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METRICS, SCALAR PRODUCT AND CORRELATION ADAPTED TO LOGARITHMIC IMAGES

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ABSTRACT

Logarithmic images, such as images obtained by transmitted light or those outcoming from the human visual system, differ physically from linear images. So, their processing and analysis require specific mathematical laws and structures. The latter have been developed in the setting of the Logarithmic Image Processing model (Jourlin and Pinoli, 1985, 1987 and 1988). This model, called LIP, has already permitted the definition of theoretical notions such as the differentiation and the integration of logarithmic images (Pinoli, 1986 and 1987), and the well-justified introduction of performing practical notions : blending of two logarithmic images (Jourlin and Pinoli, 1988), contrast at a point or associated with a region or a boundary (Jourlin, Pinoli, Zeboudj, 1989 and Pinoli, 1991). Moreover, the LIP model appears to be an accurate framework for the introduction of other powerful and useful notions. The purpose of the present paper is to establish some of them : metrics, which appear to be naturally linked with the optical density concept, allowing the calculation of the distance between two logarithmic images, and also a scalar product allowing the introduction of the orthogonality and correlation concepts.

Keywords : Correlation, image analysis, logarithmic images, metrics, orthogonality, scalar product.

1. RECALLS ON THE LIP MODEL

A logarithmic image can be completely modeled by its logarithmic grey tone function. Such functions are defined on a non-empty compact set D of the plane $\Re^2$ with range in the real interval [0, M[where M is a strictly positive real number. In the context of transmitted light the value 0 is reached for a point x of D which is totally transparent, while the value M corresponds to a point which is totally opaque. As regards the human vision, M and 0 correspond respectively to the visual dark and the glare limit (Gonzalez and Wintz, 1987, pp. 16-19).

In order to simplify the terminologies, logarithmic images and logarithmic grey tone functions will be called simply images and grey tone functions.

1.1. The vectorial structure on the grey tone function space (Jourlin and Pinoli, 1985, 1987 and 1988)

The laws denoted $\triangle\!\!\!\!\!+$ and $\triangle\!\!\!\!\!\times$ permit to define the sum of two images and also the positive homothetics associated to an image. The class of images, identified to the class of their grey tone functions, becomes then the positive cone, denoted **II**, of the set of functions defined on D and with values in the real interval]- ∞, M[which is a real vector space for the laws $\triangle\!\!\!\!\!+$ and $\triangle\!\!\!\!\!\times$. The elements of this space, denoted ***G***, are called grey tone functions ; those of]- ∞, M[are called grey tones.

Let us recall the definition of these laws :

$$f \,\triangle\!\!\!\!\!+\, g = f + g - \frac{f\,g}{M} \qquad \forall\ (f, g) \in G^{2} \tag{1}$$

$$\alpha \,\triangle\!\!\!\!\!\times\, f = M - M\left(1 - \frac{f}{M}\right)^{\alpha} \qquad \forall f \in G,\ \forall\ \alpha \in \Re \tag{2}$$

Remark : The inclusion $F(D, \Re) \supset$ **II** allows "reading" an image *f* as an element of the space $F(D, \Re)$ of real-valued functions defined on D. In this latter case, classical addition and scalar multiplication hold. In order to avoid any confusion, an image is denoted f when considered as an element of the space $F(D, \Re)$.

1.2. The set of grey tones is an Euclidean space (Pinoli, 1985)

The set of grey tones]- ∞, M[is also a real vector space, denoted ***E***, with respect to the laws $\triangle\!\!\!\!\!+$ and $\triangle\!\!\!\!\!\times$:

$$E = (\,]-\infty, M[\,, \triangle\!\!\!\!\!+\,, \triangle\!\!\!\!\!\times) \text{ is a real vector space.} \tag{3}$$

Remark : The elements of the space $\boldsymbol{E}$, that is to say the grey tones, denoted by s, t, ..., must be used with the laws $\triangle\!\!\!\!+$ and $\triangle\!\!\!\!\times$. When they are considered as elements of the real number space $\Re$, the classical addition and scalar multiplication hold. In order to avoid any confusion in this latter case they will be denoted s, t, ...

$\boldsymbol{E}$ becomes an Euclidean space for the scalar product denoted $(.\,|\,.)_E$ and defined for two grey tones s and t according to :

$$(s\,|\,t)_E = \mathrm{M}^2 \ln\left(1-\frac{\mathrm{s}}{\mathrm{M}}\right)\ln\left(1-\frac{\mathrm{t}}{\mathrm{M}}\right) \tag{4}$$

Consequently $\boldsymbol{E}$ becomes a real Banach space for the norm denoted $||.||_E$ and defined for every grey tone s by :

$$||s||_E = ((s\,|\,s)_E)^{1/2} = \mathrm{M}\left|\ln\left(1-\frac{\mathrm{s}}{\mathrm{M}}\right)\right| \tag{5}$$

and then a metric space for the distance denoted $d_E(.,.)$ and defined for two arbitrary grey tones s and t by :

$$d_E(s,t) = \mathrm{M}\left|\ln\left(\frac{\mathrm{M-s}}{\mathrm{M-t}}\right)\right| \tag{6}$$

where $|.|$ designates the absolute value function in $\Re$.

1.3. Integration of grey tone functions (Pinoli, 1987)

For the general theory of integration in Banach spaces the reader can refer to Bourbaki (1965 and 1967), Kolmogorov and Fomine (1977), or Dunford and Schwartz (1988).

Let λ be a measure on the spatial support D, equal to the Lebesgue measure in the continuous setting and equal to the cardinal measure in the discrete setting, that is to say when D is respectively a continuous or a discrete set.

A grey tone function f is integrable with respect to the measure λ on the spatial support D, if and only if, its underlying function f is such that the real-valued function $\ln[(1 - f/M)]$ is also λ-integrable with the classical Lebesgue-Stieltjes meaning.

Thus f being an λ-integrable grey tone function, yields :

$$\int_D ||f(x)||_E \, d\lambda(x) = \mathrm{M}\int_D \left|\ln\left(1-\frac{\mathrm{f(x)}}{\mathrm{M}}\right)\right| d\lambda(x) \tag{7}$$

and :

$$\int_D f(x)\, d\lambda(x) = M - M \exp\left(\int_D \ln\left(1 - \frac{f(x)}{M}\right) d\lambda(x) \right) \tag{8}$$

If we only consider a λ-measurable subset D_0 included in D, we obtain corresponding formulas if we replace the set D in (7) and (8) by the subset D_0.

The set of λ-integrable grey tone functions is a real vector space, denoted $L^1(E)$, for the vectorial laws $\mathbin{\triangle\kern-0.75em{+}}$ and $\mathbin{\triangle\kern-0.75em{\times}}$. The set of classes of λ-integrable grey tone functions according to the equivalence relation "equality λ-almost everywhere" is a real Banach space, denoted $\mathcal{L}^1(E)$, with respect to the norm, defined by :

$$\| f \|_{\mathcal{L}^1(E)} = \int_D \| f(x) \|_E \, d\lambda(x) \tag{9}$$

that is to say explicitly :

$$\| f \|_{\mathcal{L}^1(E)} = M \int_D | \ln(1 - f(x)/M) | \, d\lambda(x) \tag{10}$$

1.4. Integration at power p for p belonging to $[1, +\infty[$ (Pinoli, 1987)

Let p be an arbitrary real number belonging to the interval $[1, +\infty[$. The set of λ-measurable functions f, such that the real-valued functions $\|f\|^p_E$ are classically λ-integrable, is a real vector space, denoted $L^p(E)$, for the vectorial laws $\mathbin{\triangle\kern-0.75em{+}}$ and $\mathbin{\triangle\kern-0.75em{\times}}$. The set of classes in the space $L^p(E)$ according to the equivalence relation "equality λ-almost everywhere" is a real Banach space, denoted $\mathcal{L}^p(E)$, with respect to the norm defined for a λ-integrable grey tone function f by :

$$\| f \|_{\mathcal{L}^p(E)} = \left(\int_D \| f(x) \|_E^p \, d\lambda(x) \right)^{1/p} \tag{11}$$

that is to say explicitly :

$$\| f \|_{\mathcal{L}^p(E)} = M \left(\int_D | \ln(1 - f(x)/M) |^p \, d\lambda(x) \right)^{1/p} \tag{12}$$

For p=2, the space $\mathcal{L}^2(E)$ becomes a real Hilbert space with respect to the scalar product defined for two grey tone functions f and g by :

$$(f \mid g)_{L^2(E)} = \int_D (f(x) \mid g(x))_E \, d\lambda(x) \tag{13}$$

or, explicitly by :

$$(f \mid g)_{L^2(E)} = M^2 \int_D \ln\left(1 - \frac{f(x)}{M}\right) \ln\left(1 - \frac{g(x)}{M}\right) d\lambda(x) \tag{14}$$

The spaces $L^p(E)$ for p belonging to the interval [1, + ∞[being normed vector spaces, they possess underlying metrics. These ones will be studied in the following paragraph.

2. METRICS ADAPTED TO LOGARITHMIC IMAGES

The integration notion previously exposed allows distances between grey tone functions and consequently between images to be defined.

For the general theory of metric spaces the reader can refer to Bourbaki (1958-1961), Dunford and Schwartz (1988) or Kreyszig (1988).

2.1. Definition

For any real number p belonging to the interval [1, + ∞[, the corresponding $L^p(E)$ space is a complete metric vector space (that is to say a Frechet space) with respect to the metric, denoted d_p, defined for two arbitrary grey tone functions f and g belonging to $L^p(E)$ by :

$$d_p(f, g) = \| f \triangle g \|_{L^p(E)} \tag{15}$$

that is to say explicitely by :

$$d_p(f, g) = M \left(\int_D \left| \ln\left(\frac{M - f(x)}{M - g(x)}\right) \right|^p d\lambda(x) \right)^{1/p} \tag{16}$$

Remark : The metric space structure being heriditary, the set of images belonging to $L^p(E)$ is also a complete metric space with respect to the same metric.

Before to expose the corresponding expressions in the discrete case, that is to say the discrete metrics, it appears of great interest, in a first time, to establish the link between the metrics d_p and the optical density notion (see Dainty and Shaw, 1974), and furthermore to demonstrate that such metrics generalize a metric proposed by Pratt (1978, p. 168).

2.2. Links with the optical density function

For an arbitrary real number p belonging to the interval $[1, +\infty[$ the distance $d_p(f, g)$ between two arbitrary images f and g in $L^p(E)$ can be expressed in terms of the optical density function of the modulus of the difference between f and g. Indeed, the optical density function of an image h being the real-valued function, denoted D_h, defined on the domain D by:

$$D_h = -M \ln(1 - h/M) \qquad (17)$$

that of the image $|f \triangle g|_G$ is the real-valued function defined on the spatial domain D by:

$$D_{|f \triangle g|_G} = M \left| \ln\left(\frac{M-f}{M-g}\right) \right| \qquad (18)$$

So, the distance $d_p(f, g)$ equals the integral in the classical space $L^p(\mathfrak{R})$ of the optical density of the image $|f \triangle g|_G$.

2.3. The metrics d_p generalize a metric proposed by Pratt (1978, p. 168)

The distance between two arbitrary elements y_1 and y_2 of the space E, that is to say between two arbitrary grey tones, is defined by (see (6)):

$$d_E(y_1, y_2) = M \left| \ln\left(\frac{M-y_1}{M-y_2}\right) \right| \qquad (19)$$

Putting $Y_1 = M - y_1$ and $Y_2 = M - y_2$, yields:

$$d_E(y_1, y_2) = M \left| \ln\left(\frac{Y_1}{Y_2}\right) \right| \qquad (20)$$

where $|\ln(Y_1 / Y_2)|$ corresponds with the distance between Y_1 and Y_2 proposed by Pratt, taking into consideration the non-linear sensitivity of the human visual system, which is known to be logarithmic (Stockham, 1972).

The metric d_E and consequently the metrics d_p introduced in the LIP model bring an important progress since they are rigorously defined in compatible vectorial structures.

2.4. Expressions in the discrete case

In many practical situations, the grey tone functions are defined on a discrete domain D, generally obtained after a discretization process. So, it is important to establish in the discrete case the corresponding expressions of the previously defined metrics.

Suppose that D is a discrete set in $\Re^2$, then the measure λ on D is the cardinal measure. The discrete metrics corresponding to the formulas (15) and (16) are defined, for two arbitrary discrete grey tone functions f and g belonging to the space $L^p(E)$, by :

$$d_p(f, g) = M \left(\sum_{x \in D} \left| \ln \left(\frac{M - f(x)}{M - g(x)} \right) \right|^p \right)^{1/p} \quad (21)$$

where the real number p belongs to the interval $[1, +\infty[$.

2.5. Applications

The notion of distance is very useful in image processing in all the situations involving a quantitative comparison between images.

For example, when a given unknown image f is to be compared with a set of images of known origin $(g_i)_{i=1..n}$, the closest match between the unknown f and each of the known image g_i is obtained by selecting the smallest distance value between f and the images $(g_i)_{i=1..n}$.

In the context of the LIP model, this problem can be formalized by using a previously defined metric d_p. The solution g_{i_0} is such that :

$$d_p(f, g_{i_0}) = \min_{i = 1..n} d_p(f, g_i) \quad (22)$$

Generally the chosen metric is $d_2(.,.)$, which is an Euclidean distance.

Another exemple of the distance notion's usefulness exists in the area of image thresholding (See Pratt, 1978, or Gonzalez and Wintz, 1987). Indeed, for a given image f, one can create an associated thresholded image, denoted f_t, with respect to the threshold t, by defining :

$f_t(x) = t$ if $f(x) > t$

and $f_t(x) = 0$ if $f(x) \leq t$.

The optimal threshold t_0 being then obtained such that the distance between the original image f and its thresholded image f_t is minimal.

In the context of the LIP model, this optimization problem may be expressed by using a previously defined metric d_p. It consists in finding the threshold t_0 belonging to the interval $[0, M[$ such that :

$$d_p(f, f_{t_0}) = \inf_{t \in [0, M[} d_p(f, f_t) \quad (23)$$

Remark : The thresholds t_0 and t are really grey tones, that is to say elements of the space E defined above.

An illustration of the problem, but in the classical setting of linear images, is exposed in the paper of Jourlin and Labouré (personal communication, 1987).

3. SCALAR PRODUCT AND CORRELATION

For the general theory of Hilbert spaces the reader can refer to Berberian (1961), Schwartz (1979) or Kreyszig (1989).

3.1. Scalar product

The space $L^2(E)$ of square λ-integrable grey tone functions is topologically the powerfullest of all the $L^p(E)$ spaces. Indeed its structure is of Hilbertian nature with respect to the scalar product, denoted $(.|.)_{L^2(E)}$, defined for two grey tone functions f and g belonging to $L^2(E)$ by (see the formula (14)) :

$$(f\,|\,g)_{L^2(E)} = M^2 \int_D \ln\left(1 - \frac{f(x)}{M}\right) \ln\left(1 - \frac{g(x)}{M}\right) d\lambda(x) \tag{24}$$

Remark : The scalar product between two grey tone functions f and g belonging to $L^2(E)$ appears to be equal to the classical integral of the product of their optical density functions.

This scalar product is a real-valued mapping. In the important particular case involving two images f and g belonging to the Hilbert space $L^2(E)$, this scalar product becomes positive :

$$(f\,|\,g)_{L^2(E)} \geq 0 \text{ for two images } f \text{ and } g \text{ belonging to } L^2(E) \tag{25}$$

Before to expose some results about the orthogonality concept, let us introduce the angle between two non-zero grey tone functions f and g belonging to $L^2(E)$, which is denoted θ_{fg} and defined by :

$$\cos\theta_{fg} = \frac{(f\,|\,g)_{L^2(E)}}{||f||_{L^2(E)}\,||g||_{L^2(E)}} \tag{26}$$

This angle will play an interesting role in the section 3.3. related to the correlation.

3.2. Orthogonality

An important specific concept of Hilbertian structures is that of orthogonality. In the present setting, two grey tone functions f and g belonging to the real Hilbert space

$L^2(E)$ are called orthogonal if their scalar product equals zero, namely :

f and g are orthogonal if $(f\,|\,g)_{L^2(E)} = 0$ (27)

Consequently, their angle θ_{fg} is a square angle.

If f and g are two images belonging to the space $L^2(E)$, then they are orthogonal (in $L^2(E)$) if and only if they are almost nowhere strictly positive on D at the same time (See (25)).

One of the major consequences of the concept of orthogonality is the possibility to define and use Hilbertian basis, that is to say orthonormal complete systems. So, an arbitrary grey tone function f belonging to $L^2(E)$ can be expressed in terms of such a basis.

3.3. Correlation

An important notion in image processing (Ballard and Brown, 1982, Gonzalez and Wintz, 1987), in signal processing (Oppenheim and Schafer, 1975), or in pattern recognition (Duda and Hart, 1973), and more generally in functional analysis, is the notion of correlation.

In the setting of the LIP model, this powerful and useful notion can be introduced, by using the scalar product. Precisely, the correlation between two grey tone functions f and g belonging to the real Hilbert space $L^2(E)$ is denoted $\Gamma_{fg}(.)$ and defined at any point y in the domain D by :

$$\Gamma_{fg}(y) = (f\,|\,g_y)_{L^2(E)} \tag{28}$$

where g_y is defined as follows :

$$g_y : D \to E$$

$$x \to g(y+x)$$

Explicitly yields :

$$\Gamma_{fg}(y) = \int_D (f(x)\,|\,g(y+x))_E \; d\lambda(x) \tag{29}$$

$$= M^2 \int_D \ln\left(1 - \frac{f(x)}{M}\right) \ln\left(1 - \frac{g(y+x)}{M}\right) d\lambda(x)$$

Remark : If f and g are the same grey tone function, the correlation function is called the auto-correlation function ; if f and g are different, the term cross-correlation is normally used.

The correlation at a point y between f and g appears as a quantitative comparison between f and the translated g_y of g. One might have utilized the metric d_2 to evaluate the "difference" between f and g_y, namely $d_2(f, g_y)$, but the correlation notion is more adapted since it takes into account the Hilbertian nature of the space $L^2(E)$. Indeed the distance $d_2(f, g_y)$ and the correlation $\Gamma_{fg}(y)$ are explicitly linked by the following formula :

$$d_2(f, g_y)^2 = ||f||^2_{L^2(E)} + ||g_y||^2_{L^2(E)} - 2\Gamma_{fg}(y) \tag{30}$$

Remark : One observes then that a problem of minimization with respect to the metric d_2 is equivalent to a problem of maximization with respect to the correlation Γ.

However, the correlation is not totally an adequate "measure" of difference and it must be replaced by the normalized correlation. Precisely, the normalized correlation between two non-zero grey tone functions f and g belonging to the Hilbert space $L^2(E)$ is denoted $\gamma_{fg}(.)$ and defined at any point y in the spatial support D, such that the translated function g_y is non-zero almost everywhere on D, by :

$$\gamma_{fg}(y) = \frac{(f \mid g_y)_{L^2(E)}}{||f||_{L^2(E)} ||g_y||_{L^2(E)}} \tag{31}$$

So, the normalized correlation between f and g appears to be equal to :

$$\gamma_{fg}(y) = \cos \theta_{fg_y} \tag{32}$$

where θ_{fg_y} designates the angle between f and g_y (See (26)).

3.4. Expressions in the discrete case

Suppose that D is a discrete set in $\Re^2$, then the measure λ on D is the cardinal measure. The discrete expressions of scalar product, correlation and normalized correlation, corresponding to the formulas (24), (29) and (31), are defined for two non-zero discrete grey tone functions f and g belonging to the space $L^2(E)$ respectively by :

$$(f \mid g)_{L^2(E)} = M^2 \left(\sum_{x \in D} \ln\left(1 - \frac{f(x)}{M}\right) \ln\left(1 - \frac{g(x)}{M}\right) \right) \tag{33}$$

$$\Gamma_{fg}(y) = M^2 \left(\sum_{x \in D} \ln\left(1 - \frac{f(x)}{M}\right) \ln\left(1 - \frac{g(y+x)}{M}\right) \right) \tag{34}$$

and :

$$\gamma_{fg}(y) = \frac{\sum_{x \in D} \ln(1 - \frac{f(x)}{M}) \ln(1 - \frac{g(y+x)}{M})}{\left(\sum_{x \in D} (\ln(1 - \frac{f(x)}{M}))^2\right)^{1/2} \left(\sum_{x \in D} (\ln(1 - \frac{g(y+x)}{M}))^2\right)^{1/2}} \quad (35)$$

3.5. Applications

One of the main applications of the correlation in image processing lies in the area of template matching, where the problem is to detect within a given image f a replica of an object g of interest (see Pratt, 1978, Ballard and Brown, 1982, Gonzalez and Wintz, 1987). If the template match is sufficiently close, the object is detected and localized within the given image f and labeled as the template object g. In an Hilbertian structure the template matching is obtained by means of the correlation.

In the context of the LIP model, this optimization problem may be expressed by using the normalized correlation (see the previous remark). It consists in finding the point(s) y_0 in the domain D such that :

$$\gamma_{fg}(y_0) = \sup_{y \in D} \gamma_{fg}(y) \quad (36)$$

4. CONCLUSION

The present paper has used well-known powerful and useful notions and concepts : metrics, scalar product, orthogonality, correlation, normalized correlation, closely adapted to study logarithmic images. All the results and formulas have been exposed both in the continuous and the discrete case, and some important applications have been presented in the area of image comparison, image thresholding and template matching. Another important notion can be introduced in the LIP model : the Fourier transformation mathematically associated to logarithmic images. This transformation is based on the notion of product and mainly on that of complex logarithmic images (Pinoli, 1988, 1989, 1990).

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