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MULTISCALING AND MULTIFRACTALITY IN IMAGE ANALYSIS

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ABSTRACT

This paper contains a contribution to the problem of the multifractal characterisation of complex structures arising in image analysis. The early spinodal decomposition of Cahn (Cahn,1965) has been used to illustrate the approach. This model, resulting from a purely random process, contains a characteristic length scale λ and is by construction a non (multi)-fractal object . It is shown that a blind use of the box counting method may lead to spurious multifractal curve, while the scaling of the information entropy provides a method to characterize the morphology of the model.

Keywords : morphology, heterogeneous materials,multiscaling,multifractal.

INTRODUCTION

Recently a number of authors have used the concepts of multifractal distribution to attempt to characterize complex structures by image analysis (Hansen et al.,1988; Blacher et al.,1991; Blacher et al.,1993). As some of these authors have noted, this method which is used to quantify the morphological differences of samples resulting from different methods of preparation or exhibiting different physical or geometrical properties, suffers from a number of practical difficulties. In most cases, the fractal and generalized dimension can be defined in a limited scale range. In these regions the "experimental points" in the log-log plots which determine the fractal and generalized dimension often present some fluctuations or deviations with respect to a straight line and it is sometimes difficult to be sure if the observed fractality is genuine or due to size effects, transient corrections to the power law asymptotic behaviour or some crossover effects. In order to discuss quantitatively this problem we have considered a model of early spinodal decomposition (Cahn,1965) used recently to simulate the properties of microemulsions (Berk ,1987) and porous media (Bradley et al.,1990). This model, which results from a purely random process and presents a complex structure, contains a characteristic length scale λ which in real materials can be determined by neutron diffraction. The scaling properties of this system,which is by construction a non (multi)-fractal object, have been studied using the classical box counting method. This example illustrates how a blind use of the box counting may lead to a scale dependent multifractal curve, and how the more general scaling of the information entropy in the crossover region around λ can provide a method to characterize the morphology of the model.

THE CAHN MODEL

In the theory of spinodal decomposition (Cahn,1965), a dynamic nonlinear equation is obtained by assuming that the time variation of the concentration c(**r**,t) (the relative amount of one phase to the other) is determined by minimisation of the local free energy. To simulate the morphology of a two-phase system during the earlier stages of spinodal decomposition, where c(**r**,t) remains smooth and has no big jumps, Cahn associates the interface between the two components with a level set of a random standing wave $S_N(\mathbf{r})$, composed of N sinusoids having fixed wavelength λ, but random directions $\mathbf{k}_n$, phase ϕ_n, and amplitudes A_n:

$$S_N(\mathbf{r}) = \frac{1}{(N<A^2>)^{\frac{1}{2}}} \sum_{n=1}^{N} A_n \cos(k\hat{\mathbf{k}}_n.\mathbf{r}+\phi_n), \qquad (1)$$

where $k=2\pi/\lambda$ and $<A^2>$ is the mean square sinusoidal amplitude. For an isometric partition, a two-phase interface coincides with the zero set of $S_N(\mathbf{r})$, since over a large space (1) is positive as often as it is negative. In real systems, like microemulsions and porous media, there exists on the one hand experimental evidence that morphology corresponds to a disordered bicontinuous structure and on the other hand a prominent scattering peak indicates the existence of a characteristic length scale in the system. The gaussian random process (1) generates a structure with a sharp k value which would give rise to a non realistic sharp scattering peak. Then, a distribution of k values was introduced in (1), the distribution law and the dispersion parameters can be obtained from scattering measurements.

Two-dimensional images of 512x512 pixels were built using Eq. (1) with N=200 terms and the value of k was chosen to simulate the three different morphologies characterized by: a) a fixed wave number, b) wave number values taken from a gaussian distribution, c) wave number values taken from a log-normal distribution. In all cases the directions $\mathbf{k}_n$ and the phases ϕ_n were chosen uniformly at random and the resulting values of $S_N(\mathbf{r})$ were used to generate one of the two components (level 0 or 1) at each position **r** in an isometric partition. To investigate the scaling properties, we built a set of successive enlargements of images simulating microscopic views of the same morphology. This can be achieved by constructing successive images all of linear size L but with increasing characteristic size λ (decreasing k). Figures 1 and 2 show two image sets corresponding to cases a) and c), obtained using fixed wave numbers k=0.05 (λ=126 pixels), 0.1 (λ=63 pixels), 0.3 (λ=21 pixels), 0.5 (λ=13 pixels), 0.8 (λ=8 pixels), 1.0 (λ=6 pixels) respectively and wave numbers taken from a log-normal distribution with mean values k=0.05, 0.1, 0.3, 0.5, 0.8, 1.0 and dispersion σ=0.05. A simple inspection of these figures shows that the structure of images becomes more and more complicated from the case a) to the case c). In the last case, as can be seen from the successive enlargements, the structure is completely inhomogeneous.

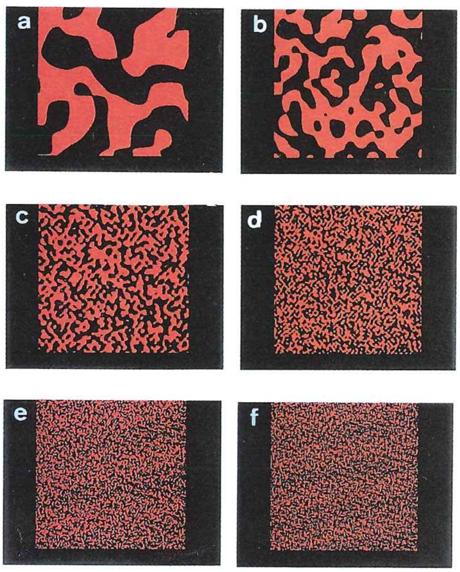

Figure 1
Binary images obtained using the Cahn algorithm with fixed wave numbers.

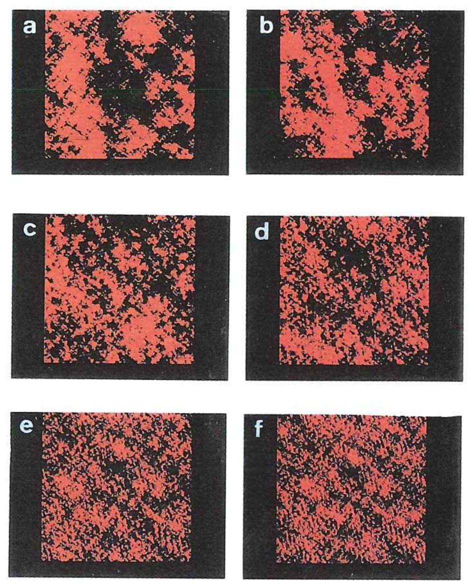

Figure 2
Binary images obtained using the Cahn algorithm with a log-normal distribution of wave number.

SCALING PROPERTIES AND UNIVERSAL CURVES

The box counting technique is applied to the analysis of the scaling properties of the morphologies created by the method described in the two previous sections. The 2-d plane was covered with N boxes of side $\delta = l/L$, where l is the linear size (calculated in pixels) of the boxes and L the linear size (calculated in pixels) of the whole image. Let p_i denote the proportion of the total mass of the object inside the i-th box determined by counting the number of black or white pixels in the box and $\mu_i(q,\delta)=(p_i(\delta))^q/\Sigma(p_j(\delta))^q$ the normalized q-th powers of the probabilities p_i for the boxes of size δ .The Hausdorff dimension F(q) of the measure $\mu(q)$ is defined as (Chhabra et al, 1989),

$$F(\delta,q)=\frac{S(\delta,q)}{\log \delta} \qquad \text{where} \qquad S(\delta,q)=-\sum_i \mu_i(\delta,q) \log \mu_i(\delta,q),$$

then,

$$\langle F(q)\rangle = \lim_{\delta \to 0} F(\delta,q) \tag{2}$$

For q=1, $S(\delta,q)$ is the usual Shannon entropy for the probabilities p_i at the scale δ. For $q\neq 1$ the $S(\delta,q)$ can be considered as the entropies corresponding to the probabilities $\mu_i(q)$. If $S(\delta,q)$ vs log δ has a slope equal to $D\neq 2$ for all values of q in a given length region the system is said to be fractal with dimension D in that length region. If the slopes are well defined and different for all q in a given length region, the system is said multifractal and the geometrical structure is characterized by a spectrum of dimension $F(q) \leq D$. If F(q) varies with the scales, the system is not multifractal and one can only hope that this variation can provide information on the underlying morphology.

The box counting analysis was made on each image set (Figures 1a-f and 2a-f), with boxes of linear dimension $\delta = l/L$ where L = 512 pixels is the linear size of the image and l is the linear size of the boxes ranging from 4 to 512 pixels. In a statistical sense, each image set corresponds by construction to the same object at different resolutions, so that it is possible to rescale the results obtained on each image to obtain a universal curve. The rescaling follows from referring the size of boxes not to the fixed linear size of the window (L) in which the image is drawn but to the characteristic length λ of the object :

$$S'(\delta',q)=S(\delta,q) +2 \log (\lambda/L)$$

where $\delta'=l/\lambda$ and $\delta=l/L$; and

$$F'(\delta',q)=S'(\delta',q) / \log \delta' \tag{3}$$

The superimposed curves are shown in Figure 3a .The length of the scaling region (where the slope in the log-log plot is linear) strongly depends on λ and q . For $\delta' >> l/\lambda$, a power law scaling with an euclidean dimension d = F(q) = 2 is observed for all q. The same occurs for

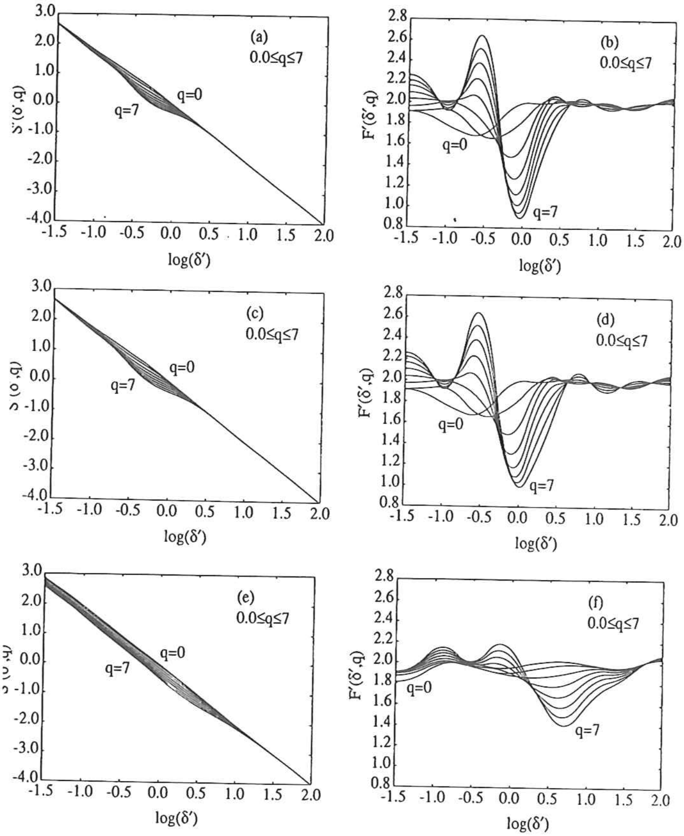

Figure 3 : Universal curves S'(δ',q) and Hausdorff estimates F'(δ',q) plotted against log δ' for 0≤q≤7, for a morphology characterised by (a,b) a fixed wave number, (c,d) a gaussian distribution of wave numbers, and (e,f) a log-normal distribution of wave numbers.

δ' <<*l*/λ. In the first case, one sees the image as a homogeneous grey object. In the second case, one counts half of the pixels i.e the white ones. Between these two situations, there is a crossover region. In Figure 3b we have drawn the variation of F'(δ',q) as a function of log (δ').The interpolation method is performed to obtain the best fit on the crossover region. As q increases, the minimum of F'(q) becomes more and more pronounced and its position converges asymptotically to log δ'= 0. For q>2, the maximum is followed by a minimum which reflects the narrowing of the crossover region as q increases. To investigate the influence of the morphology of the model on the entropy S(δ,q) in the crossover region around *l*, he same analysis was performed on images constructed using a gaussian distribution (case b) and a log-normal distribution (case c , Figures 2a-f) of wave number values.k Figures 3c-d show the results obtained for the case b. As expected, the curves are almost the same except for a spreading of the crossover region as expected are obtained. This phenomenon is even more pronounced in the case c, Figures 3e-f, where the crossover region stretches over the whole curve. For small δ', because of the properties of the log-normal distribution, the morphology appears to be almost scale independent. The variation of the F'(q,δ') is smoother and the less pronounced minimum appears for a value of δ' corresponding to the maximum of the log-normal distribution.

CONCLUSIONS

The fractal analysis of morphologies induced by the Cahn algorithm of spinodal decomposition and discussed in the previous section leads to the following conclusions:

1)To find a power law scaling for all moments of the probability distribution, one has to start with boxes of linear size more than an order of magnitude larger than λ.

2)In morphologies organised around a characteristic or a dominant wavelength, the range of the crossover region might be wide (one or several orders of magnitude) and multifractal analysis may yield misleading results. In Figure 3a a linear fit of the values of S' from δ'=1 upwards or from δ'=0.5 upwards would give rise, to different (scale dependent) false multifractal curves F(q).

3)In the crossover region there is no meaningful linear fit. This is confirmed if one plots the quantity F'(q) as a function of δ' (Figures 3b,d).

4)The position of the minimum of F'(q) tends to the value of dominant wavelength as q increases. This quantity can be determined by an image analysis statistical technique using the box counting method. More generally, the variation of F(q) with δ can be considered as a characteristic of the morphology.

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