in Henry sites ( $D_D$ ) is always much higher than the diffusivity coefficient in Langmuir sites ( $D_H$ ). In addition, F factor ranges from 0.02 to 0.2.

In order to compare and analyze the contribution of mobile molecules of the Langmuir site from the total sorption, all the polymers in Tables 4 were evaluated and figures were plotted according to the F factor in each reference.

|                 | -                               | CO <sub>2</sub>       |       |  |                   |                       |       |      |
|-----------------|---------------------------------|-----------------------|-------|--|-------------------|-----------------------|-------|------|
| Designation     | D <sub>D</sub> *10 <sup>8</sup> | $D_{\rm H}^{*10^{8}}$ | Б     |  | $D_{D}^{*}10^{8}$ | $D_{\rm H}^{*10^{8}}$ | F     | Ref. |
|                 | (cm <sup>2</sup> /s)            | $(cm^2/s)$            | Г     |  | $(cm^2/s)$        | $(cm^2/s)$            | - F   |      |
| PIM-1           | 1296.600                        | 34.120                | 0.026 |  | 364.000           | 3.340                 | 0.009 | [16] |
| CPI2-200        | 97.500                          | 3.030                 | 0.031 |  | 19.300            | 0.980                 | 0.051 | [21] |
| CPI2-425        | 507.000                         | 24.300                | 0.048 |  | 78.300            | 5.140                 | 0.066 | [21] |
| CPI2-g-γ-CD-200 | 96.500                          | 2.650                 | 0.027 |  | 19.400            | 1.130                 | 0.058 | [21] |
| CPI2-g-γ-CD-425 | 1317.000                        | 79.200                | 0.060 |  | 183.800           | 15.400                | 0.084 | [21] |
| 6F-6FpDA        | 25.500                          | 1.530                 | 0.060 |  | 2.7800            | 0.236                 | 0.085 | [22] |
| 6F-1,5-NDA      | 10.100                          | 0.655                 | 0.065 |  | 0.883             | 0.033                 | 0.037 | [23] |
| PSF             | 4.790                           | 0.581                 | 0.121 |  | 0.692             | 0.106                 | 0.153 | [24] |
| DMPSF           | 2.846                           | 0.452                 | 0.159 |  | 0.520             | 0.051                 | 0.098 | [24] |
| BPSF            | 4.718                           | 0.674                 | 0.143 |  | 0.652             | 0.082                 | 0.126 | [25] |
| TMSPSF          | 30.973                          | 1.202                 | 0.039 |  | 3.521             | 0.221                 | 0.063 | [25] |
| PC              | 6.220                           | 0.485                 | 0.078 |  | 1.090             | 0.126                 | 0.115 | [26] |
| 6F-TADPO        | 11.960                          | 1.004                 | 0.084 |  | 1.120             | 0.029                 | 0.026 | [40] |
| BTMSPSF         | 8.502                           | 0.327                 | 0.038 |  | 3.980             | 0.087                 | 0.022 | [41] |

**Table 4:** Diffusivity parameters of different glassy polymers for CO<sub>2</sub> and CH<sub>4</sub>

For example, the results of two commercials and two synthesized polyimide are shown in Figure 7 and Figure 8, respectively. These figures demonstrate the ratio of sorbed mobile molecules in the Langmuir site to the total sorption in the polymer for CO<sub>2</sub> and CH<sub>4</sub>. In addition, the ratio of this molecule was drawn in proportion to the total mobile molecules and to Henry site molecules. The results of various polymers indicated that the contribution of mobile molecules in the Langmuir state to the total sorbed molecules in the polymer is not more than 5%, which is very small in value. However, these molecules should not be considered unimportant, because the contribution of this ratio to the total mobile molecules ranged from 5 to 70% for different polymers at low pressures, which was a considerable contribution. If the contribution of these molecules were measured compared with the sorbed molecules in the Henry site, it can be seen that, despite the existing subjective perception that the sorbed molecules in the Henry site have the main contribution in permeation, the contribution of these molecules at low pressures is not only comparable, but also up to about 1.5 times more than that of the sorbed molecules in the Henry site (in some polymers). As is clear, the contribution of these molecules rapidly disappears with increasing the pressure, which is clearly evident for CO<sub>2</sub>. However, the mobile molecules in the Langmuir site have a significant contribution up to 11-15 atm for CH<sub>4</sub>, considering the importance of sorption in the Langmuir sorption model. The other difference that can be seen in the contribution of these molecules to CO<sub>2</sub> and CH<sub>4</sub> is that the difference between the ratios of these molecules to the total mobile molecules from the ratio of these molecules to the molecules in the Henry site is very high for CO<sub>2</sub> at the initial pressure. However, this initial difference for CH<sub>4</sub> is not more than 0.015-0.1. The interesting point is that, despite the large initial difference between these two ratios, this difference quickly decreases with increasing the pressure. In addition, the two curves coincided at a not very high pressure. However, that small difference for CH<sub>4</sub> is maintained at high pressures and the difference reduction occurs very slowly.

The next case is that the contribution of mobile  $CO_2$  molecules in the Langmuir site is more than the same amounts for  $CH_4$  in a certain polymer to the total mobile molecules. However, by increasing pressure, for  $CO_2$  its importance is reduced much faster than  $CH_4$ .



**Fig.7:** Contribution of mobile Langmuir sorption from total sorption for CO<sub>2</sub> and CH<sub>4</sub> in Polycarbonate and Polysulfone



Fig.8: Contribution of mobile Langmuir sorption from total sorption for  $CO_2$  and  $CH_4$  in PIM-1 and CPI2-g- $\gamma$ -CD-425

### 4. Conclusion

Investigation on the solubility of CO<sub>2</sub> and CH<sub>4</sub> in various polymers with the dual-mode sorption model showed that the sorption model of these gasses was different in terms of appearance within the normal range of sorption tests in addition to the difference in solubility.  $P_{50/50}$  parameter was much smaller for the sorption of CO<sub>2</sub> than CH<sub>4</sub> in various polymers, showing that the sorption contribution in the Langmuir site was less important than the Henry site. This parameter can be used as a scale of closeness of sorption model to the pure Henry model or the pure Langmuir model. This study attempted to provide a correct definition of *F* factor, used in references, in the form of the multiplication of mobile concentration ratios ( $C_{Hmobile}/C_H$ ) by the diffusivity coefficients in two modes ( $D_H/D_D$ ). In addition, the contribution of each sorption mode from the total sorption as well as its impact on the permeation flux of the total membrane was determined. Meanwhile, a better description was tried to be achieved for the partial immobilization model.

## 5. Nomenclature

- *b*: Langmuir affinity constant (atm<sup>-1</sup>)
- *C*: Total sorption amount of polymer ( $cm^3$  gas (STP) / $cm^3$  polymer)
- $C_D$ : Concentrations of the penetrant sorbed at the Henry site (cm<sup>3</sup> gas (STP) /cm<sup>3</sup> polymer)
- $C_H$ : Concentrations of the penetrant sorbed at the Langmuir site (cm<sup>3</sup> gas (STP) /cm<sup>3</sup> polymer)

 $C'_{H}$ : Langmuir saturation constant (cm<sup>3</sup> gas (STP) /cm<sup>3</sup> polymer)

*D*: Average diffusivity coefficient ( $cm^2/s$ )

D(C): Concentration dependent diffusion coefficient (cm<sup>2</sup>/s)

 $D_D$ : Henry's law diffusion coefficient (cm<sup>2</sup>/s)

 $D_H$ : Langmuir mode diffusion coefficient (cm<sup>2</sup>/s)

F: Mobile fraction of the Langmuir mode species

*J*: permeation flux ( $cm^3 gas (STP) / s$ )

 $k_D$ : Henry's solubility coefficient ((cm<sup>3</sup> gas (STP)) / (cm<sup>3</sup> polymer-atm))

- *p*: Pressure of the feed (atm)
- *P*: Average permeability coefficient (Barrer)
- S: Solubility coefficient ((cm<sup>3</sup> gas (STP)) / (cm<sup>3</sup> polymer-atm))
- y: Polymer thickness (cm)

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