ACTA STEREOL 1991; 10/1: 7-23
STERMAT'90
ORIGINAL SCIENTIFIC PAPER

GRANULOMETRY AND GRANULOMORPHY BY IMAGE ANALYSIS

Jean-Louis **CHERMANT** and Michel **COSTER**

LERMAT, URA CNRS n°1317, ISMRa-Université
6 Boulevard du Maréchal Juin
14050 CAEN Cedex, FRANCE

ABSTRACT

The principles of size distribution analysis on binary images are presented in the case of i) sets of individualized particles (pulverulent particles observed by projection) and ii) materials with interconnected or dispersed phases (observed on sections). A critical study on the choice of the method and of the size criterion to be used is then presented, taking into consideration the bias given by the frame of measurement.

An extension of the granulometric analysis to functions is presented. These new developments allow to investigate directly the texture of grey tone images (un-thresholded).

A granulomorphic method is also proposed to investigate the shape of particles as it is closely related to their size.

Several examples illustrate these different methods.

Keywords : Granulometric analysis, granulomorphy, size distribution, mathematical morphology, grey tone functions, texture, shape.

INTRODUCTION

The aim of image analysis is to quantify information contained in an image. This quantification is made, either by measurement of mean parameters, or by establishing a size distribution function depending on metric parameters.

These mean parameters or functions can be classified according to the nature of the quantified morphological information. For an image containing different objects, they are :

- their importance with regard to the background of the image,
- their size,
- their shape,
- their dispersion.

Among these, size is the most used parameter. The *size distribution function* or *granulometric function* is most often utilized to investigate the size of a structural features. It is obtained by classifying the objects according to a *criterion of size*. The indicator function can be either a numbering or a geometrical measure on a surface.

When using a physical method (sieving, sedimentation, laser diffusion, ...) the criterion of size and the method of classification are unique and determined from the underlying physical principle.

In image analysis the size criterion and the classification methods are multiple : this is due to the fact that there does not exist a unique geometrical definition of the size and then as there are many ways to sift the objects. This is one of the reasons which led Matheron (1975) to develop the fundamental axioms of granulometry which will be presented.

The first part of the present paper will be devoted to the derivation of the various methods from the aforementioned axioms. Then a recall will be made of the classical techniques of set granulometric analysis, followed by a comparative study. This will lead us to introduce the notion of *granulometric analysis of functions*, by use of which image analysis affords the possibility to characterize the size of texture of un-threshold images. We note un-threshold image because it is an image which cannot be thresholded like a binary image.

In the second part it will be shown how these axioms can also be utilized to classify particles according to their shape, in introducing the notion of *granulomorphy*.

MATHERON'S AXIOMS

Any kind of granulometric analysis implies a sifting of objects with an operator T_λ depending on a parameter of size λ. For example if X is a set of disconnected objects, the transformation $T_\lambda(X)$ will retain objects for which the length is greater than λ. In order to be valid as a granulometric tool, the transformation T_λ applied to the sets X or Y, must verify the three following properties :

- T_λ must be increasing : $X_1 \subset X_2 \Rightarrow T_\lambda(X_1) \subset T_\lambda(X_2)$,
- T_λ must be anti-extensive : $T_\lambda(X) \subset X$,
- T_λ must be independent of the order of the operations ; the meaning of that last condition can be explained by reference to the two transformations $T_{\lambda 1}$ and $T_{\lambda 2}$ (with $\lambda_1 \neq \lambda_2$) : the resultant transformation must give a result independent of the order of the operators :

$$T_{\lambda 1}(T_{\lambda 2}(X)) = T_{\lambda 2}(T_{\lambda 1}(X)) = T_{\sup(\lambda 1, \lambda 2)}(X)$$

This relationship, which implies the idempotence of the operator, is the most specific among these three properties.

Matheron's axiomatics are then a valuable guide to realize an extension of the granulometric methods, as will be presented later on.

GRANULOMETRIC METHODS ON BINARY IMAGES

Granulometric methods cannot be properly devised without taking into account the specificities of image analysis by which they will be implemented.

Any granulometric analysis possesses a statistical character, since the size distribution which has to be established pertains to a set limited by one or several fragmentary observations. This observation is made through the frame of measurement the size of which interacts with the analyzed set. Thus the bias introduced by this frame will have to be taken into account.

In materials science, as in biology or in medicine, the analysis is very often performed on binary images, i.e. on images for which the objects have been extracted by a threshold or a boundary detection. The choice of the method of analysis and of the size criterion will depend on the topological nature of the set to be analyzed. In fact two cases can arise :

1- the set X is composed of disconnected elements (grains, precipitates, cells, ...) : in these conditions two methods of examination are possible : observation of a projection for pulverulent materials or of a section (thin slide) for solid media ; for this type of topology, the granulometric analysis is called *individual granulometric analysis*,
2- the set X is an interconnected phase : for this type of topology, the observation of a section (or of a thin slide) of a solid media is possible ; the granulometric analysis is then called *set granulometric analysis*.

Individual granulometric analysis

When the set to be analyzed is made of individualized objects, each object can be analyzed in a sequential way.

Any metric magnitude associated with the object can operate as a size criterion. The method of sifting will correspond either to a classification as a function of the measure, or to an operation of mathematical morphology (Serra, 1982 ; Coster & Chermant, 1985 ; 1989), as the erosion-reconstruction (Coster, 1988). To be measured, the object must be totally included in the frame of measurement. Table I gives the main methods of individual granulometric analysis and figure 1 the geometrical meaning of the size criterion.

Table I . Geometrical meaning of the different size parameters used for granulometric investigations.

Size criterion	Robustness of the measure	Method
surface	strong	individual measurement
perimeter	weak	individual measurement
exodiameter	strong	individual measurement
mesodiameter	medium	individual measurement
maximum inscribable element	strong	opening
geodesic length	strong	geodesic propagation

The perimeter or surface measurement is a basic function of every image analyzer. The exodiameter (maximum Feret diameter) and the mesodiameter (minimum Feret diameter) are easily calculable from the coordinates of the boundary.

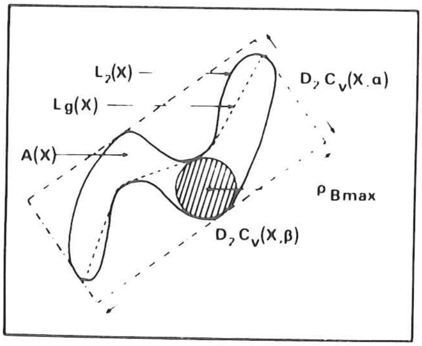

Fig. 1. Geometrical meaning of the different size parameters used for granulometric investigations. A(X) : surface area ; $L_2(X)$: perimeter ; $D_2C_v(X,\alpha)$: mesodiameter ; $D_2C_v(X,\beta)$: exodiameter ; Lg(X) : geodesic length ; ρ_{Bmax}: radius of the maximum inscribable element.

The last two size criteria use mathematical morphological functions and can only be utilized with a suitable automatic analyzer. In that case, it is possible to accelerate the sifting by not using an individual analysis.

As has already been mentioned, each analyzed object must be totally included in the frame of measurement. In the case of solid media for which the structure is stationary random, the probability of inclusion depend on the ratio between the size of the particle and that of the frame : a weight equal to the reciprocal of the inclusion probability must be attributed for each object. Lantuejoul demonstrated in 1980 that this weight is equal to the ratio between the surface A(Z) of the frame of measurement Z and the surface of the frame of measurement eroded by the rectangle R_i circumscribed to the object X_i (Fig. 2), noted $A(E^{R_i}(Z))$, that is :

$$\mathcal{P}(X_i) = \frac{A(Z)}{A(E^{R_i}(Z))} \qquad (1)$$

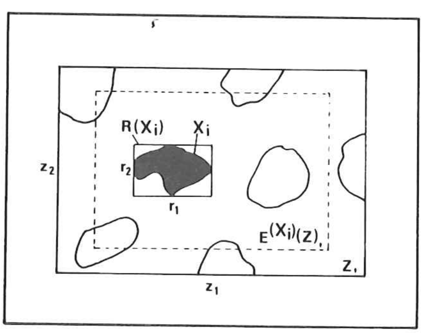

Fig. 2 . The corrective method of Lantuejoul.

For granulometric analysis of pulverulent materials, the stationarity (Serra, 1982) of the sampling cannot be assumed. In these conditions, it is better to strain the inclusion in the frame of measurement and to analyse systematically all particles in a broad domain of the sampling. If we strain the inclusion, the corrective method ($\mathcal{P}(X_i)=1,\ \forall\ X_i$) may not be used.

For any individual granulometric analysis, the size distribution functions can be established in number, F(λ), or in measure, G(λ). Generally the distribution in number is the most suitable. It can be written :

$$F(\lambda) = \frac{\Sigma\ \mathcal{P}(X_i) - \Sigma\ \mathcal{P}(X_i\ :\ \mathrm{Mes}\ X_i \geq \lambda)}{\Sigma\ \mathcal{P}(X_i)} \qquad (2)$$

Set granulometric analysis

When the set X is interconnected, i.e. when the notion of an individual object has no meaning (porous network, dendritic phase, ...), the method of sifting is necessarily a filtering by a morphological opening with a convex structuring element (square, hexagon segment, ...) of size λ (Serra, 1982 ; Coster & Chermant, 1985 ; 1989) (Fig. 3). Size distribution can be also established in number or in measure with linear openings. But they must be imperatively undertaken by measurements of surface for the other structuring elements (Serra, 1982 ; Coster & Chermant, 1985 ; 1989).

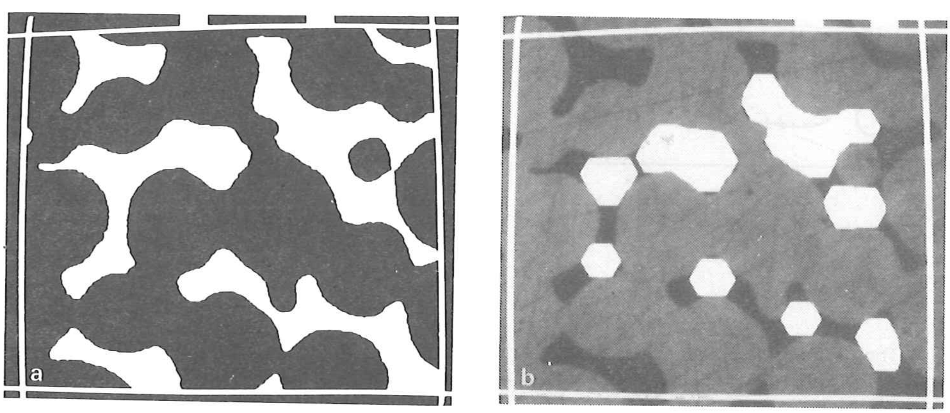

Fig. 3 . Micrography of a copper material (detected phase) sintered for 28h at 900°C (a) and corresponding image after 10 steps of hexagonal opening on the porous phase (b).

The bias, due to the frame of measurement, will be corrected by carrying out the measurement in the frame eroded twice by the structuring element (Serra, 1982). The size distribution in measure is then given by :

$$G(B,\lambda) = \frac{A\ (X \cap E^{2\lambda B}(Z) - A\ (O^{\lambda B}(X) \cap E^{2\lambda B}(Z))}{A\ (E^{2\lambda B}(Z))} \quad (3)$$

Case of size distributions by linear opening

The most straightforward procedure for granulometry by opening with a linear structuring element ℓ relies on the function $P(\ell)$ which corresponds to the probability that a segment of length ℓ be fully included within the set X (Serra, 1982). From the operational standpoint, this function is merely the expected value of the proportion of the eroded X by ℓ (Fig. 4). Taking into account the correction of the frame of measurement, for a segment ℓ oriented in the direction α, one gets :

$$P(\ell,\alpha) = \frac{\text{Mes}\ (E^{\ell}(X) \cap E^{\ell}(Z))}{\text{Mes}\ (E^{\ell}(Z))} \quad (4)$$

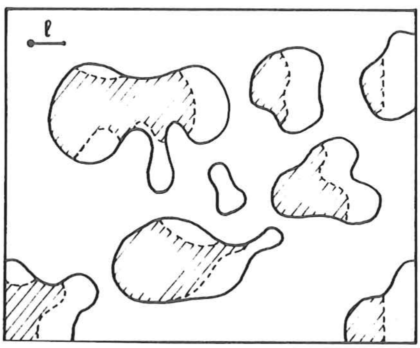

Fig. 4 . Erosion of a set X by a straight segment ℓ.

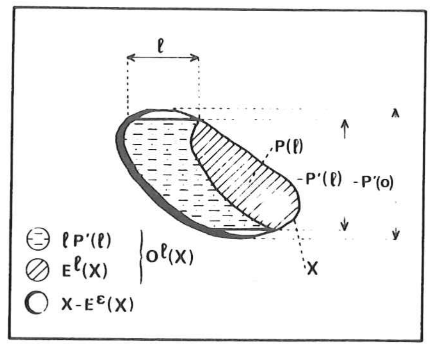

Fig. 5 . Illustration of the granulometric properties of the $P(\ell)$ function.

The $P(\ell)$ function possesses granulometric properties illustrated in Figure 5. The granulometric distribution in number is given by :

$$F(\ell) = \frac{P'(0) - P'(\ell)}{P'(0)} \quad (5)$$

and in measure by :

$$G(\ell) = \frac{P(0) - P(\ell) + \ell P'(\ell)}{P(0)} \quad (6)$$

where the prime superscript designates the derivative.

It is to be noted that the $P(\ell)$ function possesses also stereological properties : the third order moment of this function provides access to the mean volume in measure of the set X (star function in $\mathbb{R}^3$) :

$$St_3(X) = \frac{4}{P(0)} \int_0^{\infty} \ell^2 P(\ell) d\ell \qquad (7)$$

An example of $F(\ell)$ and $G(\ell)$ distributions is given in figure 6, in the case of polycrystalline silicon for solar cells. Low cell number corresponds to solar cells closed to the bottom of the ingot (they are machined parallely to the bottom of the crucible). We notice that the $G(\ell)$ function is physically more relevant than the $F(\ell)$ function : while the granulometric distribution in number is nearly independent of the position of the cell in the ingot, the distribution in measure shows that 20% of the crystals (weighted percentage) exceed 10 mm in those cells which are located close to the bottom of the ingot (Chermant et al., 1986), instead of only some percents for the cells machined at the top of the ingot.

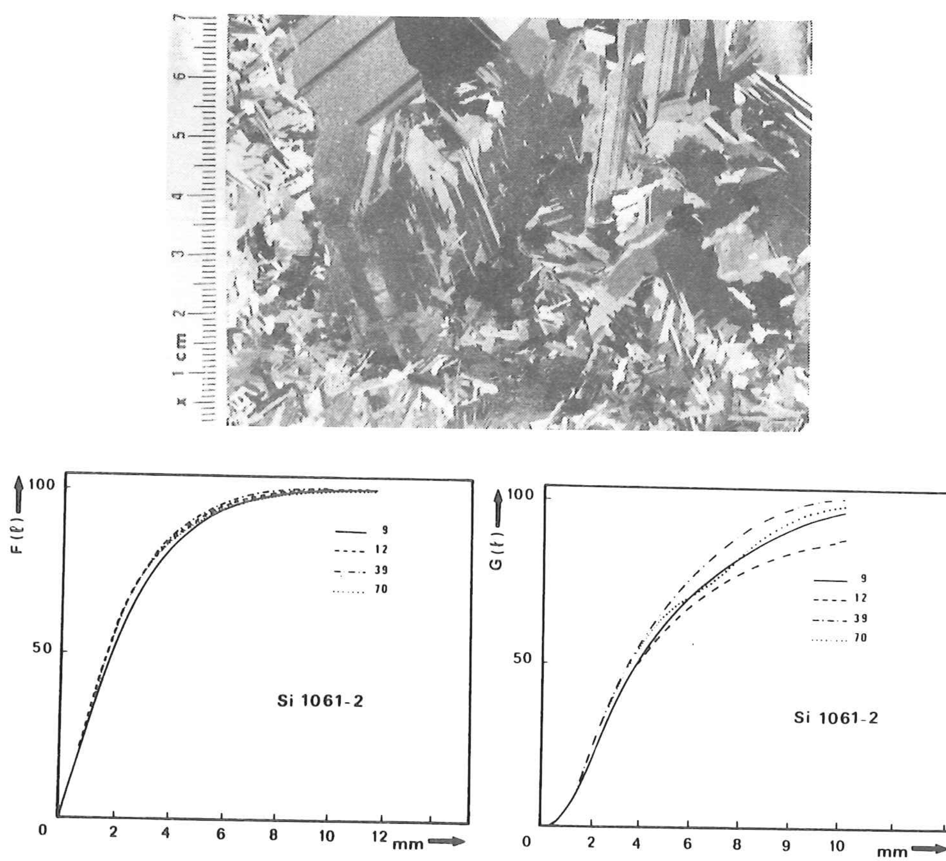

Fig. 6 . Silicon crystal size distribution curves in number, $F(\ell)$, and in measure, $G(\ell)$, for different polycrystalline solar cells.

GRANULOMETRIES ON FUNCTIONS

Matheron's axiomatics, originally established in a set framework, can be, of course, extended to functions. This possibility becomes of much avail when no segmentation of the image in grey tone levels is possible. That is the case of granular rough surfaces or of fractured surfaces observed by scanning electron microscopy (SEM). Images of this nature have generally unimodal histograms in grey tone levels (Fig. 7), i.e. there does not exist any threshold algorithm allowing to obtain a representative binary image.

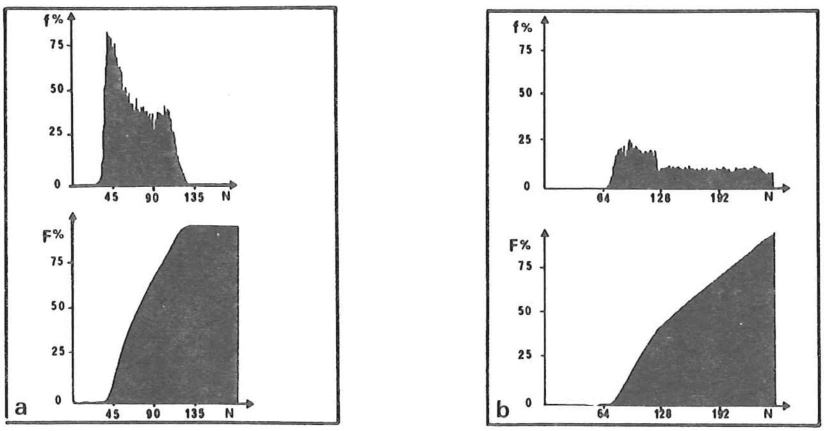

Fig. 7 . Histograms of the image in grey tone level of the figure 8a for an aperture value of 3.5 (a) and 8 (b).

Morphological transformations in grey tone levels

All the set morphological operators (erosion, dilation, opening, closing, ...) (Serra, 1989) possess their counterpart for the functions. So the definition of the erosion applied to a function f(x) is given by :

$$E^B f(x) = \inf \left\{ f(u) = u \in B_x \right\} \quad (8)$$

and the dilation by :

$$D^B f(x) = \sup \left\{ f(u) : u \in B_x \right\} \quad (9)$$

The eroded and dilated sets of f(x) are respectively the inferior and superior value of the function f(x) in the domain B centered on x. These elementary operators allow to define morphological openings and closings on the functions :

$$O^{B}f(x) = D^{\check{B}}E^{B}f(x) = \sup_{y \in \check{B}_x} \left\{ \inf_{z \in {}_y} [f(z)] \right\} \qquad (10)$$

$$F^{B}f(x) = E^{\check{B}}D^{B}f(x) = \inf_{y \in \check{B}_x} \left\{ \sup_{z \in {}_y} [f(z)] \right\} \qquad (11)$$

Matheron has generalized his axiomatics to functions. Thus an application carrying out on a grey tone function and depending on a positive parameter λ is a granulometry if this mapping is an opening according to the third principle of the Matheron's axioms. By duality an anti-granulometry is defined in replacing opening by closing.

We have thus a means to analyse the size of a texture represented by an holomorph function f(x) (i.e. a function for which there exists only one value f(x) for a given x), where x represents a point of the space $Z \in \mathbb{R}^2$ (image space) and f(x) its radiometric value.

The granulometric distribution function is given by :

$$G(f,\lambda) = \frac{\int_Z (f(x) - O^{\lambda B}f(x))dx}{\int_Z f(x)dx} \qquad (12)$$

This distribution function is normalized ($0 \le G(f,\lambda) \le 1$), if the minimum global value of f(x) in the Z support is equal to zero. In the opposite case, it would be necessary to modify the radiometric values of the pixels by linear anamorphosis :

$$f_1(x) = f(x) - E^{Z}f(x) \qquad (13)$$

The granulometric distribution is then established on the function $f_1(x)$. If the structuring element B_x is a flat structuring element, the granulometric distribution on the function f(x) is independent of every anamorphosis and particularly of the previous one.

This last property is very important as it shows that the granulometric distribution of an image in grey tone levels is robust, regarding the variations in the acquiring conditions (signal amplification, contrast, ...).

Example : granulometric analysis of carbide particles

Steels with a high carbon content, obtained by extrusion of microcrystalline powders, contain a high ratio of carbides (Mordike et al., 1986 ; Michelland-Abbé, 1990). Due to their small size, these carbide crystals can only be observed by scanning electron microscopy. To reveal their granular structure, the specimens must be chemically etched in order to take off the grains. Then the corresponding image is a topographic one for which the grey tone levels are function of the local orientation of the surface. It is intuitively clear that image texture will possess size characteristics in close relation to the granular texture. A granulometry in grey tone levels by opening will therefore provide indirect access to the carbide grain size distibution.

This method of granulometric analysis has been implemented on a SUN 3/140 with an image processor Imaging Technologies series 151, and a VISILOG environment. The morphological opening has been performed with a square grid. Figure 8 illustrates the image transformation for different opening steps.

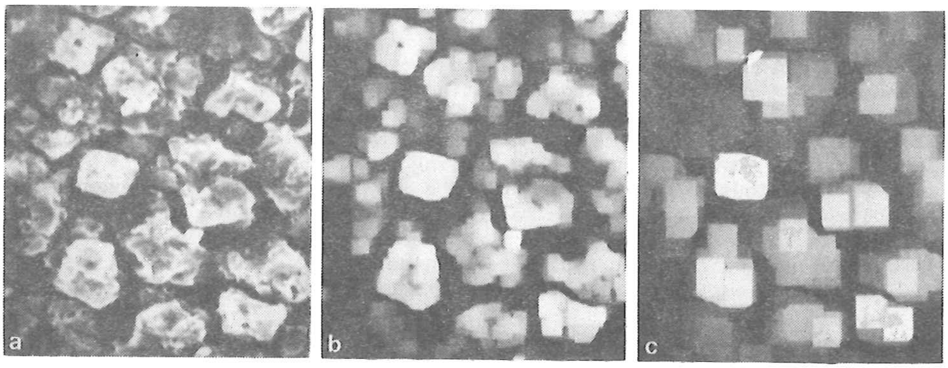

Fig. 8 . Video image of a microcrystalline steel (a) and images after an opening of size 5 (b) and 10 (c).

Before developing that approach as a general tool in materials science, we had to assert its robustness in a practical situation and see whether the data pertaining to the etched material was representative of the granular structure.

a) Robustness of the granulometric measurements

As a first step, we have verified that variations in the conditions of data acquisition had no influence on the results of the granulometric analysis. This experiment has been realized from SEM photographs via a CCD camera. From figure 9, we note that an anamorphosis on the grey tone levels

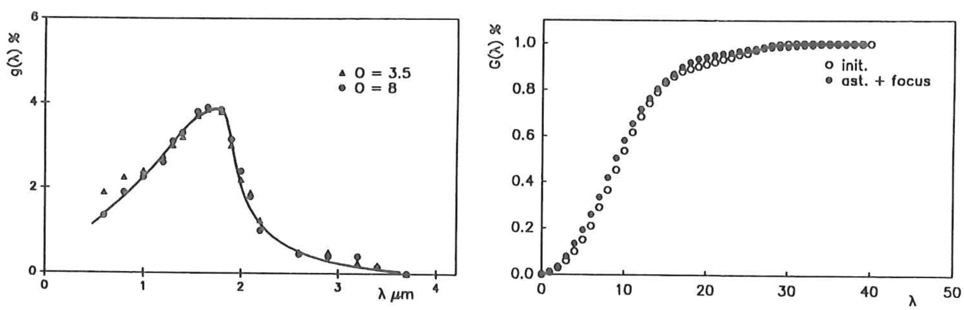

Fig. 9 . Granulometric density, g(λ), and granulometric distribution, G(λ), for an area of the same specimen photographed with different aperture values (O=3.5 or 8) and different values of astigmatism and focus.

produced by a change in the diaphragm value and in the camera gain does not change the size distribution ; it is the same when astigmatism and focus values are changed on the SEM (Fig. 9).

b) Comparison with a semi-automatic method

As explained above, the basic hypothesis at the root of this investigation was that the texture (global aspect of the image) was induced by the granular nature of the carbide crystals. So a texture granulometry must be correlated to a granulometry of the carbide crystals.

To obtain the size distribution of the carbides, the only practical solution, for the material under study, was to resort to a semi-automatic method with a digitizing table, leaving up to the experimenter the care of assessing by himself the limits of the crystals on the table where the SEM image is projected. The boundary limit of each carbide grain has been manually drawn with the help of the reticle of the table. A granulometric sifting was then realized according to the crystal surface area.

Figure 10 shows that the results obtained in that way are in remarkable agreement with that given by the opening method in grey tone levels

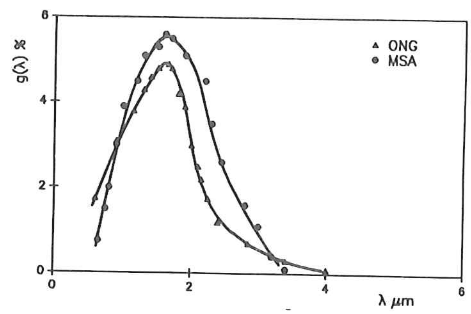

Fig. 10 . Results of the size distribution by semi-automatic method (MSA) and of the texture by grey tone level openings (ONG) on the same sintered steel.

(the difference in size is less than 10%). This strongly vindicates Matheron's concept of generalized granulometry by morphological transformations of functions.

c) Results

As an application of that type of analysis, it has been found possible in our laboratory (Michelland et al., 1989a ; Michelland-Abbé, 1990) to follow the change in the carbide size in microcrystalline steels as a function of the atomization conditions of the powder, of the carbon contents and thermal treatment. It was shown that the carbide size decreased with thermal treatment.

FROM GRANULOMETRY TO GRANULOMORPHY

As already stated, granulometric methods allow to sieve objects according to a size criterion λ. If this size criterion is replaced by a shape criterion, it is possible to extend Matheron's axiomatics. This gives rise to a whole range of new methods, which we designate by the generic term of *granulomorphy* (Michelland et al., 1989).

Just as for granulometry of binary images, granulomorphy can be divided in two classes, namely individual analysis and set analysis.

Granulomorphy by individual analysis

In the same way as individual granulometric analysis, granulomorphy by individual analysis implies that the particle (connex component) is totally included in the frame of measurement.

a) Definition of shape criterion

In order to be compatible with the usual concepts, the shape of an object must be defined independently of its size. The selected shape criteria to be invariant under transformation by homothety, imply that they have no dimension. The shape factor ϕ will be most often made up of two metric parameters, λ and μ, both related to the size, but taken with some suitable exponents, ℓ and m, so that their ratio is dimensionless :

$$\phi = \frac{\lambda^{\ell}}{\mu^{m}} \tag{14}$$

It is to be noted that a topological parameter, such as the genus, can serve as shape criterion.

b) Granulomorphic distribution by individual analysis

For each connex component included in the frame of measurement, the shape factor is either directly calculated or determined after image transformation.

The laws of individual analysis are the same for granulomorphy as for granulometry. The granulomorphic size distributions are generally established in number by counting the connex components whose shape factor $\phi(X_i)$ is less than or equal to a given ϕ value. The size distribution function is given by :

$$F(\phi) = \frac{Nb(X) - Nb(X : \phi\ (X\ )_i \leq \phi)}{Nb(X)} \tag{15}$$

with : $\underset{i}{X} = \sum X_{i}$,

X_i : connex component.

The Lantuejoul correction (1980) analysis holds in the same way as in granulometry.

c) Example : Granulomorphy of steel powders

Steel powders can be obtained by atomizing liquid metal in cold water. The powder is then sieved and each sieved fraction can be analyzed by granulometry and granulomorphy. Regarding the granulomorphic analysis, two different scales of analysis have been carried out : the macroscopic scale which gives information on the particle silhouettes (outline) and the microscopic scale which gives information on the roughness.

The chosen macroscopic shape factor is the eccentricity of the equivalent ellipse calculated by Serra's method (1975). This parameter characterizes the particle lengthening. To avoid the noise due to roughness, the calculation has been made on the digital convex hull using an hexagonal grid. Three distribution classes (circular C, elongated A, very elongated TA) corresponding to three mean values of the eccentricity have been determined (Fig. 11). We note on this figure that the proportion of elongated particles increases when their size increases.

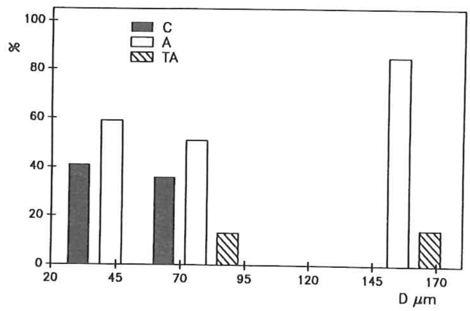

Fig. 11 . Change in the granulomorphic distribution as a function of the particle size (C : circular particles ; A : elongated ; TA : very elongated).

To investigate the powder roughness, image transformations have been used to isolate the asperities of each particle (Fig. 12). We use the following algorithm :

- thickening to obtain the hexagonal convex hull,
- "exclusive or" between convex hull and particle following by closing of half size of the ultimate eroded to obtain a result independant of the particle size,
- intersection between this last transform and the particle,
- union between the previous transform and the closed particle.

The final result is illustrated on Fig. 12. Two shape factors are then calculated :

$$F_1 = \frac{\text{surface area of asperities}}{\text{surface area of the particle}} \qquad (16)$$

$$F_2 = \frac{F_1}{\text{number of asperities}} \qquad (17)$$

One notes on figure 13 that the parameter F_1 increases with the equivalent diameter of particles while F_2 decreases abruptly for a value of $\bar{D}$ = 60 µm and then becomes stabilized.

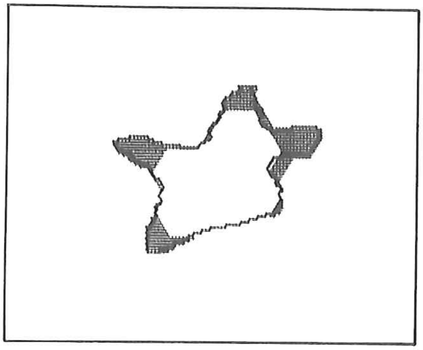

Fig. 12 . Roughness extraction of a particle.

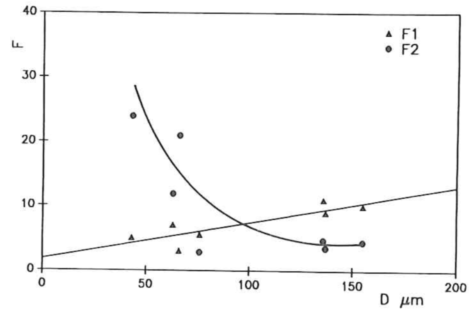

Fig. 13 . Change in the roughness parameters of the particles, F_1 and F_2, as a function of their size, D (equivalent mean diameter).

Granulomorphy by set method

a) Principle

Although the notion of shape can only be defined for a connex component, it is possible to consider a simultaneous treatment of all the connex components intersected by the frame of measurement. It includes 2 steps : i) the connex components are classified according to a granulometric transformation ψ_λ ; ii) each granulometric class is sieved using the granulometric transformation ψ_μ. This process is then related to the shape factor $\phi = \lambda/\mu$.

To undertake such an analysis without bias, the principle of the local knowledge defined for the granulometries on an eroded frame Z' must be used. As for the granulometries by morphological opening, the size distribution function must be established in measure according to the relationship:

$$G(\phi) = \frac{A_A(X \cap Z') - A_A(\psi_\phi(X \cap Z'))}{A_A(X \cap Z')} \qquad (18)$$

with : $A_A(X \cap Z')$ the fraction of surface area of X in the frame eroded by Z',

$\psi_\phi = \psi_\lambda \psi_\mu$.

Instead of the two successive granulometric transformations, it is possible to construct a granulomorphy using in a first step an homeomorphic transformation simplifying the image and in a second step a non-homotopic transformation to separate the components of different topology.

b) Example : Granulomorphy of Si in Al-Si alloys

The shape of silicon crystals in Al-Si alloys is a function of the chemical content and the thermal treatment. To analyze the shape of this phase, two granulomorphic methods have been used : the first corresponds to a topological classification and the second to two successive granulometric classes according to two different criteria of size (Gougeon, 1988 ; Michelland et al., 1989b).

In a first step the particles of silicon with and without holes are separated. The algorithm is based on a skeletonization followed by an infinite clipping. Only the loops related to components with holes then only remain, and they are used, as marker, to reconstruct the particles. Figure 14 displays the topological change as a function of thermal treatment.

* *Granulomorphic classification by crossed granulometries*

A classification according to the lengthening of particles without holes is obtained from two granulometric sieves. A first classification is based on the thickness of the ultimate eroded. For each class of ultimate eroded a classification according to the length in clipping the skeleton proportionally to the thickness is established. The particles more elongated than Λ (which is equivalent to ϕ in figure 15) retain markers after clipping and then can be reconstructed. Figure 15 illustrates this classification and figure 16 shows the change for each class of lengthening as a function of the thermal treatment. It is apparent that the particles become more and more circular during this metallurgical treatment.

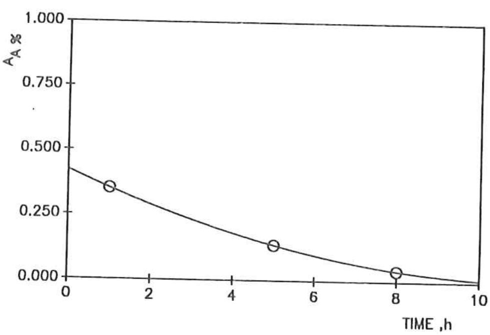

Fig. 14. Change in the surface area, A_A, of particles of silicon with holes as a function of the thermal treatment.

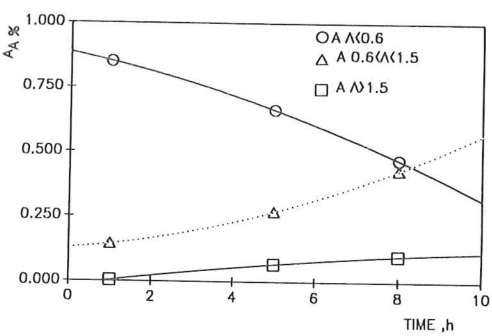

Fig. 16. Change in the surface area, A_A, of particles of silicon without holes as a function of thermal treatment. Λ is a parameter linked to the number of clipping which leads to the classification.

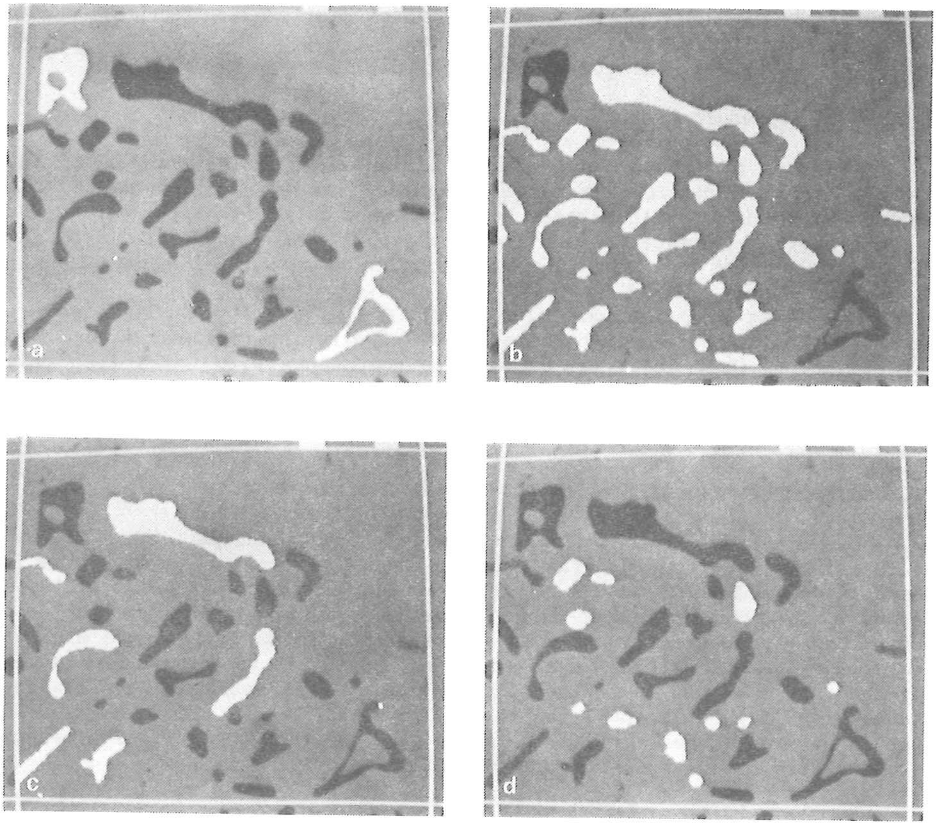

Fig. 15 . Example of classification of silicon particles in Al-Si alloys : a) detected particles with holes ; b) detected particles without hole ; c) particles of the first class ; d) particles of the third class.

CONCLUSION

In this paper we have presented granulometric and granulomorphic methods, based on the Matheron's axiomatics, based respectively on a criterion of size or of shape.

On one hand it has been also shown that granulometric analysis in grey tone levels broadens the range of investigations of materials by image analysis. Further applications include quantitative study of fractured surfaces by closings in grey tone levels (Michelland-Abbé, 1990) and more generally the characterization of rough surfaces, as for example that of ceramic films used for multilayer capacitors (Prod'homme et al, 1990).

On the other hand it has been shown that granulomorphy permits correct classification of objects to be made according to a shape criterion.

ACKNOWLEDGMENTS

Part of this work has been made under the contract PICS CNRS n°31 between the Institut für Werkstoffkunde und Werkstofftechnick of Clausthal-Zellerfeld, FRG, and the Laboratoire d'Etudes et de Recherches sur les Matériaux of Caen, France. Thanks are due to Drs Gilles Gougeon and Sophie Michelland for their authorization to present some parts of their results.

REFERENCES

Chermant JL, Coster M, Lay P, Nouet G. Morphological characterization of polycrystalline silicon. Acta Stereol 1986 ; 5/2 : 299-304.

Coster M. Les mesures granulométriques : axiomes et critères de taille, influence de la forme. Colloque Granulométrie, IFTS, Agen, June 1-2, 1988.

Coster M, Chermant JL. Précis d'analyse d'images. Les Editions du CNRS, 1985. Seconde édition, Les Presses du CNRS, 1989.

Gougeon G. Etude par analyse automatique d'images de la forme et de la texture d'alliages Al-Si. Thèse de Doctorat de l'Université de Caen, Fev. 1988.

Lantuejoul C. On the estimation of mean values in individual analysis of particles. Microscopica Acta 1980 ; S4 : 266-273.

Matheron G. Random sets and integral geometry. J. Wiley, 1975.

Michelland-Abbé S. Morphologie et comportement mécanique d'aciers microcristallins extrudés. Thèse de Doctorat de l'Université de Caen, June 1990.

Michelland S, Schiborr B, Coster M, Mordike BL, Chermant JL. Size distribution of granular materials from unthreshold images. J Microscopy 1989a; 156 : 303-311.

Michelland S, Gougeon G, Coster M, Chermant JL, Schiborr B, Mordike B.. Granulomorphic analysis in material science. Acta Stereol 1989 ; 8/2 : 381-386.

Mordike BL, Burchards HD, Kainer KU. Mechanical properties of rapidly quenched iron-based alloys. Int. Conf. Int. Met. and Semiconducting Glasses, Hyderabad, India, December 16-20, 1986.

Prod'homme M, Chermant L, Coster M, Chermant J. Etude de la texture de films céramiques par analyse automatique d'images. Troisièmes Journées Nationales d'Etudes : "Technologie des céramiques pour l'électronique et l'électrotechnique", CNET-SEE, Lannion, france, 15-16 May 1990.

Serra J. Anisotropy fast characterization. Fascicule 8 in 15 Fascicules de Morphologie mathématique appliquée. Centre de Géostatistique et de Morphologie Mathématique, Ecole des Mines, Fontainebleau, 1975.

Serra J. Image analysis and mathematical morphology. Academic Press, 1982.

Serra J. Image analysis and mathematical morphology : Vol. II. Theoretical advances, Academic Press, 1989.