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GENERALIZED FOURIER DESCRIPTORS FOR SHAPE ANALYSIS OF 3-DIMENSIONAL CLOSED CURVES

Tamás RÉTI and Imre CZINEGE
Bánki Donát Polytechnic
H-1081 Budapest, Népszinház u. 8, Hungary

ABSTRACT

A new method for shape characterization of 3-D space curves is presented. The proposed procedure based on the Fourier analysis techniques can be regarded as a generalization and further development of methods described in the literature. Its advantages are its simplicity and its ability to describe the shape of any closed space curves regardless of their nature. The space curve is parametrised by its arc length and characterized by a set of 3-D vector valued "shape functions". The shape function is unambiguously defined for any closed space curve, and contains the complete 3-D information on the curve. The three components of the shape function (called partial shape functions) are periodic with the period 2Π, and can be expanded in Fourier series. Starting with the appropriately selected partial shape functions, shape descriptors generated from Fourier coefficients are defined for shape evaluation. They are invariant under translation, rotation and dilation. In order to verify the validity of the computational model and to analyse the efficiency of the proposed procedure, experimental study has been performed using different test curves.

Keywords: form, Fourier, morphology, shape, space curve.

INTRODUCTION

In recent years 3-D shape analysis using computers has become an increasingly important research area due to its practical applications in fields such as object identification, quantitative assessment of microstructural features (Exner, 1987; Flook, 1987; Chong-Huah Lo and Hon-Son Don, 1989). In this paper, a new approach employing generalised Fourier-descriptors is proposed for shape analysis of 3-dimensional curves. The method reported here relies on the concept of the "generalised Fourier analysis" published earlier (Réti and Czinege, 1989) and can be considered as a possible extension of the techniques proposed for the shape description of closed plane curves. After summarizing the theoretical bases of the suggested method, the computation of Fourier descriptors, which are translational, scaling and rotational invariant, is outlined. Finally, some results of preliminary experiments performed to verify the validity of the proposed method are presented.

SPACE CURVE REPRESENTATION USING VECTOR-VALUED SHAPE FUNCTIONS

Let A be a continuous, piecewise-smooth closed space curve that is parametrised by its arc length parameter $0 \le s \le P$, where P is the total arc length (the perimeter) of the closed curve. Without loss of generality, we can assume that the perimeter is normalised to 2π, that is $P = 2\pi$. In what follows, we suppose that space curve A described by a vector-value function $r_A(s)$ is located in a cartesian coordinate system so that its centre of mass coincides with the origin O.

Based on the general method published earlier, for the shape characterisation of space curve A a set $\mathbb{M}_A$ of so-called "shape functions" is introduced (Réti and Czinege, 1989). A shape function $U_A(s) \in \mathbb{M}_A$ represented as

$$U_A(s) = [U_{A,1}(s),\ U_{A,2}(s),\ U_{A,3}(s)]^T$$

is a piecewise continuous, vector-valued, 3-component function with scalar variable s. The set $\mathbb{M}_A$ of shape functions is defined and constructed in a way that satisfies the following conditions:

(a) The set $\mathbb{M}_A$ of shape functions belonging to the space curve A is invariant under translation, dilation and rotation of the curve A.

(b) The components $U_{A,k}(s)$ $(k = 1,2,3)$ of the shape function $U_A(s)$ are square integrable real functions defined in the interval $(-\infty,+\infty)$ and are periodic with period 2π. The components $U_{A,k}(s)$, called "partial shape functions", can be expanded in a Fourier series.

(c) If the shape functions $U_A(s) \in \mathbb{M}_A$ then from this it follows that any shape function of the form

$$U_A(\varepsilon\, s + s_x)$$

belongs to the same set $\mathbb{M}_A$, where $s_x \in [0, 2\pi)$ and ε is equal to +1 or -1. As can be seen, this relation is a valid equivalence relation (reflexive, symmetric and transitive) and may be used to partition the set of space curves into equivalence classes.

(d) Let us suppose that the space curve A_b is a reflected and rotated version of A_a. In this case, for the kth components $U_{A,k,a}$ and $U_{A,k,b}$ of the corresponding shape functions $U_{A,a}(s)$ and $U_{A,k,b}(s)$, the relation

$$U_{A,k,b}(s) = E_k\ U_{A,k,a}(\varepsilon_k s + s_y)$$

where $s_y \in [0, 2\pi)$ and the value of integers E_k and ε_k are +1 or -1 for k = 1,2,3 is valid. Starting with the Fourier coefficients $a_{m,k}$, $b_{m,k}$, $a_{m,n}$ and $b_{m,n}$ (m = 0, 1, 2,...) obtained from the kth and nth shape function components, the following shape descriptors can be defined

$$W^{(k,n)}_{A,m} = a_{m,k}a_{m,n} + b_{m,k}b_{m,k} \qquad (1.1)$$

$$Z^{(k,n)}_{A,m} = a_{m,k}b_{m,n} - b_{m,k}a_{m,n} \qquad (1.2)$$

It should be pointed out that they are not independent of each other, that is they satisfy the following condition

$$W^{(k,k)}_{A,m} W^{(n,n)}_{A,m} = [W^{(k,n)}_{A,m}]^2 + [Z^{(k,n)}_{A,m}]^2$$

The properties of the resulting Fourier descriptors $W^{(k,n)}_{A,m}$ and $Z^{(k,n)}_{A,m}$ (m = 1,2, ..) may be summarised as follows:

a/ The shape descriptors introduced are invariant to translation and rotation of the space curve.

b/ A fundamental property is the *shift invariance*. This means that the value of shape descriptors depends only on $\mathbb{M}_A$ and is independent of the particular choice of the offset parameter s_x, which characterises the starting point of the parametrisation.

c/ The shape descriptors $W^{(k,k)}_{A,m}$ and the absolute value of $W^{(k,n)}_{A,m}$ and $Z^{(k,n)}_{A,m}$ are independent of the selection of the parameters E_k and ε_k.

d/ If a component $U_{A,k}(s)$ of the shape function $U_A(s)$ belonging to $\mathbb{M}_A$ is periodic with period $S_J = 2\pi/J$, (J = 2,3,...), then

$$W^{(k,k)}_{A,m} \ldots = Z^{(k,n)}_{A,m} \ldots = 0$$

for $m \neq 0 (\mathrm{mod}\ J)$.

e/ It follows from the known property of the Fourier coefficients that the shape descriptors tend to zero with increasing m.

DEFINITION OF PARTIAL SHAPE FUNCTIONS USED FOR SPACE CURVE DESCRIPTION

In order to characterize the shape of an arbitrary space curve, we introduce a three-component shape function $U_A(s)$ represented as

$$U_A(s) = [R_A(s), G_A(s), H_A(s)]^T$$

The basic property of the partial shape functions is that they are uniquely determined by the shape of the space curve, and allow a description of the shape, but this description depends upon the particular choice of the initial point on the curve and the "direction of the curve parametrisation".

Taking into consideration the requirements mentioned above, the three partial shape functions $U_{A,1}(s)=R_A(s)$, $U_{A,2}(s)=G_A(s)$, and $U_{A,3}(s)=H_A(s)$ which are referred to as the "distance function", the "cumulative area function" and the "pseudo-torsion function", respectively can be generated as follows:

i. The *distance function* is based on the following considerations: Let the point O be the centre of mass of the space curve, and let P be an arbitrary point of the curve A determined by the arc length parameter s. The partial shape function $R_A(s)$ is defined by the distance OP. It is obvious that $R_A(s)$ is periodic with period 2Π.

ii. For constructing the *cumulative area function*", as a first step, consider the "vector area" of a closed space curve represented by $r_A(s)$. It is known from the vector analysis, that the vector area F_A of a closed curve A is obtained as $F_A = f_A(2\Pi)$ where the vector-valued function $f_A(s)$ is defined by

$$f_A(s) = \frac{1}{2}\int_0^s [r_A(t) \times \dot{r}_A(t)]dt \qquad (2)$$

and s stands for the arc length along the space curve measured from an arbitrary initial point on the curve A. It should be noted here that the derivative of $r_A(s)$ is supposed to be a piecewise continuous function.

As a second step, let us define a unit vector e_A, which characterises the position of the curve A in 3-D Euclidean space. It is assumed that the unit vector e_A, called the "reference vector" is determined unambiguously by the space curve A.

Starting from the relationship expressed in Eq. (2), the *cumulative area function* $G_A(s)$ suitable for space curve characterisation can be defined as

$$G_A(s) = g_A(s) - \frac{g_A(2\Pi)}{2\Pi} s - g_{A,0} \qquad (3)$$

where

$$g_A(s) = e_A f_A(s)$$

and

$$g_{A,0} = \frac{1}{2\Pi}\int_0^{2\Pi} \left\{g_A(s) - \frac{g_A(2\Pi)}{2\Pi} s\right\} ds$$

It follows from the definition that $G_A(s)$ is periodic with period 2Π, and

$$\int_0^{2\Pi} G_A(s)ds = 0$$

The quantity $g_A(2\pi)$ is called the "virtual area" of the closed space curve. In the case of plane curves, $g_A(2\pi)$ is equal to the truth area enclosed.

The reference vector e_A can be chosen in several ways. In practical computation, we have generated the reference vector on the basis of vector area F_A. Let $F_{A,i}$ be the first non-zero component of the vector area F_A, where $1 \le i \le 3$. The arc length parametrisation of a closed space curve A is said to be *of positive orientation* or of *negative orientation* if $F_{A,i}$ is positive or negative, respectively. In accordance with former considerations the reference vector e_A is defined as

$$e_A = \mathrm{sgn}(F_{A,i})F_A/|F_A|$$

where $|F_A|$ is the absolute value of the vector F_A.

iii. Starting with the preselected reference vector, the *pseudo-torsion function* $H_A(s)$ used for shape characterisation, is defined by the scalar product

$$H_A(s) = e_A r_A(s)$$

Due to this definition, $H_A(s)$ is periodic with period 2π, and $H_A(s) \equiv 0$, if and only if A is a plane curve.

The shape functions are affected by the choice of the curve parametrisation. If the sense of parametrisation is reversed, then the partial shape functions are transformed as follows:

$$R_{A,r}(s) = R_A(2\pi - s) \qquad (4.1)$$

$$G_{A,r}(s) = -G_A(2\pi - s) \qquad (4.2)$$

$$H_{A,r}(s) = -H_A(2\pi - s) \qquad (4.3)$$

where $R_{A,r}(s)$, $G_{A,r}(s)$ and $H_{A,r}(s)$ are the transformed partial shape functions belonging to the space curve of revised parametrisation. This implies that the following three shape descriptors will alter their signs:

$$W^{(1,2)}_{A,m,r} = -W^{(1,2)}_{A,m}$$

$$W^{(1,3)}_{A,m,r} = -W^{(1,3)}_{A,m}$$

$$Z^{(2,3)}_{A,m,r} = -Z^{(2,3)}_{A,m}$$

and the other remaining shape factors will not be affected.

EXPERIMENTAL STUDY

In order to illustrate some features of the method described, preliminary experiments have been carried out using four test curves denoted by B0, B1, B2 and B3 (Fig.1.). A computer program written in Pascal language has been developed for the

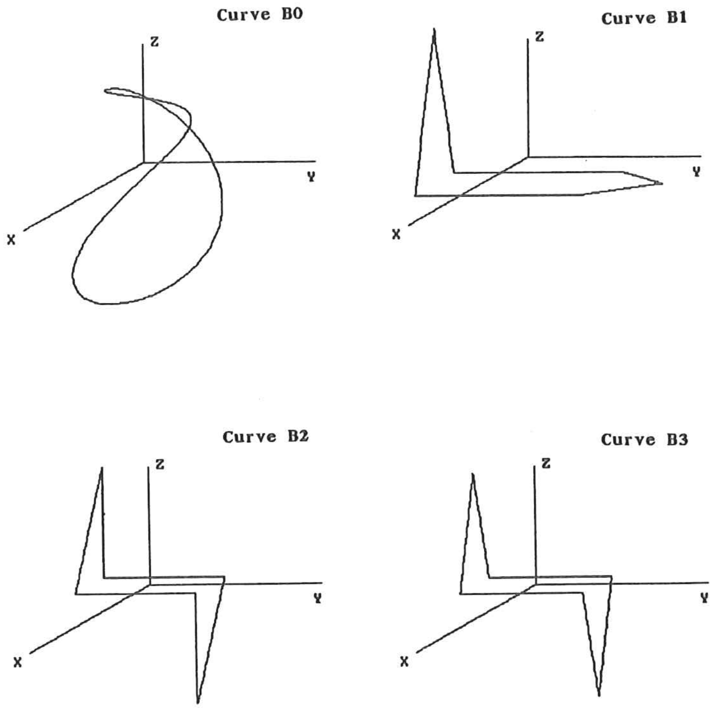

Fig. 1. Four test curves used for shape analysis

calculation and run on a microcomputer. For an arbitrary space curve the program calculates the first eight (M = 8) Fourier coefficients and shape descriptors defined according to the Eq.(1). Data input was made by specifying the coordinates of the vertex points P_j (j=0,1,..N) of the polygon, which approximated the closed space curve. By increasing the number N of vertex points, the accuracy of the approximation can be improved to the necessary extent. In the practical computation of the shape descriptors we took into consideration N = 240 sample points. As an example, Figure 2 shows the partial shape functions $R_A(s)$, $G_A(s)$ and $H_A(s)$ characterizing the space curve B1. Some computed Fourier descriptors related to four test curves are given in Table 1. Comparing the test curves in Fig. 1 the following conclusions can be drawn:

Curve B0 is unsymmetric, curve B1 possesses only one plane of symmetry, while curve B2 is centrally symmetric with respect to its point of mass centre. Curve B3 has three types of symmetry elements, since it possesses a plane of symmetry and a centre of symmetry, furthermore, it has a two-fold rotational symmetry. Analysing the data in Table 1, we can see that some

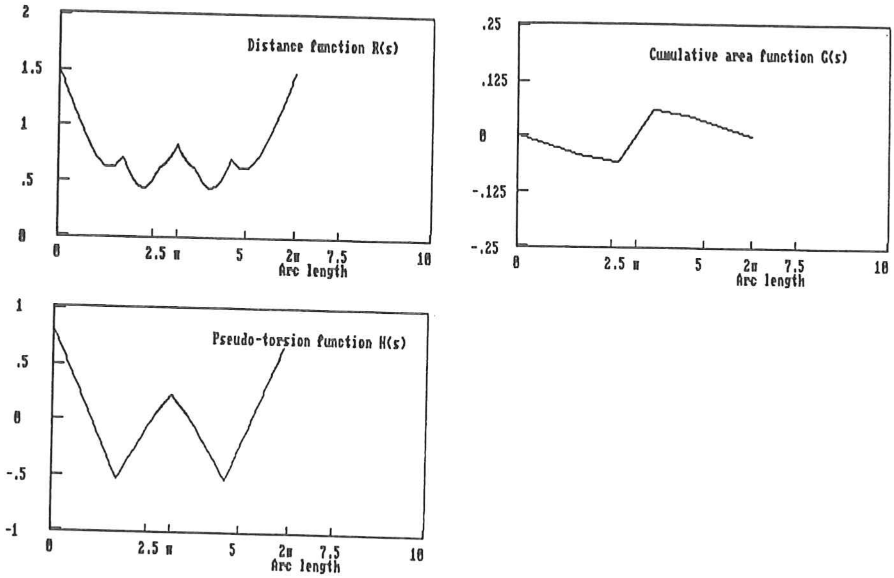

Fig. 2. Partial shape functions characterizing the space curve B1.

Table 1. Computed shape descriptors for four space curves (m = 1)

Shape descriptors	Test curves			
	B0	B1	B2	B3
$W_{B,1}^{(1,1)}$	0,10662	0,07723	0	0
$W_{B,1}^{(2,2)}$	0,05533	0,00239	0	0
$W_{B,1}^{(3,3)}$	0,29819	0,06181	0,00139	0,00131
$\lvert W_{B,1}^{(1,2)} \rvert$	$\lvert$-0,00300$\rvert$	0	0	0
$\lvert W_{B,1}^{(1,3)} \rvert$	$\lvert$-0,17812$\rvert$	0,06093	0	0
$W_{B,1}^{(2,3)}$	0,01091	0	0	0
$Z_{B,1}^{(1,2)}$	0,07675	-0,01357	0	0
$Z_{B,1}^{(1,3)}$	0,00819	0	0	0
$\lvert Z_{B,1}^{(2,3)} \rvert$	0,12798	0,01214	0	0

of the shape descriptors are equal to zero. This can be explained by the fact that there are close relationships between the symmetry properties of the curves and their Fourier descriptors (Réti and Czinege, 1993). We can further conclude that the visually similar curves B2 and B3 are also judged to be quantitatively similar (see Table 1.).

CONCLUSIONS

a. The partial shape functions introduced can be defined at every point on the space curve, even on those points whose curvatures vanish. Since $R_A(s)$, $G_A(s)$ and $H_A(s)$ are continuous functions, this fact affects the computation of Fourier descriptors advantageously.

b. Any space curve is completely determined, in both its position and orientation, by its shape function. Knowing this, the shape of the space curve can be unambiguously reconstructed.

c. Fourier descriptors contained detailed information on the symmetry properties of the space curves, and they can be used to quantitatively characterise the shape similarity between the space curves.

d. The major advantage of the method proposed lies in the convenience of writing a computer program for generating shape functions and finding shape descriptors.

Further studies are in progress to evaluate the performance of the method for a wider range of space curves, including many from the real world.

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