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SIZE EFFECTS ON LOCAL TOPOLOGICAL MEASUREMENTS

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ABSTRACT

3D structures are obtained by a random filling of the nodes of a F.C.C. network. The density of points is increased from 0 to 1 inside a given field, the field sizes ranging from 5 x 5 x 5 to 400 x 400 x 400. For each structure, the Euler-Poincaré characteristic is measured in all the spaces ($\mathbb{R}^0$ to $\mathbb{R}^3$) and studied as a function of the field size and the field shape. The shape effect in 3D space appears as a size effect in a space of lower dimension.

Keywords : Euler-Poincaré characteristic, size effects, stereological parameters, 3D image analysis.

INTRODUCTION

A structure can be quantitatively described by metric and topological parameters. However, the measurement of these parameters usually cannot be performed on the whole structure. A little part of the structure is then analysed inside a field of measurement. One important question arises : "What is the minimum field size compatible with valid measurements ?". Whatever the number of fields, if the field size is too small, the parameters obtained are not representative of the whole structure. This question is particularly important for the 3D quantitative image analysis because the 3D topological parameters depend on the number of loops present in the structure. If the loop size (inaccessible) is larger than the field, the results will be systematically biased.

This effect is studied on simple simulated structures.

SIMULATIONS AND TOPOLOGICAL MEASUREMENTS

3D structures are built up from a given number of points randomly placed, with equal probabilities, on the nodes of a face-centred cubic (F.C.C.) network. The density of points is adjusted to cover the whole density range 0-1.

If $\mathbb{R}^n$ is the space of dimension n, $N_n(X)$ designates the Euler-Poincaré characteristic (E.P.C.) of the set X in this space. On each structure, the E.P.C. is measured in all the spaces (Serra, 1982 ; Meyer, 1992) taking into account the shell-correction (Bhanu Prasad et al.,1989). The values of the E.P.C. are reported to the unit size of the space where they are measured. For practical purposes, the distance between two neighbouring points of the grid is taken as 1. The specific values of the E.P.C., N_P, N_L, N_A and N_V are respectively linked to the volumic fraction, the specific surface area, the integral of mean curvature and the integral of Gaussian curvature via classical stereological relations.

For the 3D structures made up with random points, the E.P.C. can be calculated as a function of the density (Jernot and Jouannot, 1993). In the case of N_V, the minimum is obtained for the volumic fraction 0.5. The theoretical value calculated for this minimum is $-132.58\ 10^{-3}$. The experimental mean values measured on the simulations are $-130.49\ 10^{-3}$ for a field size 200 x 200 x 200, $-131.60\ 10^{-3}$ for 400 x 400 x 400 and $-131.75\ 10^{-3}$ for 500 x 500 x 500 with a standard deviation lower than 10^{-4}. Thus, even with very large field sizes, the theoretical value is not reached . Nevertheless, the size effect remains negligible (< 1.6%) in this case but what happens when smaller and smaller field sizes are considered ?

FIELD SIZE EFFECTS

The structures are built up inside fields of various sizes and the evolution of the E.P.C. along the whole density range is followed as a function of this field size (Figure 1a,1b,1c). In 1D space, no size effect is noticeable, the mean values being identical for lines of length 5 or 1000000 (Fig.1a). In 2D space and 3D space, a size effect is clearly apparent (Fig.1b,1c). : the smaller the field size, the lower the amplitude of the curves. Anyhow, the size effect in 2D space is less pronounced than in 3D space. For similar field sizes, the values of the E.P.C. corresponds respectively to 82% (plane 10 x 10) and 72% (cube 10 x 10 x 10) of the theoretical values. It must be noticed that, for a given field size, the mean values of the E.P.C. are proportional to the theoretical values all along the density range, the constant of proportionality depending on the field size only. As a consequence, the symmetry of the curves remains in all the cases. This would not be observed, had the shell-correction not been applied : for N_V, the first maximum would be higher than the second one and the curves would be shifted towards the high values of the density.

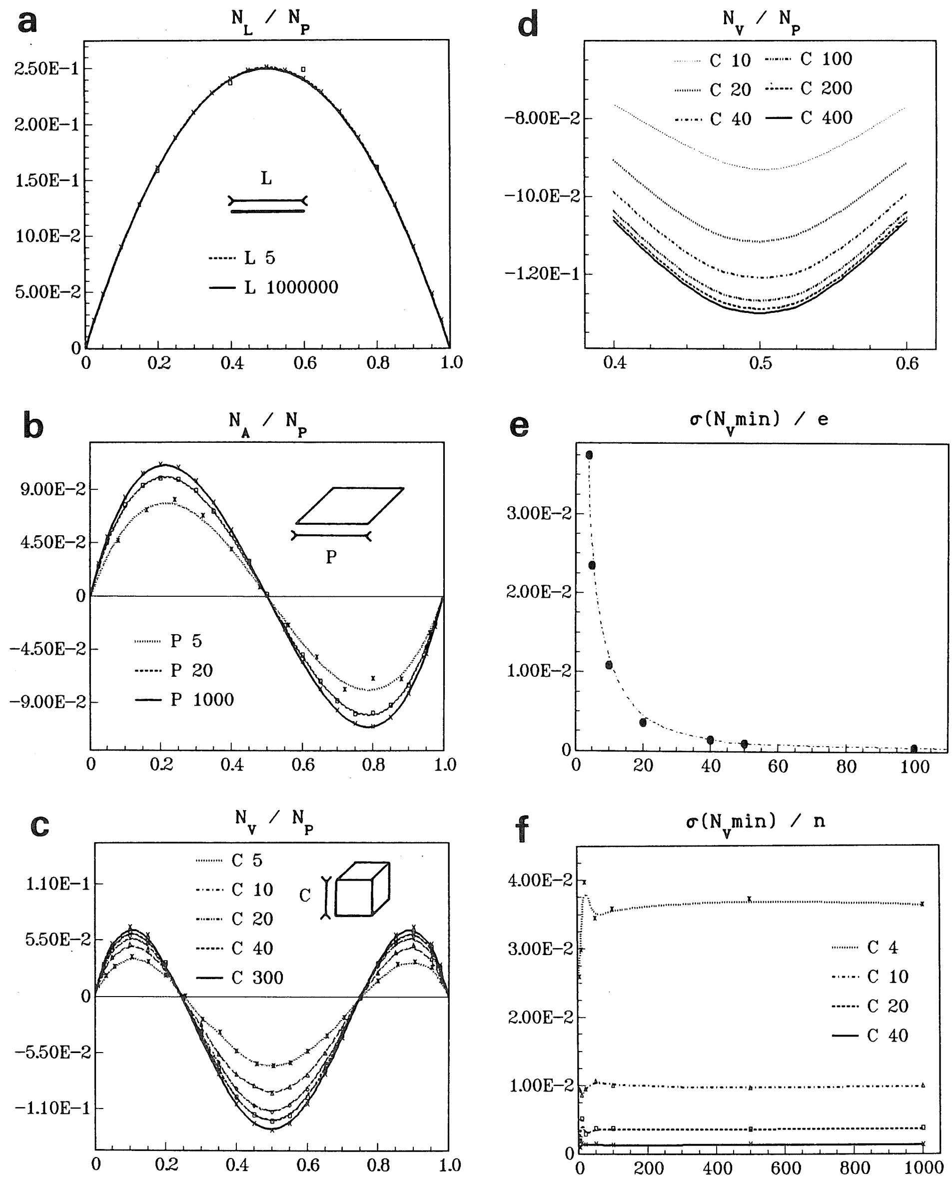

Figure 1 . E.P.C. in **(a)** $\mathbb{R}^1$, **(b)** $\mathbb{R}^2$, **(c)** $\mathbb{R}^3$ as a function of the compacity for different field sizes. **(d)**: zooming of (c) around $V_V(S) = 0.5$. **(e)** and **(f)**: standard deviation of N_V (for a compacity $V_V(S) = 0.5$) as a function of e, the edge of the cubic field (e) and n, the number of simulations (f).

From now on, we will focus our attention on the behaviour of N_V . The size effect being more apparent around a density 0.5, a zooming in this area is presented for different field sizes (Fig. 1d). It is seen on these curves, that a field size 100 x 100 x 100 leads to a good estimation of N_V (97% of the theoretical value) but this is no longer the case with a field 20 x 20 x 20 : whatever the number of fields, only 85% of the theoretical value is reached. Moreover, the field size also affects the standard deviation, σ, of measurements (Fig. 1e, 1f). After a sufficient number of simulations, this parameter keeps a constant level depending on the field size only : with e edge of the cubic field, σ actually varies as $e^{-3/2}$ (Fig. 1e). This is consistent with the theory of integral range (Matheron, 1989 ; Lantuéjoul, 1991) in which the variance varies as the inverse of the field volume.

FIELD SHAPE EFFECTS

So far, we have only considered the effect of the volume of the field on the measurements. For a given volume, what happens when the field shape is modified ? Two simple deformations of the cubes (C) are studied : flattening (sheets S) and lengthening (tubes T).

In order to separate the shape effect from the size effect, a volume of ~1600000 is used first (this volume corresponds nearly to a cubic field 117 x 117 x 117 for which the size effect is small). The fields are ranging from 5 x 5 x 64000 to 80 x 80 x 250 for tubes and 565 x 566 x 5 to 141 x 142 x 80 for sheets. The results are presented in figure 2a, 2b : the shape effect is more important for the tubes than for the sheets. Surprisingly, the curves are identical for S5/T10 (Fig. 2c), S10/T20, S20/T40 and S40/T80.

Owing to this coincidence between sheets and tubes, the interaction between the size effects and the shape effects is only studied with tubes of various volumes : for V = 64000, small tubes (t) from 5 x 5 x 2560 to 20 x 20 x 160 and, for V = 8000000, large tubes (T) from 5 x 5 x 320000 to 100 x 100 x 800. In both cases, the limit curve corresponds to cubes of volumes 40 x 40 x 40 (Fig. 2d) and 200 x 200 x 200 (Fig. 2e). All the results are summarized in figure 2f. On this figure, the lower curve (cubic fields ranging from 4 x 4 x 4 to 500 x 500 x 500) corresponds to the minimum value of $N_V$min (i.e. N_V for a volumic fraction 0.5) accessible from a given field size. This curve reflects a pure size effect or volume effect. Besides, for a given section of the tubes (dotted line), $N_V$min is the same whatever the volume considered (except for very small volumes where the size effect prevails). The value of $N_V$min is only limited by the smallest dimension of the field. Moreover, the equivalence already noticed between the sheets and the tubes can be extended to the cubes. For a given length e, if the field shape is a sheet of thickness e, a tube of section 2e x 2e or a cube 3e x 3e x 3e, very close values are obtained for $N_V$min , as can be seen in table 1.

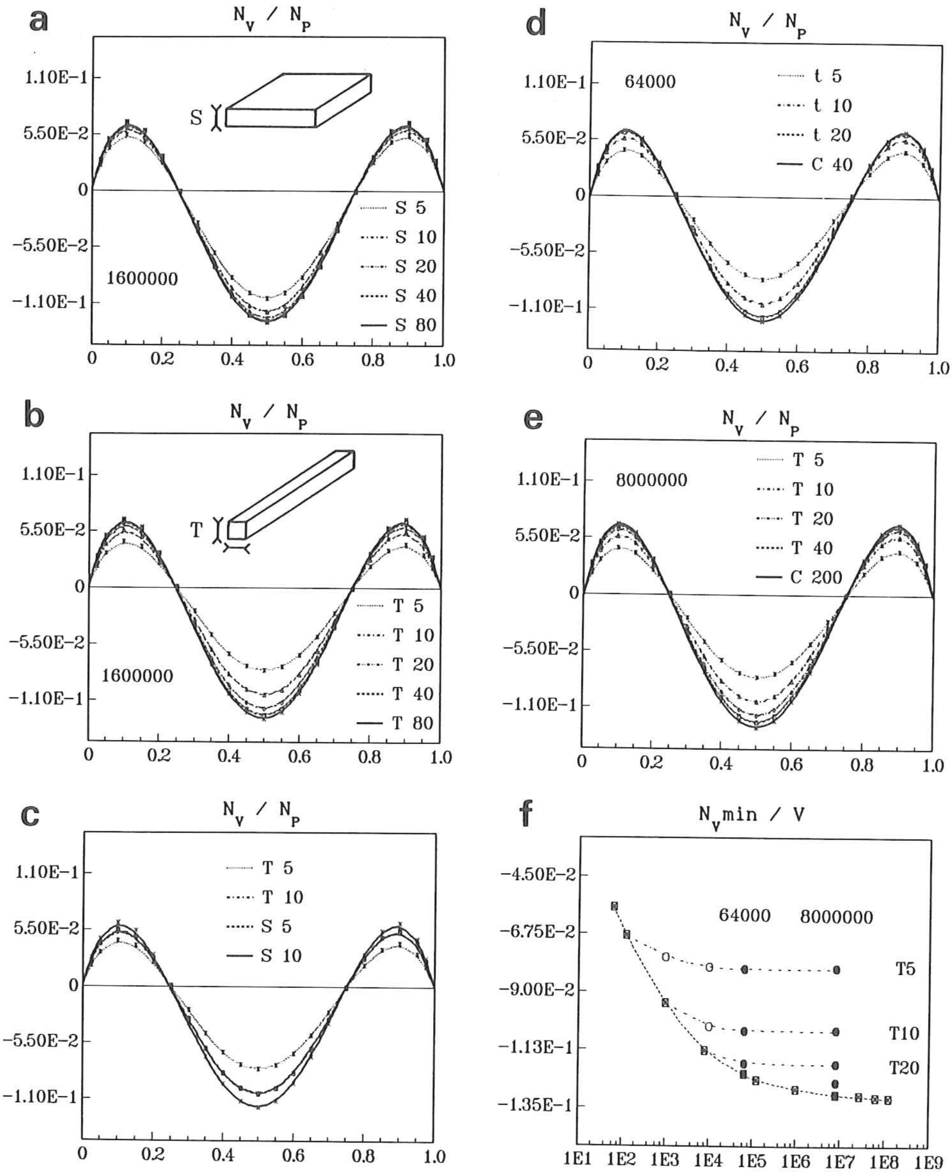

Figure 2 . E.P.C. in $\mathbb{R}^3$ as a function of the compacity for different field shapes **(a)**: sheets of increasing thickness. **(b)**: tubes of different square sections. **(c)**: comparison between (a) and (b) (same volume). **(d)** and **(e)**: tubes of various sections and volumes and comparison with cubes of corresponding volumes. **(f)**: evolution of the minimum of N_V with the field volume and the section of the tubes. The lower curve corresponds to cubic fields.

Table 1 . Comparison between the values of $N_V$min for different field shapes : sheets of thickness e, tubes of section 2e x 2e and cubes of volumes 3e x 3e x 3e. All the values in the table are multiplied by 1000. The volumes v1, v2 and v3 designate 1600000, 3380000 and 27000000.

	e = 5	e = 10	e = 50	e = 100
S_e	-105.7 (v1)	-118.7 (v1)	-128.8 (v2)	-130.7 (v3)
T_{2e}	-105.3 (v1)	-118.5 (v1)	-129.2 (v2)	-130.9 (v3)
C_{3e}	-106.2	-118.8	-129.8 (v2)	-131.1 (v3)

Thus, the shape effect observed here corresponds to a size effect in a space of lower dimension, everything happens as if the effect associated to a length e was playing its role one time for a sheet, two times for a tube and three times for a cube.

CONCLUSION

For a given structure, when the field size available for the measurements of topological parameters is too small, the results are systematically biased whatever the number of fields.

A chosen value of the variance, i.e. of the volume analysed, leads to the same results of N_V with a cube 3e x 3e x 3e, a tube ~2e x 2e x 7e or a sheet ~5e x 5e x e. Consequently, if this result can be generalized to any isotropic structure, the optimum shape of the fields will be a sheet (minimum number of serial sections) for the practical determination of the topological parameters.

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