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MORPHOLOGICAL TRANSFORMATIONS ON A RANDOMLY FILLED 3D NETWORK

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ABSTRACT

The nodes of a 3D cubic face-centred network, simulated on a computer, are randomly filled with points up to a given density. The structure obtained is then modified by two morphological transformations : dilation or closing. Two parameters can be adjusted : the density of points of the initial structure and the size of the transformation. The progressive filling of the space by transformations of increasing size is then followed by the Euler-Poincaré characteristic measured in spaces $\mathbb{R}^0$ to $\mathbb{R}^3$.

Keywords : Euler-Poincaré characteristic, 3D morphological transformations, 3D image analysis, mathematical morphology.

INTRODUCTION

Some basic quantitative parameters of image analysis (volumic fraction , specific surface , integral of mean curvature, gaussian curvature) can be obtained from the Euler-Poincaré characteristic (E.P.C.)(Serra, 1982). The densification of a material is conveniently described by these parameters, but they are usually attainable within a limited range. Moreover, is there a general evolution, whatever the densification process? At first, 3D structures of points of increasing densities were simulated on a computer (Euler-Poincaré characteristic of a randomly filled 3D network, submitted to Journal of Microscopy). Each structure was built up from a given number of points randomly placed, with equal probabilities, on the nodes of a (200 x 200 x 200) cubic face-centred (C.F.C.) network. The density of points was adjusted to cover the whole compacity range 0 - 1. Then, the specific values of the E.P.C. in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ were followed as a function of the compacity (i.e. N_L, N_A and N_V as a function of N_P).

In this paper, the preceding structures are densified by two basic transformations of mathematical morphology : dilation or closing. The effects of these transformations on the behavior of the E.P.C. are studied as a function of the compacity.

DILATION

For each structure, the initial set X corresponds to a C.F.C. network of size (100 x 100 x 100) randomly filled by points up to a given density. Then, the set X is dilated by a 3D elementary structuring element : a cuboctahedron of size one, B, corresponding to a point of the C.F.C. network with its twelve neighbours. When B is moved everywhere on the network, the dilated set, D1, is defined as the locus of the centres x of B where B hits the set X (Serra, 1982) and :

$$D1 = X \oplus B = \{x / B_x \cap X \neq \phi\}$$

This operation can be repeated n times to get the set Dn, the results of n successive dilations of size 1 or one dilation of size n being identical. Finally, the field is eroded n times in order to eliminate the border effects induced by the n dilations and the values of the E.P.C. are measured on the eroded field using the shell correction (local analysis) (Bhanu Prasad et al., 1989, 1990).

Thus, starting from a low density of points, a dilated structure, Dn, is obtained after $\underline{n}$ dilations of the points and the local value of the E.P.C. in $\mathbb{R}^0$, $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ is measured. Then, the same process is repeated with a progressive increase of the initial density of points so as to go up to a fully dense structure after dilation.

At first, a comparison is made, in figure 1, between the initial structures of points (D0) and the structures corresponding to a dilation of points by a cuboctahedron of size 1 (D1). The shapes of the curves are roughly the same for D0 and D1. However, the curves relative to D1 are no longer symmetrical and their amplitudes are lower than the ones for D0. Moreover, a shift towards the high values of the compacity is observed for the curves 1a, 1b and 1c. Then, in figure 2, the influence of the dilation size (1 to 3) on the E.P.C. curves can be observed. The effects already noticed for the D1 curves are amplified when the dilation size is increased. The decrease of the amplitude of the curves being so large, a comparison between D0 and all the dilated structures could not be made on the same graph.

The shapes of the curves 1(a,b,c) and 2(a,b,c) are identical to those obtained for an Euclidean Boolean schema (from equations given by Miles, 1976). Nevertheless, the extrema observed here are lower for the dilated digitized structures than for the Euclidean Boolean schema of corresponding size and the transitions through zero (1b,1c and 2b,2c) do not occur for the same values of the compacity. Moreover, for these transitions, a shift towards the right is observed for the dilated structures as the dilation size increases (2b,2c). This behavior is not observed for an Euclidean Boolean schema but is in agreement with the remarks of Jeulin (1991) in the case of a digitized Boolean schema.

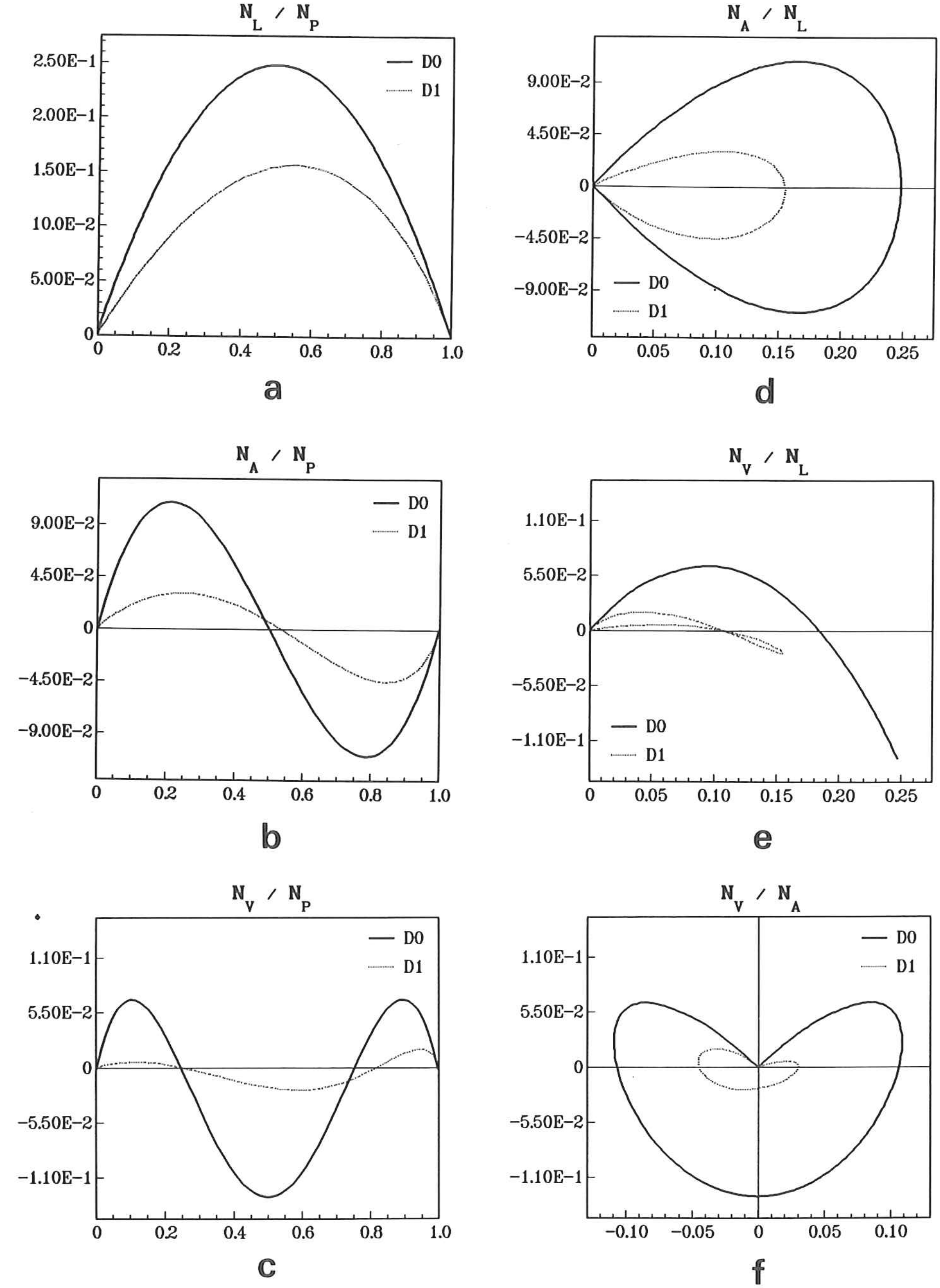

Figure 1 : Comparison between the initial structures D0 (points randomly placed on the C.F.C. network) and the dilated structures D1.
(a)(b)(c) : E.P.C. in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the compacity.
(d)(e) : E.P.C. in $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^1$.
(f) : E.P.C. in $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^2$.

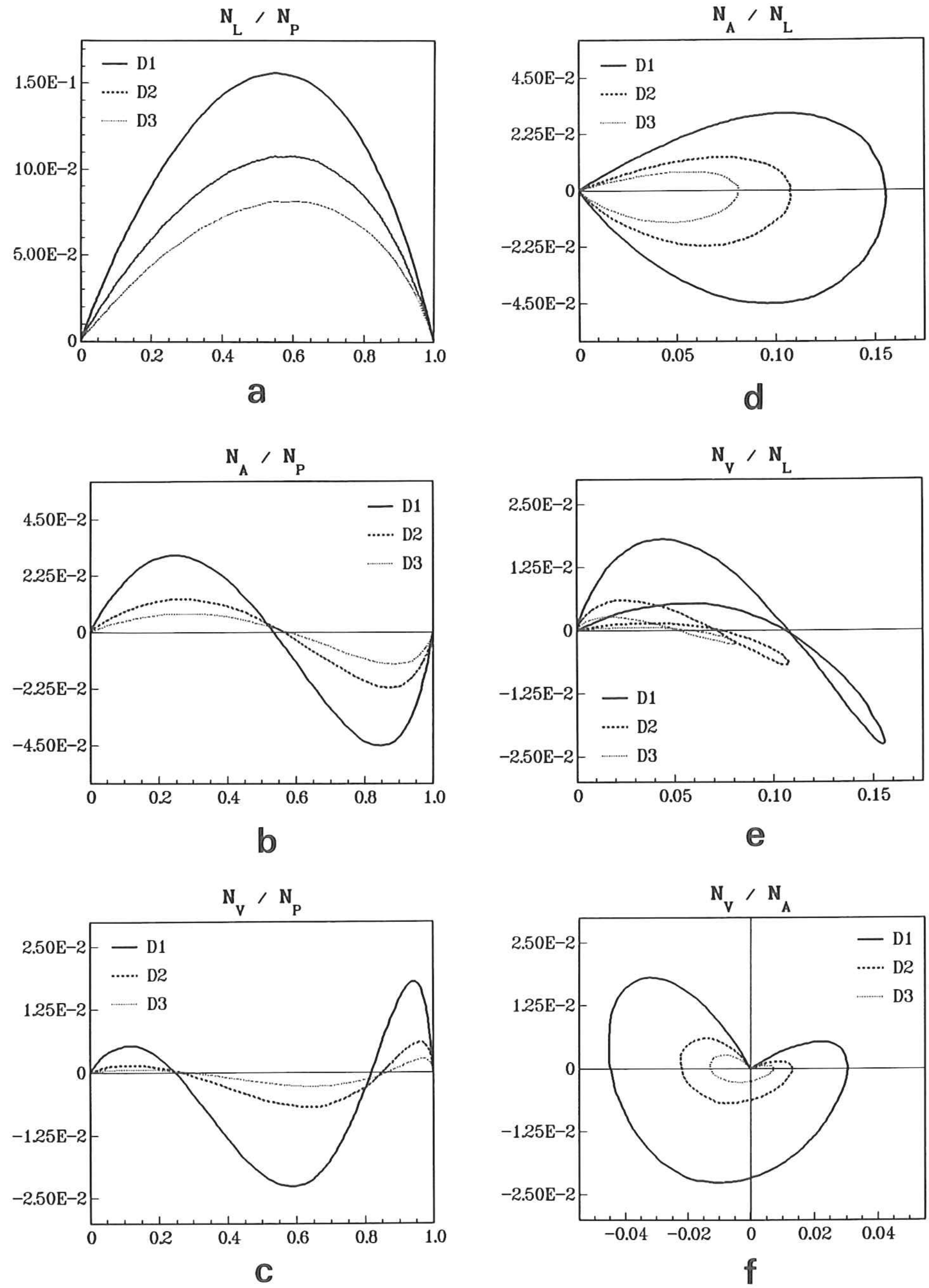

Figure 2 : Evolution of the E.P.C. with dilations of increasing size.
(a)(b)(c) : E.P.C. in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the compacity.
(d)(e) : E.P.C. in $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^1$.
(f) : E.P.C. in $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^2$.

CLOSING

Up to now, the initial set X was densified by dilation. Another morphological transformation is now applied : the closing. Similarly to a dilated set, an eroded set is defined as the locus of the centres x of B where $B \subset X$ (Serra, 1982). The closing is the succession of a dilation and an erosion using the same structuring element. For a structuring element of size one :

$$F1 = (X \oplus B) \ominus B$$

and for a structuring element of size n :

$$Fn = (X \oplus nB) \ominus nB$$

One can see that, contrarily to the dilation, the result of n closings of size 1 is not identical to the result of one closing of size n.

Starting again from a low density of points, the same process as for the dilated structures is applied to obtain the closed structures. In figure 3, the initial structures of points (F0 identical to D0) are compared to the closed structures F1. Then, the effects of the closing size on the E.P.C. curves can be observed in figure 4. The shapes of the curves are globally the same as for the preceding ones. Again, the symmetry of the initial curves disappears by closing and the amplitude decreases. Nevertheless, a shift towards the low values of the compacity is now observed for the curves (fig.3a, 3b and 3c). These effects are amplified when the closing size is increased (fig.4).

Some additional comments can be made on the very beginning of the densification. For the initial set X (or D0,F0), even at a very low compacity ($\simeq 10^{-4}$), a few points are already connected. Thus, for this range of compacity, a new point added may be isolated (N_V increases) or connected to the previous ones (N_V remains unchanged or decreases). The competition between these two phenomena leads to an equilibrium corresponding to the first maximum of N_V. After this maximum, each new point has more chances to be connected to the previous points than to be isolated and N_V decreases. The dilation of the points accentuates their possibilities of connections and the first maximum of N_V is then lower than for the initial set (fig.1). The closing of the points leads to an intermediate case : some points connected by dilation are disconnected by the following erosion. For very low concentrations of points, most of the points remain disconnected after closing and the closing curves (Fn) are identical to the initial curves (F0). This behavior is observed within a range depending on the closing size (fig.3 and fig.4) : the larger the dilation, the more difficult the disconnections.

CONCLUDING REMARKS

We have seen successively a progressive filling of the space by dilations or closings of increasing size. The differences between these two morphological transformations are pointed out in a comparison between D1 and F1 as represented in figure 5 (a,b,c).

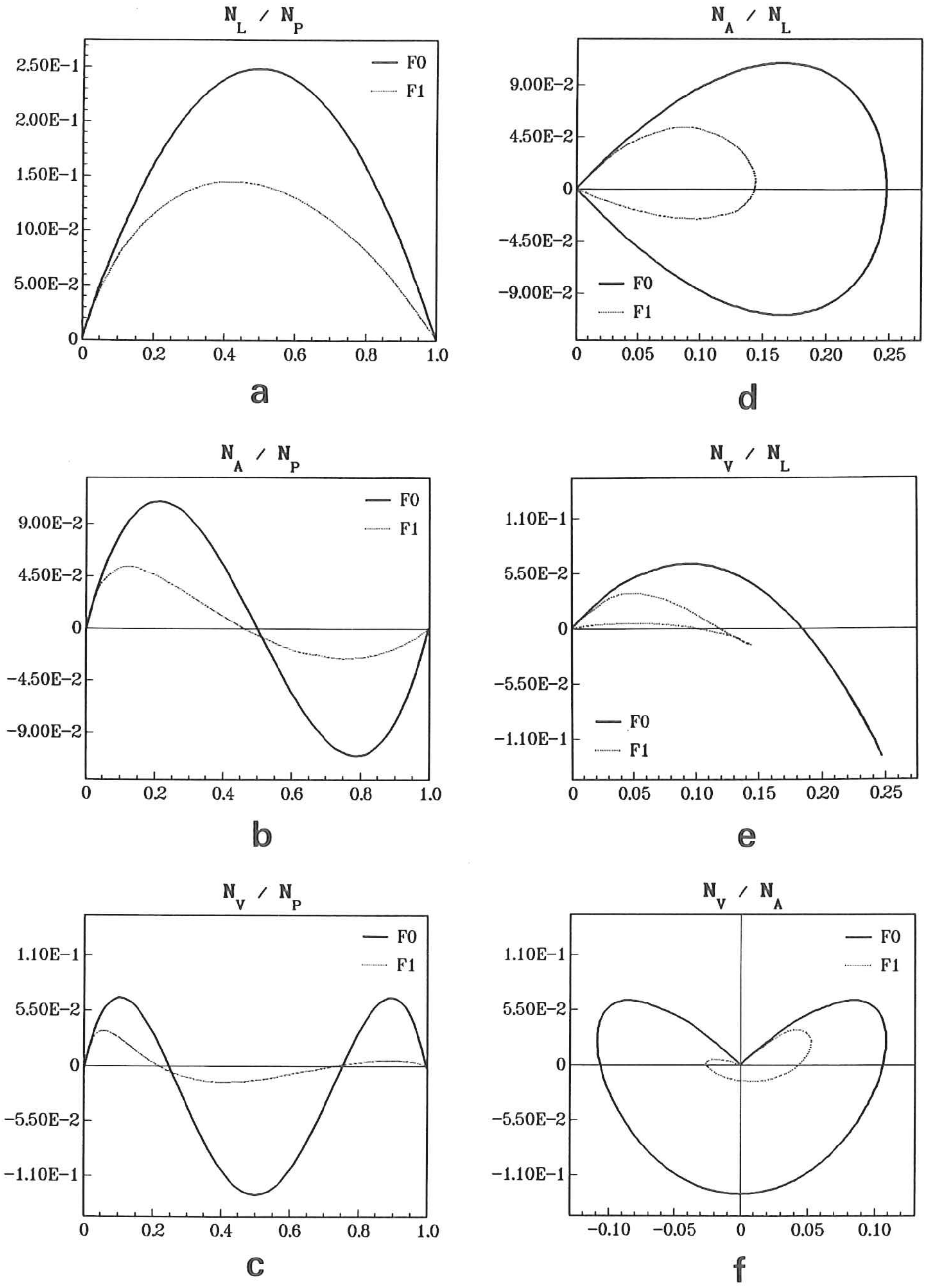

Figure 3 : Comparison between the initial structures F0 (points randomly placed on the C.F.C. network) and the closed structures F1.
(a)(b)(c) : E.P.C. in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the compacity.
(d)(e) : E.P.C. in $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^1$.
(f) : E.P.C. in $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^2$.

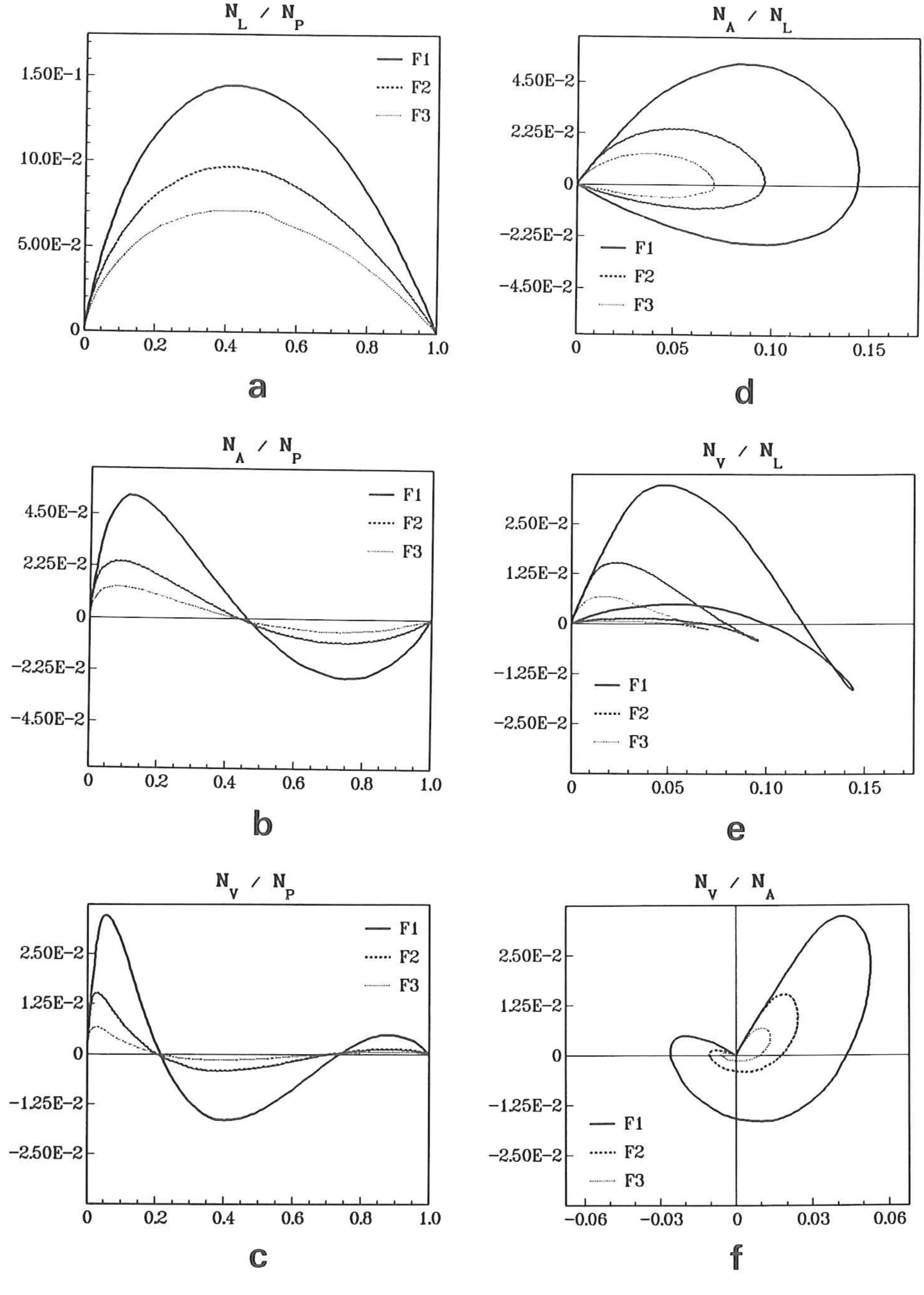

Figure 4 : Evolution of the E.P.C. with closings of increasing size. (a)(b)(c) : E.P.C. in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the compacity. (d)(e) : E.P.C. in $\mathbb{R}^2$ and $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^1$. (f) : E.P.C. in $\mathbb{R}^3$ as a function of the E.P.C. in $\mathbb{R}^2$.

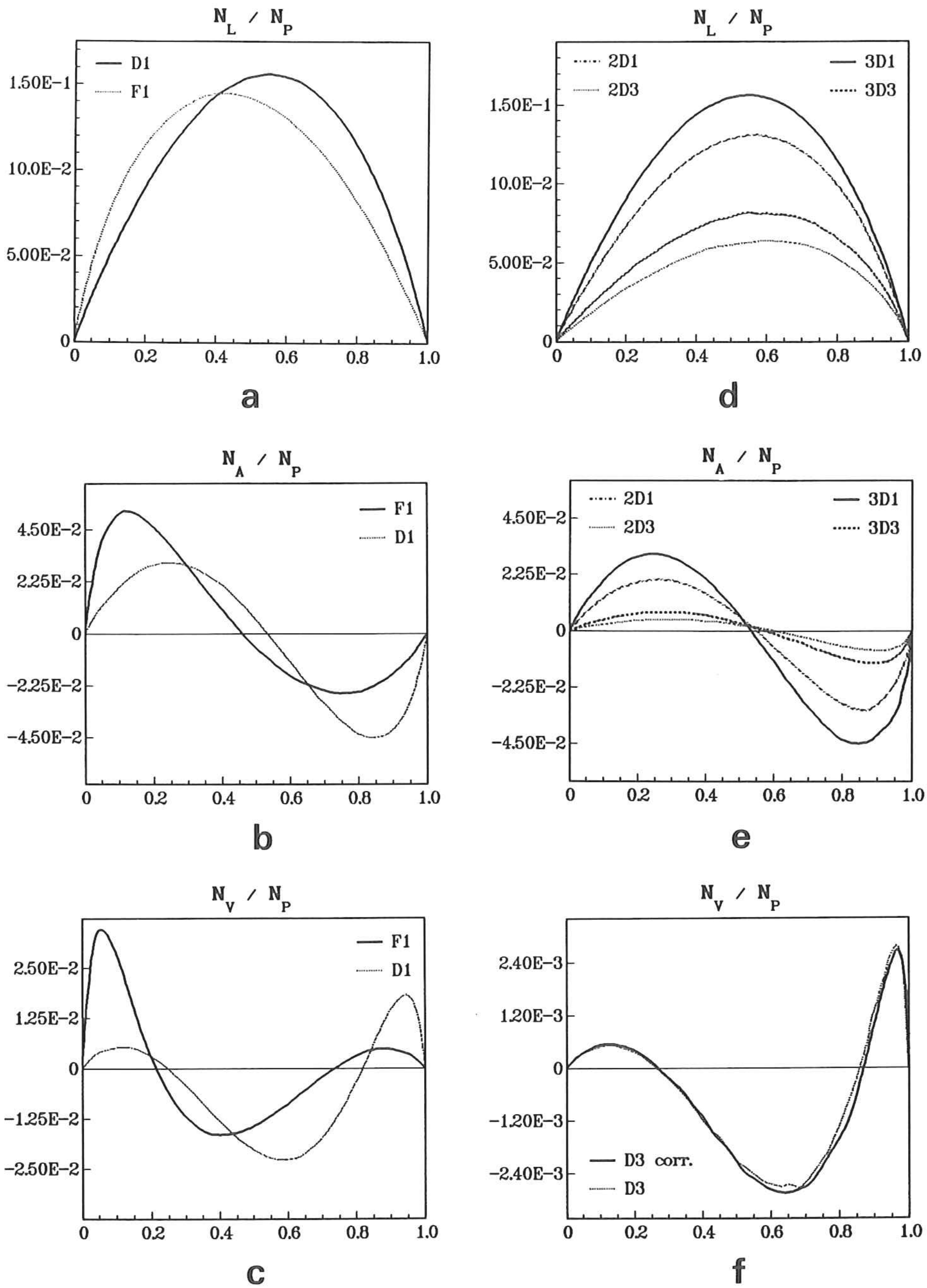

Figure 5 : Compared effects of dilation and closing on the E.P.C. in $\mathbb{R}^1$(a), $\mathbb{R}^2$(b) and $\mathbb{R}^3$(c). Comparison between the dilations in 2D space and the dilations in 3D space (d)(e). Effect of the shell correction on the E.P.C. in $\mathbb{R}^3$(f).

All the transformations have been made in 3D space. Would the curves have been similar if the transformations had been carried out in 2D space? The answer is no but the shape of the curves is similar as can be seen in figures 5d and 5e, the amplitude of the curves being lower in 2D space than in 3D space.

The preceding curves (fig. 1 to 4) were obtained from structures built inside (100 x 100 x 100) fields. The choice of the field's size results from a compromise between good precision of the measurements and reasonable computing time. Nevertheless, a smaller size could have been used as can be seen in table 1 where some results are reported as a function of the field size for D1 and D3. The good stability of these results is in accordance with the fact that the shell correction has a minor influence on the measurements (fig. 5f), the volume of the field being large enough to take into account a representative part of the structure.

Table 1 : E.P.C. of dilated stuctures for different field sizes

Field size	N_P	$10^3 N_L$	$10^3 N_A$	$10^3 N_V$
Initial compacity 0.055, Dilation of size 1				
50 X 50 X 50	0.5197	155.27	2.94	- 21.78
	0.5218	153.02	2.07	- 21.39
	0.5194	153.48	2.36	- 21.46
100 X 100 X 100	0.5206	155.12	2.70	- 21.39
	0.5201	154.66	2.85	- 21.38
	0.5213	155.28	2.30	- 21.39
200 X 200 X 200	0.5208	155.85	2.38	- 21.80
	0.5205	155.92	2.61	- 21.74
	0.5208	155.85	2.39	- 21.80
Initial compacity 0.005, Dilation of size 3				
50 X 50 X 50	0.5185	79.01	2.32	- 2.27
	0.5227	81.71	2.70	- 2.19
	0.5351	81.10	1.57	- 2.54
100 X 100 X 100	0.5216	80.58	2.20	- 2.28
	0.5234	79.48	1.96	- 2.32
	0.5252	80.53	2.17	- 2.32
200 X 200 X 200	0.5204	80.31	2.21	- 2.26
	0.5201	80.59	2.33	- 2.24
	0.5226	80.52	2.21	- 2.28

Another remark can be made : these results were obtained on a C.F.C. grid. Another type of grid can be used : for example, a square grid in 2D space or a simple cubic one in 3D space. In these cases, the curves are no longer symmetrical for the structures of points (Jeulin,1991) but their general shape is preserved. After dilation or closing, the modifications should then be qualitatively the same.

Finally, one kind of space filling has been studied : starting from different initial concentrations of points, dilated or closed structures have been obtained using a structuring element of a given size. There is another simple way to fill the space, using the same transformations : starting from one low initial concentration of points, dilations or closings using structuring elements of increasing size can be performed. What then are the paths followed by the successive transformations of the same initial set ?

REFERENCES

Bhanu-Prasad P, Lantuejoul C, Jernot JP, Chermant JL. Unbiased estimation of the Euler-Poincaré characteristic. Acta Stereologica 1989 ; 8 : 101-106.

Bhanu-Prasad P, Lantuejoul C, Jernot JP. Use of the shell correction method for quantification of three dimensional digitized images. Transactions of the Royal Microscopical Society. 1990 ; Vol.I : 297-300.

Jeulin D. Modèles morphologiques de structures aléatoires et de changement d'échelle. Thèse de Doctorat ès Sciences Physiques. Université de Caen. 1991.

Miles RE. Estimating aggregate and overall characteristics from thick sections by transmission microscopy. Journal of Microscopy 1976 ; 107 pt 3 : 227-233.

Serra J. Image analysis and mathematical morphology. London : Academic Press, 1982.

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