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A CONTRAST DEFINITION FOR LOGARITHMIC IMAGES IN THE CONTINUOUS SETTING

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ABSTRACT

The specific laws and structures adapted to the processing and the analysis of logarithmic images, such as the images obtained by transmitted light or those outcoming from the human visual system, have been defined in the setting of the Logarithmic Image Processing Model (Jourlin, Pinoli, 1987 and 1988). This model called **L.I.P.** also allowed the introduction of a natural contrast definition, which is closely linked with logarithmic images and setted up in an vectorial structure. However, only the discrete case has been good studied until now (Jourlin, Pinoli, Zeboudj, 1989). With the tools developed elsewhere (Pinoli, 1987a and 1987b), this paper extends the contrast notion to the continuous case, as it was previously announced.

Keywords : Contrast definition, image modeling, logarithmic images, human vision.

1. RECALLS ON THE L.I.P. MODEL

An image can be completely modeled by its grey tone function. Such functions are defined on a compact D of the plane $\Re^2$ with range in the real interval [0,M[where M is a strictly positive real. In the context of transmitted light the value 0 is reached for a point x of D which is totally transparent, while the value M corresponds to a point which is totally opaque. As regards the human vision, M and 0 correspond respectively to the dark and the glare limit (Gonzalez, Wintz, 1977).

1.1. The vectorial structure on the grey tone function space (Jourlin, Pinoli, 1985, 1987 and 1988)

The laws denoted $\triangle\!\!\!+$ and $\triangle\!\!\!\times$ permit to define the sum of two images and also the positive homothetics associated to an image. The class of images, identified to the class of their grey tone functions, becomes then the positive cone, denoted Π, of the set of functions defined on D and with values in the interval $]-\infty, M[$ which is a vectorial space for the laws $\triangle\!\!\!+$ and $\triangle\!\!\!\times$. The elements of this space, denoted G, are called grey tone functions ; those of $]-\infty, M[$ are called grey tones.

Let us recall the definition of this laws :

$$f \triangle\!\!\!+ g = f + g - \frac{f\,g}{M} \qquad \forall\ (f, g) \in G^2 \tag{1}$$

$$\alpha \triangle\!\!\!\times f = M - M\left(1 - \frac{f}{M}\right)^{\alpha} \qquad \forall\ f \in G,\ \forall\ \alpha \in \Re \tag{2}$$

Remark : The inclusion $F(D, \Re) \supset \Pi$ allows "reading" an image f as an element of the space $F(D, \Re)$ of real functions defined on D. In this last case, classical addition and scalar multiplication hold. In order to avoid confusion an image is denoted f when considered as an element of $F(D, \Re)$.

1.2. The set of grey tones is a normed Riesz space (Pinoli, 1985 and 1987a)

The set of grey tones $]-\infty, M[$ is also a vectorial space, denoted E, with respect to the laws $\triangle\!\!\!+$ and $\triangle\!\!\!\times$:

$$E = (\,]-\infty, M[\,, \triangle\!\!\!+, \triangle\!\!\!\times) \tag{3}$$

Remark : The elements of the space E, that's to say the grey tones, denoted s, t, ..., must be used with the laws $\triangle\!\!\!+$ and $\triangle\!\!\!\times$. When they are considered as elements of the real space $\Re$, the classical addition and scalar multiplication hold. In order to avoid confusion in this last case they will be denoted s, t, ...

E becomes an Euclidean space for the scalar product denoted $(.\,|\,.)_E$ and defined for two grey tones s and t according to :

$$(s\,|\,t)_E = \mathrm{Ln}\left[\,1 - \frac{s}{M}\right] \mathrm{Ln}\left[\,1 - \frac{t}{M}\right] \tag{4}$$

Consequently E becomes a Banach space for the norm denoted $\|.\|_E$ and defined for any grey tone s by :

$$\|s\|_E = \left[(s \mid s)_E\right]^{\frac{1}{2}} = \left| \mathrm{Ln}\left(1 - \frac{\mathrm{s}}{\mathrm{M}}\right)\right| \tag{5}$$

where $|.|$ designates the absolute value in $\Re$.

E becomes also a strictly ordered space with the natural order relation :

$$s \le t \text{ if and only if } \mathrm{s} \le \mathrm{t} \tag{6}$$

Denoting the positive part, the negative part and the module of a grey tone s respectively by :

$$s_{\triangle\!\!\!\!\!+}\,,\ s_{\triangle\!\!\!\!\!-} \text{ and } |s|_E$$

yields :

$$s_{\triangle\!\!\!\!\!+} = \mathrm{Sup}\,(\mathbf{0}, s) = \mathrm{s}_+ \tag{7}$$

$$s_{\triangle\!\!\!\!\!-} = \mathrm{Sup}\,(\mathbf{0}, \triangle\!\!\!\!\!-\, s) = (\mathrm{M\,s}_-)\,/\,(\mathrm{M} + \mathrm{s}_-) \tag{8}$$

$$|s|_E = \mathrm{Sup}\,(s, \triangle\!\!\!\!\!-\, s) = (\mathrm{M}|\mathrm{s}|)\,/\,(\mathrm{M} + \mathrm{s}_-) \tag{9}$$

and :

$$s = s_{\triangle\!\!\!\!\!+} \ \triangle\!\!\!\!\!-\ s_{\triangle\!\!\!\!\!-} \tag{10}$$

$$|s|_E = s_{\triangle\!\!\!\!\!+} \ \triangle\!\!\!\!\!+\ s_{\triangle\!\!\!\!\!-} \tag{11}$$

The module has good properties related to the laws $\triangle\!\!\!\!\!+$ and $\triangle\!\!\!\!\!\times$:

$$|\lambda \ \triangle\!\!\!\!\!\times\ s|_E = |\lambda| \ \triangle\!\!\!\!\!\times\ |s|_E \qquad \forall \lambda \in \Re,\ \forall s \in E \tag{12}$$

$$|s \ \triangle\!\!\!\!\!+\ t|_E \le |s|_E \ \triangle\!\!\!\!\!+\ |t|_E \qquad \forall (s, t) \in E^2 \tag{13}$$

An important theorem has been proved elsewhere (Pinoli, 1985 and 1987a) : E is a normed Riesz space (see Luxembourg, Zaanen, 1971 for the general theory of Riesz spaces) where the order relation and the norm induce the same topology. So, the convergent sequences in the space E are identical for the module and the norm.

The module and the norm can be used as "measure of grey". The norm is not satisfactory in a physical point of view, because $\|s \ \triangle\!\!\!\!\!-\ t\|_E$ taken as measure of the difference between the grey tones s and t is a positive real number and not a grey tone belonging to the real interval [0 , M[. Contrary to the norm the module answers to this preoccupation and must be chosen as a good "measure of grey". This fundamental idea underlies the contrast notion definition set up in this paper.

1.3. Grey tone function differentiation (Pinoli, 1986 and 1987a)

Let us recall some results of previous papers (Pinoli, 1986 and 1987a). For the general theory of differentiation in Banach spaces the readers can refer to Dunford and Schwartz (1958), Kolmogorov and Fomine (1977) or Cartan (1979).

A grey tone function f is smooth at a point x of the inner set , denoted D°, associated to the spatial support D if and only if there exists a linear and continuous application, denoted $f'(x)$ and defined from $\Re^2$ with values in the grey tones space E, such that :

$$f(x+v) \mathbin{\triangle\!\!\!\!\!-} f(x) = f'(x)(v) \mathbin{\triangle\!\!\!\!\!+} \alpha(x,v) \tag{14}$$

where $\dfrac{1}{\|v\|_{\Re^2}} \mathbin{\triangle\!\!\!\!\!\times} \|\alpha(x,v)\|_E$ tends towards zero

when $\|v\|_{\Re^2}$ tends towards zero.

A grey tone function f is smooth at a point x of the inner set D° if and only if it's underlying function f is also smooth at the same point. For any vector v of the plane $\Re^2$, yields :

$$f'(x)(v) \quad = \quad M - M \exp\left(- \frac{f'(x) \, . \, v}{M - f(x)}\right) \tag{15}$$

The derivative of a smooth grey tone function f at a point x of the inner set D° in the direction of a vector v, denoted $\partial_v f(x)$, is defined by :

$$\partial_v f(x) = \frac{1}{\|v\|_{\Re^2}} \mathbin{\triangle\!\!\!\!\!\times} f'(x)(v) \tag{16}$$

Remark :

The directional derivative $\partial_v f(x)$ is a grey tone, that's to say an element of the space E ; it must be consequently used with the laws $\mathbin{\triangle\!\!\!\!\!+}$ and $\mathbin{\triangle\!\!\!\!\!\times}$.

1.4. Grey tone function integration (Pinoli, 1987b)

For the general theory of integration in Banach spaces the readers can refer to Bourbaki (1965 and 1967), Dunford and Schwartz (1958) or Kolmogorov and Fomine (1977).

A grey tone function f is integrable according to the Lebesgue measure on the spatial support D, denoted λ, if and only if it's underlying function f is such that the function Ln [(1 - f(x)/M)] is also integrable.

For an integrable grey tone function f, yields :

$$\int_D \| f(x) \|_E \, d\lambda(x) = \int_D \left| \mathrm{Ln}\left(1 - \frac{f(x)}{M}\right) \right| d\lambda(x) \quad (17)$$

and :

$$\int_D f(x)\, d\lambda(x) = M - M \exp\left(\int_D \mathrm{Ln}\left(1 - \frac{f(x)}{M}\right) d\lambda(x) \right) \quad (18)$$

If we only consider a measurable subset D_o included in D, we obtain corresponding formulae if we replace the set D in (17) and (18) by the subset D_o.

The mean value of a grey tone function f on D_o is very useful in image processing, as exemple in image filtering. It is denoted $m_{D_o}(f)$ and defined as below :

$$m_{D_o}(f) = \frac{1}{\lambda(D_o)} \triangle\!\!\!\!\times \int_{D_o} f(x)\, d\lambda(x) \quad (19)$$

where $\lambda(D_o)$ is the area of the subset D_o.

The mean value $m_{D_o}(f)$ can be expressed explicitely as follows :

$$m_{D_o}(f) = M - M \exp\left(\frac{1}{\lambda(D_o)} \int_{D_o} \mathrm{Ln}\left(1 - \frac{f(x)}{M}\right) d\lambda(x) \right)$$

Remark :

The mean value $m_{D_o}(f)$ is a grey tone, that's to say an element of the space E and must be consequently used with the laws $\triangle\!\!\!\!+$ and $\triangle\!\!\!\!\times$.

2. RECALLS ON THE DEFINITION AND THE PROPERTIES OF CONTRAST IN THE DISCRETE SETTING

In a previous paper (Jourlin, Pinoli, Zeboudj, 1989), the definition and the properties of contrast associated to logarithmic images, defined on a discrete spatial support D of the plane $\Re^2$, have been introduced and studied. Let us remember some results.

2.1. Two neighbouring points

The contrast between two neighbouring points x and y of the spatial support D for an image f is defined by :

$$C_{(x,y)}(f) = \text{Max}\,(f(x),f(y)) \triangle\!\!\!\!- \ \text{Min}\,(f(x),f(y)) \tag{20}$$

and thus explicitely :

$$C_{(x,y)}(f) = \frac{|f(x) - f(y)|}{1 - \dfrac{\text{Min}\,(f(x),f(y))}{M}} \tag{21}$$

Remark :

Previously (Jourlin, Pinoli, Zeboudj, 1989) we have proved that this contrast notion takes into account the logarithmic sensitivity of the human visual system and follows the Weber-Fechner law (See Pratt, 1978 for more details about this law).

2.2. Two arbitrary points

The contrast defined previously is not completely satisfactory. In fact the eye is more sensitive to a grey difference between two neighbouring points than between two distant points. Consequently, the contrast definition between two arbitrary points must to be weighted by their distance.

The contrast between two points x and y of the spatial support D for an image f is defined by :

$$C_{(x,y)}(f) = \frac{1}{d(x,y)} \triangle\!\!\!\!\times \left(\text{Max}\,(f(x),f(y)) \triangle\!\!\!\!- \ \text{Min}\,(f(x),f(y))\right) \tag{22}$$

where d(x,y) is the Euclidean distance between x and y.

Remark :

In the discrete setting, one gets the previous equalities (20) and (21) for two neighbouring points x and y since $d(x,y) = 1$.

2.3. Contrast at a point

The contrast at a point x of the spatial support D can be defined by means of contrasts $C_{(x,x_i)}(f)$ previously introduced between x and it's neighbours $(x_i)_{i=1..n}$.

In fact we can define the contrast at a point x as the mean value of contrasts between x and it's neighbours, or as the larger of these contrasts.
The first definition is :

$$C_x(f) = \frac{1}{n} \mathbin{\triangle\!\!\!\times} \mathop{\triangle\!\!\!+}_{i=1..n} C_{(x,x_i)}(f) \tag{23}$$

The second definition is :

$$C_x(f) = \mathop{\mathrm{Max}}_{i=1..n} C_{(x,x_i)}(f) \tag{24}$$

2.4. Inner contrast associated to a region

The inner contrast of a region R included in the spatial support D, that's to say neglecting the influence of adjacent regions, is defined by :

$$C_{i_R}(f) = \frac{1}{\# R} \mathbin{\triangle\!\!\!\times} \Big(\mathop{\triangle\!\!\!+}_{x \in R} \big(\frac{1}{n_x} \mathbin{\triangle\!\!\!\times} \mathop{\triangle\!\!\!+}_{j=1..n_x} C_{(x,x_j)}(f) \big) \Big) \tag{25}$$

where n_x denotes the number of neighbours, denoted x_j, of pixel x belonging to region R and # R denotes the cardinal of R.

2.5. Contrast associated with a boundary

The contrast associated with a boundary F which separates two or several adjacent regions is evidently linked with those of pairs of neighbouring pixels separated by F. It is defined by :

$$C_F(f) = \frac{1}{\# V} \mathbin{\triangle\!\!\!\times} \Big(\mathop{\triangle\!\!\!+}_{(x,y) \in V} C_{(x,x_j)}(f) \Big) \tag{26}$$

where V denotes the set of pairs of pixels separated by F and # V denotes the cardinal of V.

After these recalls on the L.I.P. model and on the definition and properties of contrast in the discrete setting, we can now define and study the contrast notion in a continuous setting.

3. DEFINITION AND PROPERTIES OF CONTRAST IN THE CONTINUOUS SETTING

3.1. Two arbitrary points

The contrast between two points x and y of the spatial support D for an image f is defined by the formula (22). But it can be also expressed with the module $|.|_E$ of the grey tone space $\boldsymbol{E}$ as follow :

$$C_{(x,y)}(f) = \frac{1}{\|\overrightarrow{xy}\|} \triangle\!\!\!\!\times \; |f(x) \triangle\!\!\!\!- \; f(y)|_E \tag{27}$$

where $\|.\|$ denotes the Euclidean norm in $\Re^2$.

Remark :

It is important to note that $\|xy\|$ equals d(x,y) and that the following equality

$|f(x) \triangle\!\!\!\!- \; f(y)|_E$ = Max (f(x),f(y)) $\triangle\!\!\!\!-$ Min (f(x),f(y))

can be easely established with the properties of the law $\triangle\!\!\!\!-$ and the module $|.|_E$.

3.2. Contrast at a point x in a direction

The contrast at a point x of the inner set D° in the direction of a vector v is defined for a smooth image f by :

$$C_{x,v}(f) = |\partial_v f(x)|_E \tag{28}$$

where $\partial_v f(x)$ denotes the directional derivative of f in the direction of vector v (See formula (16)).

This definition is natural and logical. Indeed, if we replace y by x + v in the formula (27) and if we use both the formulae (14) and (16), we understand clearly the proposed definition for the contrast at a point in a direction.

So, the directional contrast notion defined in the continuous setting is naturally the limit of the corresponding discrete contrast notion :

$$C_{x,v}(f) = \lim_{\varepsilon \to 0} C_{(x,x+\varepsilon v)}(f) \tag{29}$$

where $C_{(x,x+\varepsilon v)}(f)$ is defined by the formula (27).

Remark :

The contrast notion appears closely linked to the logarithmic differentiation introduced in the L.I.P. model and naturally adapted to logarithmic images. This link is particularly evident as regards the human vision because the human visual system is known to be logarithmic (Stockham, 1972).

3.3. Contrast at a point x

We are now able to define the contrast at a point x belonging to the inner set D°. In fact, there exist two definitions corresponding to the two ones in the discrete setting (See (23) and (24)).
The first definition is :

$$C_x(f) = \frac{1}{\lambda(S(x,1))} \; \triangle\!\!\!\!\!\!\;\tiny{x}\; \int_{S(x,1)} |\partial_{\overrightarrow{xy}} f(x)|_E \, d\lambda(y) \qquad (30)$$

or, if we express it related to the directional contrast defined in (28) :

$$C_x(f) = \frac{1}{\lambda(S(x,1))} \; \triangle\!\!\!\!\!\!\;\tiny{x}\; \int_{S(x,1)} C_{x,\overrightarrow{xy}}(f) \, d\lambda(y) \qquad (31)$$

where S(x,1) denotes the sphere of the plane $\Re^2$ with the center located at point x and with radius equals 1, λ denotes the curvilinear measure on the sphere (that is to say the Lebesgue measure of the sphere) and $\lambda(S(x,1))$ denotes the area of this sphere.
Let us note that :

$$\lambda(S(x,1)) = 2\pi \qquad (32)$$

The second definition is :

$$C_x(f) = \sup_{v \in S(0,1)} |\partial_v f(x)|_E = \sup_{v \in S(0,1)} C_{x,v}(f) \qquad (33)$$

where S(0,1) denotes the sphere of the plane $\Re^2$ with the center located at the origine 0 and with radius equals 1.

If we express the formula (33) related to the derivative of the image f at the point x, yields :

$$C_x(f) = \sup_{v \in S(0,1)} |f'(x)(v)|_E \qquad (34)$$

Remark :

The precedent notions exist if the image f is smooth at point x. Moreover the contrast $C_x(f)$ of f at a point x is a grey tone, that's to say an element of the grey tone space E ; it must be used consequently with laws $\triangle\!\!\!\!+$ and $\triangle\!\!\!\!\times$.

It is important to note that the contrast notions at a point x can be obtained as limits of the respective associated discrete notions. Indeed, for the first definition (See (23) and (30)) yields :

$$C_x(f) = \lim_{n \to +\infty} \frac{1}{n} \triangle\!\!\!\!\times \mathop{\triangle\!\!\!\!+}_{i=1..n} C_{(x,x_i)}(f) \tag{35}$$

and for the second definition (See (24) and (33)) yields :

$$C_x(f) = \lim_{n \to +\infty} \mathop{\mathrm{Max}}_{i=1..n} C_{(x,x_i)}(f) \tag{36}$$

3.4. Contrast image

For a smooth image f at each point of the inner set D°, we can introduce an associated contrast image, denoted $C(f)$, and defined by :

$$C(f) : D \to E \tag{37}$$

$$x \to C_x(f)$$

where $C_x(f)$ is defined by formulae (30) or (33).

So the contrast image $C(f)$ associated to a smooth image f appears as an element of the space G of grey tone functions, and must be consequently used with the laws $\triangle\!\!\!\!+$ and $\triangle\!\!\!\!\times$.

3.5. Inner contrast associated to a region

The inner contrast of a region R included in the spatial support D, that's to say neglecting the influence of adjacent regions, is defined by :

$$C_{i_R}(f) = \frac{1}{\lambda(R)} \triangle\!\!\!\!\times \int_R C_x(f)\, d\lambda(x) \tag{38}$$

where the contrast $C_x(f)$ is defined by the formula (30) and λ denotes the Lebesgue measure in the plane $\Re^2$.

So, the inner contrast of a region R appears as the mean value of contrasts of points belonging to R.

It is important to note that the contrast C_{i_R} in the continuous setting is the limit of the inner contrast defined in the discrete setting by formula (25).

3.6. Contrast associated with a boundary

The contrast associated with a measurable boundary F which separates two or several adjacent regions is defined by :

$$C_F(f) = \frac{1}{\lambda(F)} \triangle\!\!\!\!\!\times \int_F C_x(f)\, d\lambda(x) \qquad (39)$$

where the contrast $C_x(f)$ is defined by (30), λ denotes the curvilinear measure on the boundary F and $\lambda(F)$ is the length of this boundary.

So, the contrast associated with a boundary appears as the mean value of contrasts of points belonging to it.

4. CONCLUSION

This theoritical paper has permit to define rigorously the contrast notion naturally adapted to logarithmic images in the continuous setting. Moreover, the discretes formulae of contrast (between two points, at a point, associated to a region or to a boundary, ...) previously introduced and studied (Jourlin, Pinoli, Zeboudj, 1989) are mathematically the good approximations of corresponding continuous formulae setted up in this paper. So, the L.I.P. model appears to be an accurate framework for the introduction of a rigorous definition of contrast in the context of logarithmic images. This work is now leading to the difficult problem of the definition of contrast between textures or associated to a texture.

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