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ROLE OF MATHEMATICAL MORPHOLOGY IN FILTERING, SEGMENTATION AND ANALYSIS

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ABSTRACT

In this paper, we present an overview on the possibilities of mathematical morphology during image processing and image analysis from the first steps of filtering to the final one corresponding to measure and modelling.

Key words : image processing, statistical images, mathematical morphology, measure, models, filtering, segmentation.

INTRODUCTION

The aim of image analysis is to describe an image or a set of images by some parameters or render these images by synthetic models estimated from measurements. Today, image analysis is generally performed by an automatic system. Before measurements, it is necessary to treat the image to eliminate noise and non-interesting parts, to enhance regions which must be analysed and finally to separate each set of interest.

Several methods can be used to treat image before measurements. These methods can be classified in two main groups. The first one derives from signal treatment techniques with linear operators (Fourier transforms, convolution products, ...), the second one uses mathematical morphology transformations which are non linear operators. Matheron and Serra are the fathers of mathematical morphology. Two books written by Serra (1982, 1988) resume these works in the domain of image processing and image analysis. Today other books have been published in French (Coster and Chermant, 1985; 1989) or in English by american workers (Dougherty and Giardina, 1987; Giardina and Dougherty, 1988).

The image treatment can be divided in three steps : pre-treatment, segmentation or threshold, and post-treatment. After these steps, analysis itself can be performed. The scope of this paper is to present the contribution of mathematical morphology to image treatment and image analysis and to give some examples.

IMAGE PRE-TREATMENT

In this paper, we shall not speak on the very important problem of image acquisition because it is a "hardware" problem where mathematical morphology or alternate methods are not used. Afterwards, one assumes that initial images are stored in the memory of a computer or image analyser. Mathematically speaking, an image can be represented by a function f(x) belonging to space $IR^2 \to IR$, where x is the point of the support IR^2 and f(x) the radiometric value of this point defined in IR space.

Two kinds of filters are used during the image pre-treatment step. In fact two problems must be solved before segmentation: one must eliminate the noise and enhance the objects. Nevertheless, for images without noise and having a good contrast between background and objects, this step can be avoided. In terms of signal analysis, to eliminate the noise without to destroy information, it is necessary that the noise appears to different frequencies than that of the analysed features. In this case, the filtering can be made by three kinds of filters : linear filters, adaptative filter or morphological filter.

Low pass filters

Generally the noise is in the high frequency domain, then a low pass filter must be used. In mathematical morphology, the standard filters are opening, noted $O^B f(x)$, and closing, noted $F^B f(x)$. They are built from erosion and dilation, respectively noted $E^B f(x)$ and $D^B f(x)$. For $IR^2 \to IR$ functions, B_x is a structuring function V(y) defined on the support B centred on x, ($y \in B_x$). With these notations, the following relations define respectively erosion and dilation :

$$E^B f(x) = \inf(f(x) - V(y) : y \in B_x) \quad (1)$$

$$D^B f(x) = \sup(f(x) + V(y) : y \in B_x) \quad (2)$$

Opening and closing are then defined by $O^B f(x) = D^B E^B(f(x))$ and $F^B f(x) = E^B D^B f(x)$. Unlike linear filters or adaptive filters, the morphological filters are not symmetric but they are idempotent and increasing. They modify only the narrow crests and peaks (opening) or narrow valleys and basins (closing), but this modification is generally more important than that is obtained from other filters (figures 1 to 3).

To avoid this strong modification, other more complex morphological filters can be used. By using the extensivity or anti-extensivity of morphological filters, these can be classified. Indeed, there are :

$$F^B f(x) \geq F^B O^B F^B f(x) \geq F^B O^B f(x) \text{ or } O^B F^B f(x) \geq O^B F^B O^B f(x) \geq O^B f(x) \quad (3)$$

The complex filters thus defined are more smooth than standard filters. In the domain of low pass filters, two other classes of morphological filters exist, the first corresponds to sequential filters and the second one to auto-median filters. These filters are built from four basic transformations : opening, closing, and "sup" or "inf" between two images.

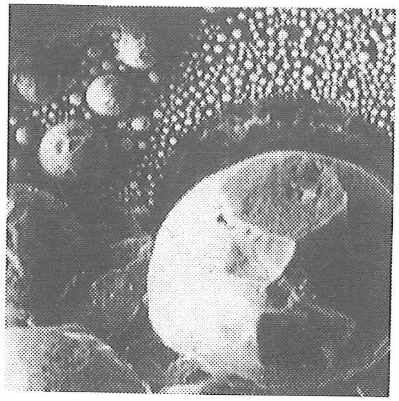

Figure 1 : Initial image.

Figure 2 : Opening image of size 5 on fig.1.

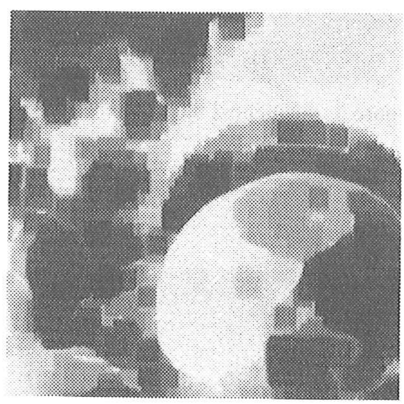

Figure 3 : Closing image of size 5 on fig. 1.

Sequential filters

Let γ_i be a granulometric transformation by opening with convex structuring element of size i and ϕ_i a granulometric transformation by closing of size i. From γ_i and ϕ_i, it is possible to build alternative sequential filters. In discrete case, the simplest sequential filter is defined by (figure 4) :

$$M^i f(x) = \gamma_i \varphi_i \gamma_{i-1} \varphi_{i-1} \cdots \gamma_1 \varphi_1 f(x) \qquad (4)$$

$M^i f(x)$ generalises the filter $\gamma_i \phi_i$, by alternative opening and closing of increasing size. By a decreasing sequence it is possible to build another sequential filter $M_i f(x)$, defined by :

$$M_i f(x) = \gamma_1 \varphi_1 \gamma_2 \varphi_2 \cdots \gamma_i \varphi_i f(x) \qquad (5)$$

Theses filters have good properties. If γ and φ have granulometric properties then M keep this property. In mathematical morphology field, alternative sequential filters are considered as good noise cleaner. They have been used by different workers (Sternberg (1986) and Destival (1986), in satellite image field; Bloch and Préteux (1986) in scanner radiography; Friedlander (1986) in cardiology; Guedj (1986) in materials science, ...).

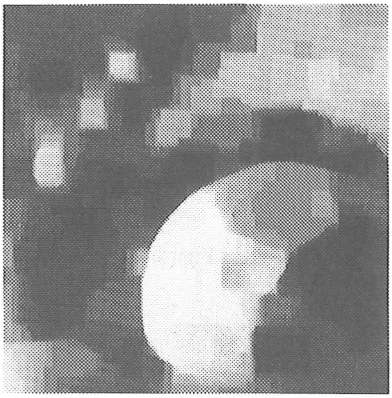

Figure 4 : Sequential filter M^5 on fig. 1.

Auto-dual filters

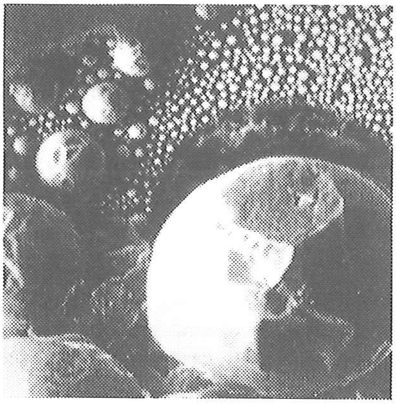

Figure 5 : Auto-dual filter on fig. 1.

Auto dual filters (figure 5) correspond to a second class of complex morphological filters. The previous filters are asymmetric because at the beginning one starts from opening or closing which are respectively anti-extensive or extensive transformation. Let T_1 be an anti-extensive filter (ex. $\gamma\phi\gamma$) and T_2 an extensive filter (ex. $\phi\gamma\phi$); it is possible to define a symmetric morphological filter called, auto dual-filter, $\beta f(x)$ given by :

$$\beta f(x) = \sup[T_1 f(x); \inf[T_2 f(x); f(x)]] \tag{6}$$

If T_1 and T_2 are dual, then :

$$\beta(-f(x)) = -\beta(f(x)) \tag{7}$$

By iterative process, one obtains an auto median filter. An example of this filter has been given by Laÿ (1984). The convergence of this filter is slow. That is the reason why the iterative process is often limited to some steps only. This class of filters can be used to eliminate noise in image pre-treatment (Prod'homme ,1992; Prod'homme and al.,1992).

High Pass Filters

Generally image enhancement is often obtained by high pass filters. In mathematical morphology, the main high pass filter is the morphological gradient. It is obtained by difference between two images. The first one must be greater than the second one (from the mathematical meaning). The most common morphological gradient is (figure 6) :

$$Grad(f(x)) = \frac{\left(D^{\lambda B} f(x) - E^{\lambda B} f(x)\right)}{2\lambda} \tag{8}$$

Figure 6 : Morphological gradient on fig. 1.

The morphological gradient image is similar to the image of the gradient modulus for a function continuous and derivable. It is often used before image segmentation by morphological methods (as we can see later on). Two reduced versions of morphological gradient can also be defined and called respectively external and internal gradient (Beucher, 1990) :

$$Grad^{+}(f(x)) = \frac{D^{\lambda B} f(x) - f(x)}{\lambda} \quad \text{and} \quad Grad^{-}(f(x)) = \frac{f(x) - E^{\lambda B} f(x)}{\lambda} \tag{9}$$

For these kinds of filters (low and high pass), the definitions are given with an Euclidean sense (i.e. without any condition). A geodesic version of these filters exists. In that case the transformations are performed within a mask defined by a binary or grey tone level image. It should be noted that any filter corresponds to Laplacian filter in mathematical morphology.

IMAGE SEGMENTATION

Image segmentation is the most important step in image processing, because that is the step where one loses the greatest quantity of information. The segmentation process can be classified in several classes : pixel-based methods, region based methods and edge based methods.

The pixel-based segmentation only takes the grey value of a pixel in order to decide whether it belongs to the object (phase) or not. The automatic threshold uses histogram analysis (Zeboudj, 1988; Prod'homme and al., 1992). Because, the segmentation depends only on the grey value of the pixel, any morphological operator can be used for this kind of segmentation.

Region-based methods focus our attention on an important aspect of the segmentation process which has been missed in the previous methods : the connectivity of the neighbouring pixels. In mathematical morphology, top hat transform (Meyer; 1978) and watershed methods (Beucher, 1990) belong to this class of methods.

The human perception of the image is frequently based on the edge detection process. In image processing, this kind of segmentation is often used, but in mathematical morphology any classical operator belongs to this type of segmentation.

Top hat transformation

Top hat transformation (figures 7 and 8) is the first morphological transformation belonging to segmentation process. Let *f(x)* be the grey tone level image at point x and t a grey tone level value. The top hat transformation combines a morphological filtering with a threshold. The standard of this transformation is defined by :

$$X=\{x : f(x)-O^{B}f(x)\geq t\} \tag{10}$$

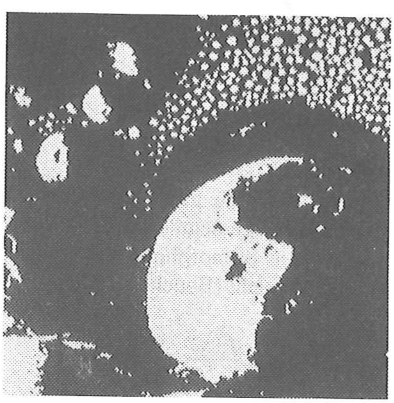

Figure 7 : Normal threshold on fig. 1.

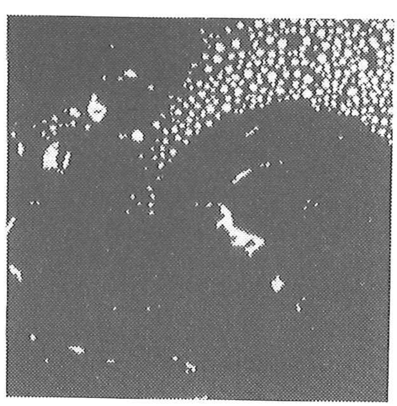

Figure 8 : Top hat transform on fig. 1.

A dual version according to complementation and using closing transformation has been also defined :

$$X = \{x : F^{B} f(x) - f(x) \geq t\} \tag{11}$$

Starting from the Meyer definition, Bonton (Bonton and al., 1986) proposes another transformation called "discriminating" top hat transformation defined by :

$$g(x) = \begin{cases} 0 \text{ if } F^{\lambda B} f(x) - O^{\mu B} f(x) \leq t \\ f(x) \text{ else} \end{cases} \tag{12}$$

This transformation gives a grey tone level image which requires a threshold to perform the segmentation. This last transformation has two important properties : segmentation of same class of particles on different background and segmentation of different particles on the same background.

Watershed segmentation

A grey tone image can be considered like a relief. This relief can be segmented in different regions called the catchment basins. If one supposes that it rains on these relief, each region is then delimited by the watersheds. To obtain a segmentation by watershed, several methods can be performed. According to the terminology in image processing, watershed segmentation can be considered as a region-growing algorithm. The earliest methods have been built from homotopic thinning (Beucher, 1983). This is the grey tone level version of the skeleton by influence zone for binary images. The grey tone level thinning starts from the regional or local minima. This first algorithm can be used on analyser system having dedicated morphological hardware increasing the speed of treatment, but it is a very time consuming process on classical computer. This is the reason why other algorithms, using modern methods in computer techniques, have been created (Vincent, 1990; Vincent and Soille, 1991). Their algorithm is based on an immersion process from the local minima in which the flooding of the water in the relief is obtained by using a queue of pixels.

Before to describe the building of catchment basin by immersion (independently of the algorithm), some transformations must be defined. Let *f(x)* be the image function, $Z \in IR^2$ its support and M the maximum of grey tone level. The threshold at level t is defined by :

$$X^{t} = \{x \in Z; f(x) \leq t\} \tag{13}$$

Let $m_t f(x)$ be the local minima and W_t the section of catchment basin at level t. To obtain these catchment basin, we must use the geodesic skeleton by influence zone of W_{t-1} inside X_t. Starting from $W_0 = m_0 f(x)$, we have the recursive relation from t = 0 to M :

$$W_t = SKIZ_{X_t}(W_{t-1}) \bigcup m_t f(x) \; W_t = SKIZ_{X^t}(W_{t-1}) \cup m_t f(x) \tag{14}$$

W_M is the set of catchment basins and the watershed of f(x) is the complementary set of the catchment basins. After that these catchment basins can be labelled by their minima.

This method of segmentation is very powerful and used in many domains of application. For true grey tone level images, watershed segmentation has been used for the analysis of electrophoretic images (Beucher, 1983), for the analysis of road traffic images (Beucher and al., 1990), for the analysis of fracture surface (Beucher, 1990). Generally the watershed algorithm is not performed directly on the initial image but on the filtered one, because it is very sensitive to noise. Another way to eliminate over-segmentation is to use some morphological transformations to connect the neighbouring minima. These transformations can be classical dilation or more complex algorithms like rh-minima (Grimaud, 1991). Often, to perform segmentation the morphological gradient of the image is used. In this last case a pre- treatment is generally performed to avoid over-segmentation.

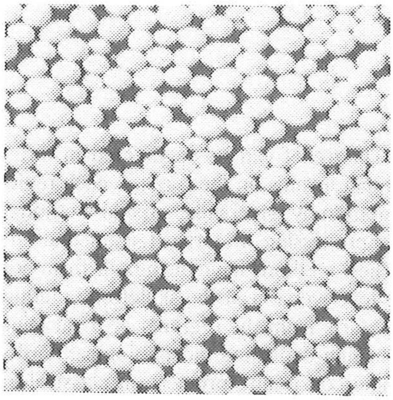

Figure 9 : Initial image.

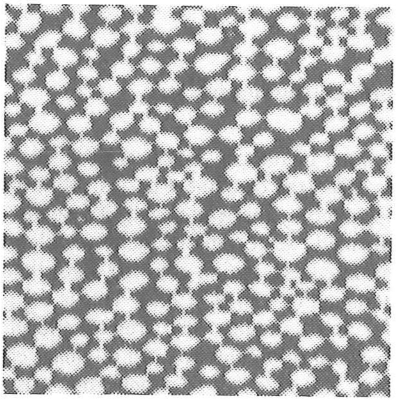

Figure 10 : Distance image on fig. 9.

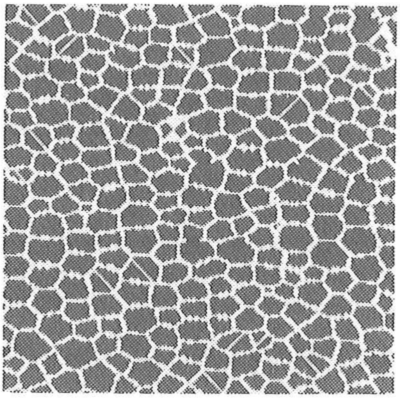

Figure 11 : Watershed of distance image on fig. 10.

The watershed segmentation can also be applied to separate convex particles in set (or binary) images (figures 9 to 11). The process is the following. First, the distance function of the set which must be segmented is performed. By this process, each pixel of the set takes a grey tone value corresponding to the distance of this pixel to the boundary of the set :

$$f(x) = \mathrm{d}(x \in \mathrm{X}, \partial\mathrm{X}) \qquad (15)$$

Then negative image is obtained from the following expression :

$$g(x) = \mathrm{M} - f(x) \qquad (16)$$

Finally, the watershed transformation is performed on *g(x)* to give boundaries between convex particles.

Today, the watershed segmentation replaces the segmentation of binary images based on ultimate erosion and SKIZ (Lantuejoul and Beucher, 1981; Chermant and al., 1981). A more complex version (Gauthier and al., 1993), using multi-mode images, directional morphology (Kurdy, 1990) and rh-minima, was performed to separate polygonal and convex grains in WC-Co system.

ANALYSIS

According to the domain where image treatment and image analysis are used, the last step is either a quantitative description or a decision. In the domain of "statistical" images this last step concerns the quantitative description. To describe quantitatively an image, two approaches are possible. The first one is a passive approach where the description is given by some parameters (often stereological parameters) or by some functions (often size distribution functions). The second one is a more active approach where the description is synthesised by a model (probabilistic models for example). For these two approaches, mathematical morphology can be used.

Stereology and mathematical morphology

The mathematical morphology is very well adapted to estimate stereological parameters. For example, to obtain specific connectivity number of a phase, $N_A(X)$, two hit or miss transformations must be performed. So with a square grid in 8 connectivity the two structuring elements are :

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} \cdot & 1 \\ 1 & 0 \end{bmatrix}$$

So, with $N_A(X)$, it is possible to estimate the integral of mean curvature of the phase X per unit volume, $M_V(X)$, using the following stereometric relationship :

$$M_V(X) = 2\pi N_A(X) \qquad (17)$$

The knowledge of $M_v(X)$ is very useful in sintering investigations with liquid phase since the coarsening of larger particles from smaller ones depends on the difference between their mean curvature according to Gibbs' equation (DeHoff, 1984; Quenec'h, 1991). For this kind of investigations the material (or biological) microstructure is known only inside a mask. The bias of local knowledge introduced by this mask can be corrected by two ways : the first one uses the theorem of mask of measurements (the measure must be performed in an eroded mask) (Serra, 1982), the second one uses shell correction method (Pinnamaneni and al., 1989).

Granulometry and mathematical morphology

The size distribution analysis is an important domain of image analysis. For individual particles, a great number of methods and measures exist without using mathematical morphology. But, mathematical morphology is a tool very well adapted to characterise the granulometry of interconnected media by using opening by a convex structuring element. In this case the size distribution function is given by the general equation :

$$G(\lambda) = \frac{Mes(f(x) \text{ or } X) - Mes(O^{\lambda B} f(x) \text{ or } X)}{Mes(f(x) \text{ or } X)} \tag{18}$$

As indicated by this equation the granulometry can be performed on set X or function *f(x)*. In the case of set analysis, *Mes* is area for bi-dimensionnal structuring element of size λ or length with a segment of size λ. Granulometry by opening is often used in quantitative metallography (Chermant and Coster, 1991). It should be noted that P(l) function, which corresponds to area fraction of eroded set by segment of length l, gives by derivation linear granulometries. For the function, the measure is the volume of the sub-graph of the function (Coster, 1992). Granulometries on grey tone images (by opening or closing) have been used to analyse the texture of ceramic films (Prod'homme and al., 1992), deep etched surfaces of extruded steels (Michelland-Abbé and al., 1992) or fractured surfaces of steels (Michelland-Abbé and al., 1991).

Modelling and mathematical morphology

If it is possible, the modelling is the more synthetic way to describe quantitatively the analysed structures. Probabilistic models are good tools to describe quantitatively the statistic images obtained from metallographic sections or others ways (Jeulin, 1991;1992). These models are characterised by functionnals called Chocquet's capacities depending on compact K. These functionnals are known analytically for several models like Boolean models, dead leaves models or mosaic model. By mathematical morphology, it is possible to estimate the Choquet's capacities T(K) because they correspond to a measure on eroded set by compact K according to their definition :

$$T(K) = 1 - Q(K) = P(K \cap X \neq \emptyset) = 1 - P(K \subset X^c) \tag{19}$$

These probabilistic models have been tested with success on sintered materials (Quenec'h and al., 1992, 1993) and on powder materials (Jeulin and Terol Villalobos, 1992).

CONCLUSION

Mathematical morphology can be used during all steps of image processing and image analysis. It is a very convenient tool for statistical images since after filtering, segmentation and measures, it is possible to describe the images by a model .

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