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DETECTION OF DOMINANT POINTS ON A DIGITAL CLOSED CURVE

Marie José Laboure , Jean Azema , Thierry Fournel

Laboratoire TSI . Unité CNRS 842 . 23 Rue du Dr Michelon .
42023 Saint - Etienne cedex . France

ABSTRACT

A recursive algorithm for detecting dominant points on a digital non convex closed curve is presented . It is based on convex hull procedure and gives alternatively convexity or concavity points .

Keywords : shape analysis , digital curve , dominant point .

1. INTRODUCTION

Information on the shape of a planar closed curve γ is concentrated at dominant points : these points are the ones of γ which "summarize" the shape ; they are often high curvature points ; by joining them an optimal polygonal approximation of the curve is obtained . Several approaches to the problem of their detection have already been made . These methods consist in either finding these points directly by angle and corner detection or approximating the digital curve by a piecewise linear polygone whose vertices are dominant points (See in Teh & Chin (1989) a complete bibliography and the description of these algorithms).

The present approach is very different . A recursive algorithm , using convex hull procedure is described . At each step we obtain alternatively "convexity or concavity dominant points" without any curvature computation .

2. PRELIMINARIES

Let γ be a closed non convex curve in the euclidean plane. Let K be the compact body of boundary γ. A point x of γ is said to be a convexity point (respectively a concavity point) if there exists a neighbourhood V of x such that [y z] is included in K (resp. in the complementary of K) for any y , z of $V \cap \gamma$. An arc γ' of γ containing only convexity points (resp. concavity points) will be named a convex (resp. concave) region of γ (see figure 1). Let us denote by conv K the convex hull of K and by γ_c its boundary. γ_c includes common parts of γ and external parts of γ ; these segments will be named the added segments (see figure 2).

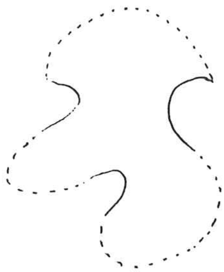

Fig. 1. - - - *convex region* — *concave region*

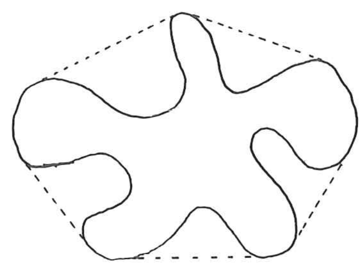

Fig. 2 . added segments : [Ai Bi] ....

3 . DETECTION OF FIRST ORDER CONVEXITY DOMINANT POINTS

We have studied in a previous paper (Labouré et al (1989)) the set of farthest points on Conv K denoted here by E^0 (see figure 3). Let us remind that E^0 is the set of points of γ_c which realize the maximal distance with at least one point of γ_c .

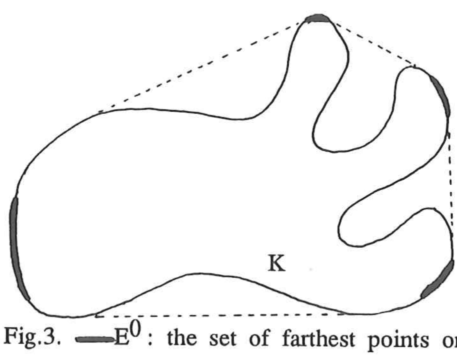

Fig.3. ▬ E^0 : the set of farthest points on the convex hull of K

It has been proved that farthest points on Conv K cannot be interior to a segment of γ_c because they are always extreme points. Farthest points are high convex curvature points of γ_c and γ, since osculator centre (when it is defined) in such a point is in Conv K. Thus E^0 corresponds to the most significant convexity points. We say E^0 is the set of first order convexity points.

4. DETECTION OF FIRST ORDER CONCAVITY DOMINANT POINTS

Let us denote by D^1 the set of cavities which are obtained by difference between K and Conv K:

$$D^1 = \overline{\text{Conv K - K}}$$

(where the symbol ——— refers to topological adherence in order to obtain a compact set D^1)

D^1 represents the set of cavities D_i^1 which are bounded by the added segments $[A_i B_i]$. Let us now examine one of these cavities D_i^1 bounded by $[A_i B_i]$ (see figure 4).

Let h_i^1 be the maximal distance between the support line of $[A_i B_i]$ and the boundary γ_i^1 of D_i^1. The set E_i^1 of points realizing this maximal distance h_i^1 is the set of farthest points of D_i^1 from the line $(A_i B_i)$.

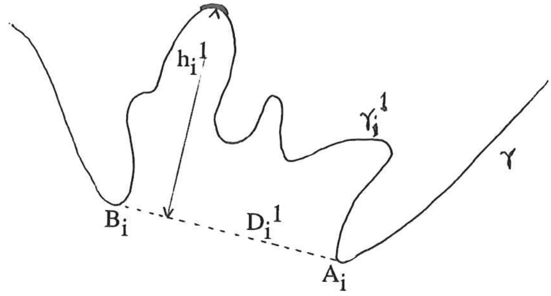

Fig. 4.

— E_i^1 : points realizing maximal distance between $(A_i B_i)$ and the edge of the cavity

It is clear that E_i^1 corresponds to a convex region on γ_i^1, which implies it corresponds to a concave region on γ. We consider that this region is significant if its depth h_i^1 is not too small compared to the size of K. For the implementation we compute the ratio of h_i^1 by the maximal euclidean diameter of K (which has the advantage to be independent of scale) . If this ratio is lower than an arbitrary value λ related to the context we do not take into account the cavity D_i^1 : the concavity E_i^1 is not significant . The union of significant concavities E_i^1 is denoted by E^1 and corresponds to the deepest areas of γ in Conv K ; we name them first order concavities on γ.

5 . DETECTION OF HIGHER ORDER DOMINANT POINTS

The convex hull of each cavity D_i^1 is bounded by the segments $[A_i B_i]$. Let us denote by γ_{ci}^1 the boundary of conv D_i^1 . The precedent procedure may be applied again on the second order cavities defined by

$$D_i^2 = \overline{\text{Conv } D_i^1 - D_i^1} .$$

To each cavity D^2_{ij} of D_i^2 , bounded by the added segment $[A_{ij}\ B_{ij}]$ we associate the farthest points from $(A_{ij}\ B_{ij})$. These points are concavity points of γ_i^1 and therefore convexity points on γ.

Let us eliminate as above non significant convexities. Thus we obtain second order convexity points E_i^2 . Their union is denoted by E^2.

Let us recursively apply this procedure to the cavities obtained by difference between the precedent order cavity and its convex hull . The process stops when a non significant cavity is found . By construction we get $E^0, E^2, \ldots E^{2K} \ldots$: convex regions (respectively E^1, E^3, ... $E^{2K+1} \ldots$: concave regions) on γ , from the most exterior to the most interior convexity (respectively from the most interior to the most exterior concavity) (see figure 5) .

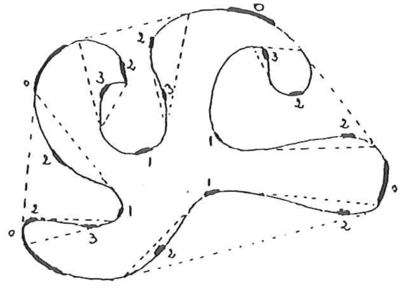

Fig.5. — all the dominant points with their order number

6. THE COMPLETE SET OF DOMINANT POINTS

We have implemented the recursive research of $E^0, E^2, \ldots E^K$. It has produced good results about dominant points but the shape of K was not all defined by these sets in many cases. So we have decided to add to the set $\cup E^k$ of dominant points, the extremities of the added segments when they are related to significant cavities. They are indeed very important points since they are the extreme points which precede a curvature change on γ. They are particularly necessary at the first step since farthest points are not sufficient in many cases (see for example figure 6).

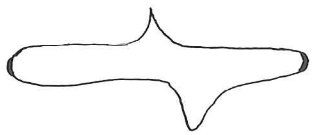

Fig. 6. — E^0 : Farthest points

Starting from $k = 0$, let F^k denote the union of E^k with the set Ex^k of the new extremities of the added segments at the step k (that means the extremities which have not already been found as dominant points at a previous step). Thus we obtain the sets F^k of dominant points. If k is odd F^k is a set of concavity points, else F^k is a set of convexity points.

7. IMPLEMENTATION, RESULTS AND CONCUSION

The simultaneous determination of the convex hull and the added segments extremities of a simple n points polygonal contour γ given by their Freeman code on a grid is done in O(n) time (Chen C- L (1989)) . Finding the farthest points on Conv K (i . e E^0) requires $O(n^2)$ time. For each following step the convex hull procedure is applied to disjointed pieces of γ, so it requires again O(n) time. Finding all the sets E^i ($i > 0$) takes O(n) time. The complexity of the method depends only on the number of steps (which is, most of the time in the applications, lower than 4). Thus the method gives fast results.
We can see some results on figure 7 (image of a turbulent jet) where dominant points represent about 2.6 % of the initial contour. By joining the adjacent vertices (i.e the dominant points) a polygon is drawn, which approximates the original shape.

This algorithm is less interesting for convex contours : in this case only farthest points detection is performed , and polygonal approximation may be very far from the initial contour : see for example the case of a flattened triangle (Labouré et al 1989) , then the polygonal approximation is a segment ! Results are better when cavities are numerous .
We have computed the ' error ' between the initial curve and its polygonal approximation by the evaluation of the maximal distance between them (i. e Hausdorf distance) and the ratio between this distance and the maximal euclidean diameter of K (around 3 % on this example).
In conclusion we can say that the interests of this method are : ---- independance of scale (which was not the case of the precedent existing methods) --- no curvature computation (which would take much more time) ---- a small number of points is sufficient to re - build accurately the original shape .

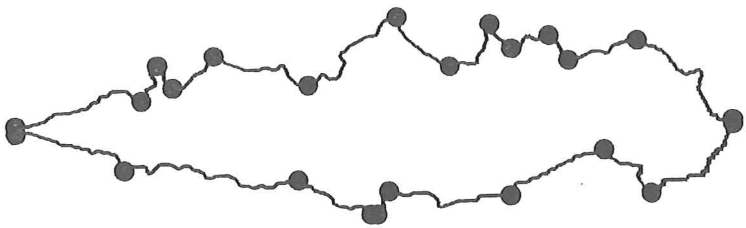

Fig. 7. Superposition of the original shape (turbulent jet) and its polygonal approximation by joining dominant points.

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