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STEREOLOGY IN GEOSCIENCES: ACHIEVEMENTS, DIFFICULTIES AND LIMITATIONS

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ABSTRACT

The methods of stereological analysis find more and more applications in the quantitative evaluation of the geometrical structure of rocks and soils. To elucidate the origin of these materials the knowledge of their structures supplies significant information. Modal analysis of rocks is realized on a wide scale. A systematic progress is observed in the approach to the problem of mineral liberation. Some models are verified experimentally. Discovering the relations between the structure of the rock undergoing comminution and the new structure induced by the network of the crushing surfaces continues to be the program of investigations. The methods of a quantitative evaluation of the morphology of the discontinuity surfaces, both the natural ones, i. e. existing "in situ", as well as those induced in the strength tests are being developed. Methods of a quantitative evaluation of the pore structure of the soil are implemented. The intensification of the experimental studies, however, depends at least on two factors: progress in the methods of contrasting polished sections of rocks built of transparent minerals and the development of the methods of automatic analysis of complex structures of rocks and soils.

Keywords: rock structure, soil structure, stereological analysis, mineral liberation, discontinuities in rocks.

INTRODUCTION

The term "geosciences" has been used in the title to indicate those scientific disciplines which, for various reasons, are concerned either with the quantitative analysis of the structure of rocks or soils or with the characteristics of the geometry of the comminution products of rocks regarded as collections of individual particles. The geometrical structures of rocks/soils are, in particular, the object of interest of geology, mineralogy, rock mechanics, mineralurgy, soil mechanics, agrophysics, mechanics of flow through porous media. The interest in the structure of rocks/soils may be dictated by the scientific attitudes or, especially in modern times, by the pragmatic attitudes resulting from the conviction or expectation that the geometrical structures of rocks/soils have a significant effect on the behavior of these materials in the processes occurring in nature, "in situ", or in the industrial processes. There are reasons to believe that the knowledge of the geometrical structure of the rocks/soils can supply important information about their origin.

The model has never been verified experimentally not only because of the lack of the estimators of N_V. The main difficulty in verifying it, is the tacit assumption that the network of the crushing surfaces is independent of the contact network of both minerals. This assumption eliminates the possibility of taking into consideration the physical properties of the minerals and of the rock itself. In the integral - geometrical model there appear only the global parameters. For their estimation one need not to utilize the complete geometrical information available on the grain cross - sections of the rock samples and the cross sections of particles embedded in a mounting medium.

The development of the models of mineral liberation to date has not been achieved, in principle, through the explicit generalization of the assumptions adopted in the model of Bodziony (1965 c). In this respect the study published by Davy (1984) is an exception. The actual development of the models was attained by taking into consideration the more and more complete geometrical information accessible on the sections. Use is being made, in particular, of the length distributions of the various types of chords (linear analysis) and the distributions of the areas of the cross-sections of the particles (areal analysis) and of their moments of higher orders. New concepts have been introduced, such as e. g. linear and areal liberation. The characteristics of the second order have been also utilized recently.

From among the studies referring to the liberation models and employing the stereological methods we will mention the most significant ones: Giger (1968), Coleman (1969, 1973, 1978, 1987), Andrews and Mika (1975), Steiner (1975), Jones et al.(1978), Barbery et al.(1979), King (1979, 1982, 1983), Bodziony and Kraj (1985), Bérubé et al. (1985), Barbery and Leroux (1988). An extensive review of the models of mineral liberation was published by Barbery (1985). Not all the models are characterized by absolute correctness of mathematical reasoning. King's model (1983) and the model of Barbery et al. (1979) have been criticized by Moore (1985). Some mathematical misunderstandings have been also found in Barbery's model by the author himself (see Barbery et al. 1985).

The development of automatic image analysis has created a chance for a more and more accurate verification of the liberation models. This can be observed in the studies published in the nineteen-eighties. It is to be expected that specialists in the field of mathematical morphology will pay more attention to the specific features of the rock structures and develop methods enabling automatic stereological analysis of rocks of more complex structures.

The study of Davy (1984) deserves special attention. Its main results is the proposal of a new index of mineral liberation in a polymineral rock. Coleman demonstrated, using two particular collections of grains, that the index proposed by Davy is sensitive to the kind of the data (from the linear or areal analysis), on the basis of which it is calculated. This is contrary to the thesis of the author. However, Davy's new concepts of characterizing the comminution process and the comminution product may be of much greater importance that the proposed liberation index. Davy has formulated a certain idea of introducing a relation between the structure of the rock undergoing comminution and the comminution process itself (the structure of the crushing surfaces). It is to be expected that the ideas conceived by Davy will by developed.

SURFACES OF DISCONTINUITIES IN ROCKS: ROUGHNESS

There is no exaggeration in the opinion that metallography is the domain of the development of the methods of the quantitative evaluation of the geometrical structures formed by the discontinuity surfaces in materials. The metallographic bibliography of this problem is very rich. Two topics are ahead of the others: (a) characteristics of the spatial orientation of the discontinuity surfaces forming the structure, and (b) characteristics of the morphology of a single discontinuity surface. The rock mechanics, in principle, takes up and adopts the methods of metallography for the investigations of the discontinuity structures and single discontinuities in rocks. A significant progress in this field has taken pores weakens the rock. On the other hand, the network of the contact surfaces of crystals or minerals may either strengthen or weaken the material. One can imagine a case when this network has no effect on the comminution process. Comminution consists in the creation of new surfaces, called the crushing surfaces, along which the rock loses its cohesion. Comminution even of a single piece of rock in practice excludes the possibility of its reconstruction. This fact is the main, so far not overcome, difficulty in the study of the behavior of rock during the comminution process. Let us imagine, however, that the reconstruction has been attained. The network of the crushing surfaces forms then a new, induced substructure. The principal task of stereology in this respect consists in the elaboration of methods of discovering the relations between the structure of a not comminuted rock and the new structure induced by the network of the crushing surfaces, Fig. 3.

Let us consider a piece of monomineral rock whose structure is formed exclusively of a network of crystals. We can distinguish two extreme cases of the network of the crushing surfaces: (a) intercrystalline, and (b) transcrystalline. A real network of the crushing surface represents, in general, an intermediate case: the surfaces have locally the inter- or the transcrystalline character. Bodziony (1967) formulated the basis for a stereological method of calculating the division of the crushing surfaces into parts of intercrystalline or transcrystalline origin. This method has been applied by Górski (1972) for the analysis of the comminution of dolomite.

A technically important effect occurring during the comminution of polymineral rocks is the mineral liberation. In the simplest case of a rock built of two minerals: A- useful one and B (usually: $V_v(A) << V_v(B)$) as the rock matrix, the comminution product comprises three types of particles. Particles of type A and type B are the so-called liberated particles, built exclusively of mineral A or B, respectively. Particles of AB type are the so-called composite particles, built of both minerals. The simplest characteristics of the liberation of the given mineral is the so-called liberation degree or index. It represents the quotient of the volume of this mineral contained only in the liberated particles as divided by its volume in the whole comminution product, i. e. in the volume of the rock undergoing comminution . The product of comminution is usually divided into classes according to the particle size. The mineralurgists expect to provide them with a method which would enable to determine the degree of the liberation of the given mineral and to determine the distribution of its volumetric content in the group of composite particles in each class of the particle size. This, generally speaking, is the essence of the problem of mineral liberation.

It is obvious that intercrystalline comminution gives a better liberation effect. This is one of the reasons why mineralurgy is interested in the mechanism of rock comminution. The liberation degree increases with increasing comminution degree, i. e. with the increase in the area of the newly exposed crushing surface. The liberation effect is the direct result of the spatial location of the network of the crushing surfaces in the rock structure, and in particular with regard to the mineral contact network.

The problem of mineral liberation was formulated for the first time by Gaudin (1939), who supplied also the first geometrical model of liberation. The literature of the problem numbers at present about two hundred articles published in various professional journals. The application of stereological method was initiated by J. Bodziony (1965 a, b, c). At the assumption of a two - component rock and convexity of the grains of the useful mineral in the rock and of the particles of the comminution product, it was possible, basing on the Blaschke - Santalo relation; (Blaschke, 1955), to derive a formula for the relative number $N_v(A+AB)$ of the liberated and the composite particles in the comminution product.

$$N_V(A+AB) = N_V(P)\ V_V(G) + \frac{1}{4\Pi} M_V(P)\ S_V(G) + \frac{1}{4\Pi} S_V(P)\ M_V(G) + V_V(P)\ N_V(G) \qquad (1)$$

In this integral - geometrical liberation model all four global stereological parameters are utilized both with respect to the rock structure (G- grains) and to the network of the crushing surfaces (P- particles).

The application of stereological analysis for the determination of the volumetric content of minerals in a rock cannot be questioned. The modal analysis is the basic method with geologists and mineralogists. Thin sections are used in the modal analysis of transparent minerals, and polished sections - in the analysis of non - transparent minerals. Automatic modal analysis meets with great difficulties especially in the presence of "unwanted" substructures of the rock regarded as "stereological noise". Speaking in general, the methods of contrasting polished sections of rocks have not been well developed so far. Undoubtedly, the metallography has attained a much higher work comfort.

Investigations concerning the correlation between the mechanical properties of the rocks (e.g. compression and tensile strengths) and their geometrical structure have not been carried out in a wider scale so far. Here, there should be mentioned the study by Erkan (1971) on the effect of the relative surface S_V of all the components of granite on its compression strength P. Fig.1 shows the main result obtained by Erkan. It appears that the microstructology of rocks in the sense of Rhines (1986) continues to be important research problem.

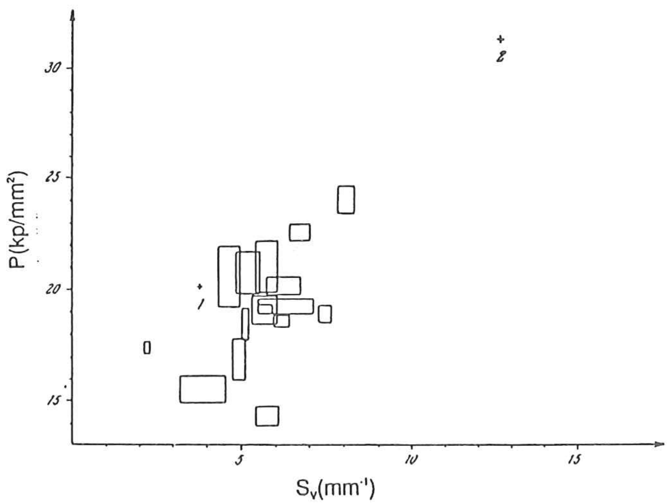

Fig. 1. Relationship between compressive strength of granite cube specimens and total surface density, (Erkan 1971). Granite from Schwarzwald: ▭,▯ denote the range of deviations; 1-granite from South Africa; 2- norite from Sweden.

COMMINUTION OF ROCKS: MINERAL LIBERATION

Comminution of brittle materials, including rocks and coal, is of great importance in industry. The economic aspect of this process which consumes a significant percentage of energy produced in the world is not less important. From the point of view of the rock structure and stereological analysis it appears that comminution (crushing and grinding) is still a process not fully recognized. In the present review the discussion of the effect of the physical properties of rocks and of the crushing facilities on the progress of the comminution process is deliberately left out, Fig.2.

Let us consider a piece of rock undergoing comminution. As already mentioned in the Introduction, its structure may comprise several substructures. The networks of discontinuities and of

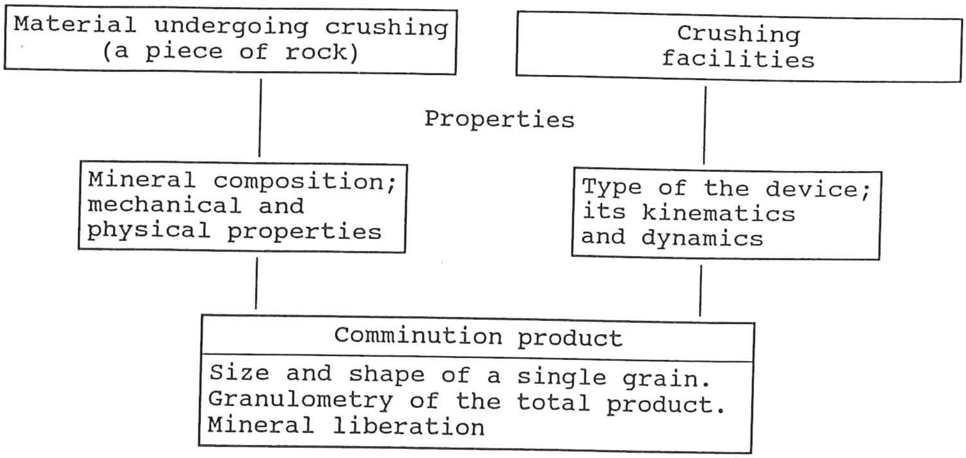

Fig. 2. Comminution of rocks. Technological point of view.

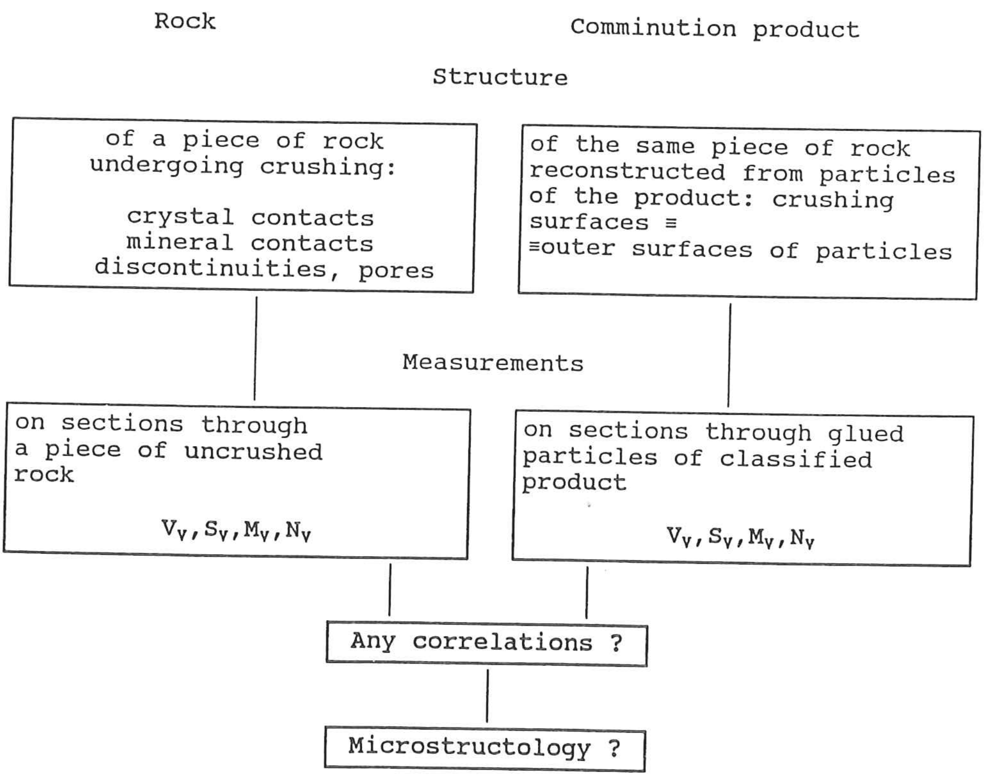

Fig. 3. Comminution of rocks. Stereological point of view

projection method. This method originated by König (1934), however, has not found wider application as yet (see Bodziony and Kraj, 1978).

Evaluation of stereological parameters is always connected with the problem of the estimation of errors. This aspect of stereological analysis still escapes the notice of most authors. There are at least two reasons for it: (a) the authors attention concentrates on the technique of preparing the sections of the examined material and attaining the object of the investigations, and (b) no handbook on the application of statistics for stereological analysis, of high didactic value, has been published so far. An example of high sensitivity of the authors to error estimation are the studies by Gentier and Riss (1987 a,b,1989).

Now we shall review the applications of stereological analysis in the geosciences with special consideration given to some problems. They are: the global stereological parameters: modal analysis of rocks, comminution of rocks: liberation of minerals, discontinuity surfaces in rocks: roughness, stereology of soils.

GLOBAL STEREOLOGICAL PARAMETERS: MODAL ANALYSIS

It is well known that the first quantitative evaluation of the spatial geometrical structures of grained materials were connected with geology and mineralogy. The point was the assessment of the volumetric content of the particular minerals in rocks. Delesse (1848), Rosiwal (1898) and Glagolev (1931) gave the estimators for the volumetric content of the distinguished mineral in the rock on the basis of areal (A_A), linear (L_L) and point (P_P) stereological analysis, respectively. Rosiwal gave simple recommendation concerning the required range of measurements to obtain the desired accuracy of the linear analysis. It is worth noticing that Rosiwal did not restrict the measurements to those made with the use of a grid of straight lines; he also included in his considerations the measurements along curved lines. Glagolev developed the method of estimating errors on the basis of the properties of Bernoulli's distribution, actually used also at present.

Estimators for the assessment of the specific surface S_V have been derived in connection with the applications of stereological analysis in metallography. The estimator for the assessment of M_V was supplied by Bodziony (1965 b) at the assumption of the convexity of particles. Basing on the methods of integral geometry and Crofton's formulae there was derived then, for the first time, a complete set of the estimators of the global parameters of the spatial structure. In the Table the ratios represent suitable densities in planar, linear and point analysis, respectively. The global parameters have been used by Bodziony (1965c) to formulate an geometrical - integral model of the liberation of minerals.

Table. List of estimators of the global parameters of the structure (Bodziony 1965 b).

Magnitude (specific)	Mathematical definition		Estimators					
Volume $\bar{V}$	$\frac{\Sigma V_i}{V_R}$	$\frac{cm^3}{cm^3}$	$\frac{\Sigma f_{ir}}{\Sigma f_r}$	$\frac{cm^2}{cm^2}$	$\frac{\Sigma s_{ir}}{\Sigma s_r}$	$\frac{cm^1}{cm^1}$	$\frac{z_G}{z_R}$	$\frac{cm^0}{cm^0}$
Surface $\bar{F}$	$\frac{\Sigma F_i}{V_R}$	$\frac{cm^2}{cm^3}$	$\frac{4}{\Pi}\frac{\Sigma l_{ir}}{\Sigma f_r}$	$\frac{cm^1}{cm^2}$	$4\frac{\Sigma n_r}{\Sigma s_r}$	$\frac{cm^0}{cm^1}$		
Total medium curvature $\bar{M}$	$\frac{\Sigma M_i}{V_R}$	$\frac{cm^1}{cm^3}$	$2\Pi\frac{\Sigma m_r}{\Sigma f_r}$	$\frac{cm^0}{cm^2}$				
Number $\bar{N}$	$\frac{N}{V_R}$	$\frac{cm^0}{cm^3}$						

Attempting to formulate the definition of the geometrical structure of rocks/soils is a fairly risky task. Let us note that rocks/soils are, in general, porous materials, built of crystals or grains of one or more minerals. The structure of these materials comprises in most cases several substructures. Examples of such substructures are: (a) the network of crystals, (b) the network of the particular minerals, forming part of the network of crystals, (c) the network of pores, (d) the network of natural discontinuities, (e) the network of induced discontinuities.

To a single spatial element of the network (including spatially degenerated one) there may be assigned in a unique way: The volume V [l^3], the surface area S [l^2], the total mean curvature M [l^1] and the total gaussian curvature C [l^0]. According to Hadwiger's (1955) classification [V,S,M,C] are the so-called basic functionals of convex bodies and they possess important properties: additivity and invariance with respect to a group of motions in the space. The relation : convex body - [V,S,M,C], however, has no one-to-one correspondence even for the class of convex bodies. Hadwiger (1955) gives a cylinder and a cone as examples, i. e. bodies of different shapes defined quantitatively by the same four numbers [V,S,M,C]. At this occasion it is worth noticing that the total mean curvature M of a convex body - either of C^2 class or of a polyhedron - has an elegant geometrical interpretation given by Minkowski (1903). M represents the average width of a body in the collection of all directions in space, with the accuracy of the coefficient 2Π. Thus M is directly related to the particle size in the sense of Feret (1932), (see Bodziony, 1965 d). Feret's diameter defined by relation $D_{F\bullet} = 1/2\Pi$ M is thus the basic functional of a convex body; the total Gaussian curvature in the class of convex bodies, and even in the compact ones is constant: $C = 4\Pi$; and judging from this point of view it is a trival quantity; however, it leads directly to the notion of the relative number of particles N_v.

From the point of view of geometry and, statistics the great variety of structures of rocks/soils is the result of a random distribution of the crystals of the particular minerals (and pores), taking place in the process of their genesis. The absence of one-to-one correspondence in the relation: body - [V,S,M,C] even in the class of convex bodies is a warning that we must not expect the existence of such a correspondence between the structures of rocks/soils and their quantitative characteristics. It appears that the one-to-one correspondence of structures and their quantitative characteristics is limited to the class of a geometrical similarity of the structures.

The main quantitative characteristics of the structures of rocks/soils, and more generally, of any grained material is based on Hadwiger's functionals. To each structure (or substructure) there may be, in general, assigned a set of four parameters [V_v,S_v,M_v,N_v]. The index V indicates that the parameters are the expected values, i. e. calculated on the average per unit volume of the material. They are the results of averaging in some definite region of space. For this reason they are often called densities or global parameters. To distinguish them from other types of structure characteristics they are also named the first order parameters.

The structures of rocks/soils are examined in the spatial regions of various orders of range: starting from regions comparable with the size of gas particles (investigations of porous materials) to regions comprising many kilometers (problems of geotectonics).

Stereological analysis of rocks/soils develops or applies the methods of a quantitative estimation of unknown real parameters characterizing the spatial structure on the basis of geometrical information contained in flat sections of these materials. Depending on the purpose of the investigations the sections of the structures may be represented by natural cliffs, side walls of underground workings of a mine and on the surface, exposures of soil, and finally, polished sections or thin sections prepared from the material samples collected in various ways. Further on we will concentrate mainly on the stereological analysis of polished sections or thin sections.

Stereological analysis of the comminution products of rocks is based on geometrical information obtained from polished sections or thin sections of particles embedded in a mounting medium. The quantitative characteristics of the geometry of single particles is possible basing on the so-called place in the period of the last 15 years.

The rock mechanics distinguishes - speaking in general - two types of discontinuities: (a) natural discontinuities, formed "in situ", and (b) discontinuities induced by mining and geotechnical activity, also "in situ", or the discontinuities induced by subjecting the rock constructions or rock samples to some definite states of stresses.

Kraj and Kruszyński (1981) and Kraj (1986) have developed a method of determining the spatial orientation distribution of discontinuities in rocks based on stereological analysis of the discontinuity traces visible on polished sections of granite samples. They established also that the orientation distribution of induced fractures in the same collection of samples by means of the so - called Paulmann's test (1966) is strongly correlated with the orientation distribution of natural fractures.

Gentier and Riss (1987b) determined the angular distribution of normals to the fracture surface elements of granite basing on the Scriven and William's method.

The principal feature of a single fracture examined by means of stereological methods is roughness. This problem has been discussed in the excellent doctoral dissertation by Gentier (1986) and in the studies by Gentier and Riss (1987a, 1987b, 1989). Gentier (1986) says that: "Roughness is not a precise notion, rather poorly defined and especially difficult to be expressed by a single value. The complexity of the definition of roughness, reflected in the diversity of the traditionally calculated indices is due to the fact that in the notion of roughness there are included such numerous morphological characteristics as amplitude, curvature, angularity and waviness (periodicity)". Exactly the same difficulties are encountered when trying to supply the definition of the grain size comprising a class of bodies wider than that of geometrically similar bodies. The example given by Hadwiger explains the situation. Defining roughness by means of a set of several parameters may be justified if they are weakly correlated with each other. This condition is of essential importance (see Bodziony 1981). The accuracy of the definition of roughness, however, will be determined by its applicability for the purposes of microstructology.

Gentier (1986) carried out extensive experimental studies on the morphology of natural fractures in the granite from Guéret (amplitude, angularity, curvature) and elaborated the results using both the methods of standard stereology and the methods of fractal and of spectral analysis, the investigations comprised both parts of a natural fracture surface and were conducted by means of a profilemeter with contact sensor. Gentier arrived at a conclusion that the morphology of a natural fracture surface is not directly associated with the size and the shape of grains. This is an important, although rather unexpected result. In this context it is doubtful whether, basing on the examination results of granite, i. e. a polymineral rock, in which the volumetric content of the dominating mineral, quartz, amounts only to about 50%, one can speak about typical size and shape of the particles. Investigations of the morphology of fractures formed in monomineral rocks with equisized grains might supply a more solid foundations for the thesis that the morphology of the fracture surface is not correlated with the size and shape of grains of which the rock is built.

Gentier's doctoral dissertation may be used as an excellent guidebook for any one who intends to develop the analysis of the morphology of discontinuities in rocks.

Gentier and Riss (1987a, b,1989) use as a basis the standard definition of areal roughness: $R_A = S/S'$ (where S - area of a certain piece of the fracture surface, S'- area of the projection of this piece on an projection plane) and linear roughness: $R_L = L/L'$ (where L - length of the profile of a piece of the fracture surface obtained on a section of this piece by a plane normal to the projection plane, L'- length of the profile projection on the projection plane). Basing on the investigation results of the surface morphology of a natural fracture of granite, Gentier and Riss have derived a new relation:

$$R_A = \frac{4}{\Pi} R_L - 0.304749 \, R_L(R_L-1) - \frac{4}{\Pi}(1-\frac{\Pi}{4}) \tag{2}$$

which reflects more accurately the relation $R_A = f(R_L)$ for natural granite fractures than other formulae, known so far. The aim of the investigations of Gentier and Riss is to obtain such quantitative characteristics of the discontinuities of rocks which might be useful for the analysis of the rock mass stability in quarries and in underground workings.

The morphology of the discontinuity surfaces may be examined with success by means of fractal analysis. More and more publications are devoted to this topic. Xie Heping (1990) was the first to publish a monograph dealing with the application of fractals in rock mechanics. This method is also used to some degree by Gentier (1986). Huang et al. (1992), however, observe that "fractal dimension as an invariant scaling parameter to describe the aspect of joint surface roughness does not seem warranted, but its use as one of a number of parameters describing the properties of joint surfaces can have positive benefits". Pape et al. (1987) proposed the so - called pigeon hole model of fractal properties for the simulation of the pore surfaces in sedimentary rocks , (see Fig. 4). For the purpose of an experimental verification of the model they propose the determination of the surface area of pores by the method of gas adsorption (BET methods), the method of fluid filtration, the methods of the microscopy of sections, and, finally, the direct visual methods for appropriate ranges of the resolution lengths. The authors do not make any reference to the difficulties in combining the results obtained by various methods. It is yet too early to pass a definite judgement about the real importance of the fractal methods for rock mechanics. This will be decided by the applicability of the fractal parameters in the microstructology of rocks.

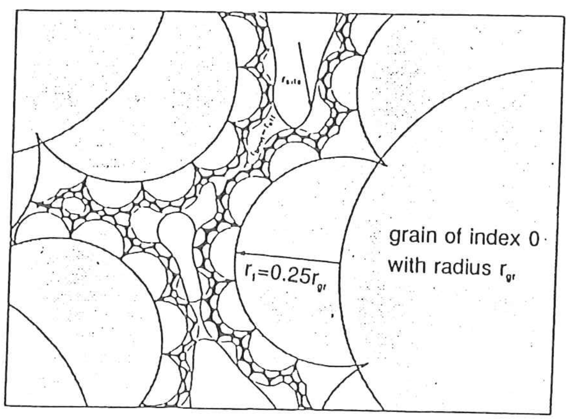

Fig. 4. Schematic drawing of a sedimentary rock according to the pigeon hole model, according to Pape et al. 1987.

STEREOLOGY OF SOILS

One can find certain analogies between the structures of some rocks and those of soils. However, the differences prevail. Maybe, greater analogies could be found in the structures of nonhomogeneous powders. Soils are made up, in principle, of grains of light, transparent minerals, and were formed as a result of the weathering processes of rocks and selective sedimentation. They are an element of the biosphere. Their physical properties were developed during a period of time comparable with some periods of the formation of lithosphere. A great effect on the present properties of soils is exercised by the agrotechnical activities. This refers also to the soil structure. Using the simplest classification the soil structure is built up of mutually complementing substructures of pores and solid phase. A most direct effect on the formation of the structure of pores is exercised by mechanical tillage and by the atmosphere. The atmospheric factors stabilize the structure of pores disturbed by repeated application of the means of mechanical tillage. They produce great, rapid deformations of pores of discontinuous character. For this reason, unlike in the case of rocks, in the investigations of the soil structure it is necessary to take into consideration not only its local variation, but also its cyclic variation with time . This aspect of the problem dictates the formulation of proper research tasks.

For some time now agrophysics has been trying to employ stereological methods of the evaluation of the structure of pores to test their applicability for the study of certain important properties of the soils. The most important here are the properties connected with maintaining or recovering the state of equilibrium of the water content in the soil. They include: (a) permeability, (b) ability to retain water (retency) and (c) ability of water penetration from the bottom towards the top layers of soil.

Agrophysicists expect the stereological analysis to solve their problems which are known in the quantitative evaluation of the pore structure of other materials. The objects of interest of agrophysics are: evaluation of global parameters V_v, S_v, characteristics of the pore throats, size distribution of pores, nonhomogeneity and anisotropy of the pore structure. Porosity V_v has additional importance in the physics of soil: it may be utilized as a measure of the volumetric deformation of soil!

However, there still exist barriers to the application on a wide scale of the already existing stereological methods for the evaluation of the structure of pores in soils. Here we mean the whole procedure preceding the moment of placing the objects (polished or thin sections) on the microscopic stage. This procedure includes: (a) Standardization of the procedure of soil sampling. It should enable the comparison of the investigation results obtained in various laboratories; (b) Development of methods of filling samples of various degrees of dampness with glue. Drying of samples prior to their gluing produces in most cases changes in the pore structure; (c) Development of better methods of contrasting the polished sections prepared from different kinds of soil for the purpose of large magnifications and analysis of the micropores.

The development of agrophysics has opened a new area for the development and application of the methods of stereological analysis.

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