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THE DAVY INDEX OF MINERAL LIBERATION

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ABSTRACT

A simulation is carried out to explore the statistical properties of the Davy index of mineral liberation. It is found that, when applied to data obtained on stereological sections, the index measures the liberation of the sections and not the liberation of the particles.

Keywords: Mineral liberation, comminution, composite materials, stereology, Davy index of liberation.

INTRODUCTION

The objective in mineral technology is often the separation of intergrown minerals to obtain particles which contain concentrations of particular minerals, for example, a valuable mineral component or phase such as gold. Comminution, the breaking of mineral bearing rocks by crushing and grinding, is expensive in terms of its use of energy and this cost generally bears directly on the value of the mineral product. The crushing is to facilitate handling, the grinding to prepare the particles for separation. Ideally the comminution should produce liberated mineral particles containing a single mineral phase. The phase structure will vary according to the amount of comminution. Generally the finer the material is ground the greater the amount that occurs as liberated particles. Unfortunately the separating processes become inefficient when fed with very small particles. The objective is thus the production of more liberated particles at coarser sizes with the least energy expended in doing so. For this, methods are needed for assessing when the grinding can be halted.

Methods of separation include dense-liquid separation, magnetic separation, and neutron activation analysis, each of which can be difficult to use or assess. For example, the liquid in the dense-liquid separation may be dangerous, possibly toxic, or expensive, or the minerals might have specific gravities similar to one another. As an alternative in the matter of assessment the population of particles can be sampled and the structures of the individual particles of the sample identified and measured. This will allow a statistical estimation of the structure of the population. The only widely applicable methods are stereological, using measurements made on sections through the particles. Devices such as image analysers can be employed to make these measurements.

The opening of particles is essential, if we leave aside the possible use of 3-d scanners, since opaque particles, which appear to be just one mineral, on being broken open may contain a second. As a case in point, in beach sands, leucoxene may completely encase a grain of ilmenite. The particles will often be mounted in a plastic or metallic matrix which can then be sectioned.

The degree of liberation of a mineral is the proportion of a mineral that occurs as liberated particles. It tells us nothing about the sizes or internal structure of the composite particles, those that are unliberated. These composite particles behave during processing in ways that depend very much on how the component minerals are intergrown and how the rocks break in relation to the phase boundaries. The degree of liberation is thus an inadequate descriptor. The descriptor should reflect the amount of intermingling of phases within the particles, and take account of their relative sizes. We would not wish to argue that a population of particles, which contained many small liberated ones together with just a few large composite ones, was well liberated. The mix of sizes is often dealt with by sieving. This separates the particles into size fractions each of which can be treated as if the particles within it were of a single size, the results being recombined to give results over the whole sample.

Because of the irregular structure of most real particles (as opposed to the hypothetical populations of regularly shaped and sized particles invented for mathematical modelling) we cannot hope to obtain exact distributions for the patterns of liberation and phase structure as the comminution proceeds. Indices giving an overall summary could however be of practical importance.

The theoretical basis for the study of mineral liberation was set out in the 1960s (for example in Bodziony (1965)) and has continued to the present time (for example in Bodziony and Kraj (1985)). Yet the comminution processes of mineral separation are even now not well enough understood as to yield to simple theoretical analysis. A contribution to the theory by Davy (1984) is the formulation of indices for assessing liberation. The indices take into account the intermingling of minerals in the particles as yet unliberated. These indices do not seem to have been adopted by experimenters, or by researchers or by mineral technologists. There is a long list of possible reasons. For a start, the paper appeared in a mathematics journal and used methods which belong very much to the theoretical side of stereology. Secondly, the exact properties of the indices have not been fully worked out. These properties depend on the material being processed, how the fracture surfaces relate to mineral phase boundaries, *etc.*, so they require assumptions regarding the physical properties of the material and the processes of crushing. There is a dynamic element present also, which relates the value of the indices to the degree of crushing. Further, there seem to be few if any theoretical models for which the processes of crushing and grinding are well enough understood for the indices to be tested. Finally the paper gave no experimental data for which the indices were calculated and examined.

Here, as part of a wider study, a simple simulation is carried out to look at the statistical properties of these indices. It uses one of those hypothetical populations of regularly shaped and sized particles referred to above.

THE DAVY INDEX

The material will be modelled with two phases, α and β. The index is symmetric in the two, though in some applications it is usual to take α to be the phase of interest and β to be a conglomerate of all the non-α phases. The index is on a scale from zero to one. Value zero corresponds to when the proportion of phase α is the same for each particle regardless of its size or shape. It represents a perfect intermingling of phases for every particle at every stage of comminution. The index takes its maximum possible value, one, when every particle is liberated. As the crushing proceeds we would expect movement towards the value one.

We restrict consideration to one, two and three dimensions, though the mathematical framework does not demand this. We let d be "dimension". In dimension 1 (d = 1) X will be the total intercept length of a line section through a particle; in 2-d (d = 2) it will be the total intercept area of a plane section; while in 3-d (d = 3) it will be the unsectioned particle volume.

Looking just at phase α within a particle, X_α will be the total intercept length within α-phase (d = 1), the total intercept area within α-phase (d = 2), or the total volume of α-phase (d = 3), with X_β defined in the same way. Then

$$X = X_\alpha + X_\beta.$$

The use of a log scale or a square root transformation with the object of improving the statistical properties of the index destroys this relationship.

The *index of liberation* proposed by Davy is

$$\Lambda = 1 - T,$$

where

$$T = \frac{E(X_\alpha X_\beta)E(X^2)}{E(X_\alpha X)E(X_\beta X)} .$$

This involves expectations (probability weighted population averages) which require assumptions about the material being crushed and the population of particles being produced.

The index Λ should be written as a function of the amount of crushing κ. Davy defines κ as a scale invariant which is inversely proportional to the amount of fracture surface per unit volume of material. As the crushing proceeds this increases causing κ to reduce, with zero as its limit.

Finding a model for simulation is difficult since the model must reflect the pattern of particle fracture. We cannot hope to obtain a model which is both simple and at the same time general enough to apply widely. In Stoyan *et al* (1987) there is a very brief section near the end of the book on spatial tessellations concentrating on the Poisson-Voronoi tessellation. There are also references in Davy (1984). Recent work in Møller (1989) and Møller *et al* (1989) develops this further, but it is not practical enough for our purposes and could well mask the aims of the study, namely, to provide insight into the statistical properties of the estimated index of liberation. In a more substantial study a simulation based on the actual crushing of a standard material of known morphological structure and fracture properties could be made.

THE SAMPLE INDEX

The sample index $\hat{\Lambda}$ will be used to estimate Λ. For a sample of n particles, the ith of which has $X = x_i$, $X_\alpha = x_{\alpha i}$, and $X_\beta = x_i - x_{\alpha i} = x_{\beta i}$,

$$\hat{\Lambda} = 1 - \hat{T},$$

where

$$\hat{T} = \frac{(\sum_i x_{\alpha i} x_{\beta i})(\sum_j x_j^2)}{(\sum_k x_{\alpha k} x_k)(\sum_m x_{\beta m} x_m)} = \frac{cd}{ab}$$

say, where

$$a = \Sigma x_{\alpha k} x_k, \qquad b = \Sigma x_{\beta m} x_m, \qquad c = \Sigma x_{\alpha i} x_{\beta i}, \qquad d = \Sigma x_j^2$$

and

$$a + b = d.$$

My initial concern was that the sample index involves four second order sample moments, and these in statistical practice often have unstable behaviour. Estimates involving second order moments typically have sampling distributions that depend on fourth order moments, and the fourth powers of observations can have poor statistical behaviour.

THE SIMULATION

Each particle is modelled as an equivalent sphere, with the same volume V. The composite particles are identical with a planar phase interface making the volume of α-phase ξV. If proportion α are α-phase liberated, and proportion β are β-phase liberated, this gives proportion $\gamma = 1-\alpha-\beta$ that are composite. Then a simple calculation (for $d = 3$) gives

$$\Lambda \quad = \quad 1 - \frac{\gamma\xi(1-\xi)}{(\alpha+\gamma\xi)\{1-(\alpha+\gamma\xi)\}} \; .$$

We shall estimate Λ from line sections which are *IUR* (uniformly random with the invariant measure) (Coleman, 1981*a*, 1981*b*). An IUR line section through a sphere gives an intercept whose two ends are uniformly at random on the surface of the sphere. This gives a straightforward simulation. A computer program was written in BASIC and run on a home computer, the Atari 520ST. The input is the set of values (α,β,ξ). For each set 1200 independent line sections were created and measured. For every 100 line sections, an estimate of Λ was made. The data were combined so that Λ was also estimated from samples of 300, 600, and the entire 1200. The results were compared with the true value above.

In Table 1 the true value of Λ is given (*i.e.* $d = 3$),

the sample estimate for $d = 3$, $\quad \hat{\Lambda}_3 = 1 - T_3$,

the sample estimate for $d = 1$, $\quad \hat{\Lambda}_1 = 1 - T_1$,

and an estimate $\quad \hat{\Lambda}_1^* = 1 - 2T_1$.

Each estimate was based on 1200 spheres.

Table 1. Estimates $\hat{\Lambda}_3$, $\hat{\Lambda}_1$ and $\hat{\Lambda}_1^*$, of the Davy index of liberation are compared with the exact value Λ in simulation runs of 1200 spheres. The proportions of phase α liberated spheres, phase β liberated spheres, and of composite spheres are α, β and $\gamma=1-\alpha-\beta$ respectively. The composite spheres have a planar phase interface giving ratio ξ for the relative volume of α phase to total volume. The estimate $\hat{\Lambda}_3$ for $d=3$ involves only the proportions of spheres that are observed to be α liberated, β liberated and composite. The estimates $\hat{\Lambda}_1$ and $\hat{\Lambda}_1^*$ involve the intercept lengths of the IUR line sections in the two phases.

Run	α	β	ξ	Λ	$\hat{\Lambda}_3$	$\hat{\Lambda}_1$	$\hat{\Lambda}_1^*$
1	.1	.15	.3	.2821	.274	.616	.232
2	.1	.15	.3	.2821	.284	.615	.230
3	.1	.15	.3	.2821	.299	.655	.310
4	.1	.15	.3	.2821	.306	.610	.219
5	.2	.05	.65	.2058	.198	.586	.172
6	.2	.05	.65	.2058	.206	.603	.206
7	.2	.05	.65	.2058	.200	.578	.155
8	.25	.15	.75	.4643	.470	.720	.440
9	.25	.15	.75	.4643	.465	.716	.433
10	.25	.15	.75	.4643	.449	.722	.443
11	.2	.3	.4	.5	.494	.751	.503
12	.2	.3	.4	.5	.497	.746	.492
13	.2	.3	.4	.5	.490	.731	.462
14	.1	.15	.75	.3711	.380	.674	.349

DISCUSSION

Table 1 shows clearly without statistical analysis that the sample estimator of Λ based on line sections does not estimate Λ, so only a summary is given. In retrospect we can see that it is estimating the index of liberation for the line sections themselves rather than for the particles. This was an unexpected result at the time.

Table 2. Estimates of ratios of intermediate statistics used in estimating the Davy index of liberation Λ from line intercept data, showing remarkable stability and suggestive of a correction factor in the estimation.

Run	R_α	$\hat{R}_\alpha$	$R_{\alpha\beta}$	$\hat{R}^*_{\alpha\beta} = 2\hat{R}_{\alpha\beta}$
1	.325	.308	.158	.164
2	.325	.325	.158	.169
3	.325	.320	.158	.150
4	.325	.313	.158	.168
5	.688	.698	.171	.175
6	.688	.702	.171	.166
7	.688	.679	.171	.184
8	.700	.705	.113	.116
9	.700	.698	.113	.119
10	.700	.714	.113	.114
11	.400	.414	.120	.121
12	.400	.416	.120	.123
13	.400	.380	.120	.127
14	.663	.654	.141	.147

The estimates $\hat{\Lambda}^*_1 = 1 - 2T_1$ were derived empirically by observing the ratios

$$\Sigma x_{\alpha i} x_i \;:\; \Sigma x_{\beta m} x_m \;:\; \Sigma x_{\alpha k} x_{\beta k} \;:\; \Sigma x_j^2, \qquad i.e. \qquad a : b : c : d.$$

In Table 2

$$R_\alpha = (\mathrm{E}X_\alpha X)/(\mathrm{E}X^2), \qquad R_\beta = (\mathrm{E}X_\beta X)/(\mathrm{E}X^2) = 1-R_\alpha,$$

$$R_{\alpha\beta} = (\mathrm{E}X_\alpha X_\beta)/(\mathrm{E}X^2),$$

$$\hat{R}_\alpha = (\Sigma x_{\alpha i} x_i)/(\Sigma x_j^2) = a/d, \qquad \hat{R}_\beta = 1-\hat{R}_\alpha = b/d,$$

$$\hat{R}_{\alpha\beta} = (\Sigma x_{\alpha k} x_{\beta k})/(\Sigma x_j^2) = c/d, \qquad \hat{R}^*_{\alpha\beta} = 2\hat{R}_{\alpha\beta}.$$

Then

$$\Lambda = 1 - R_{\alpha\beta}/(R_\alpha R_\beta),$$

$$\hat{\Lambda} = 1 - cd/ab = 1 - \hat{R}_{\alpha\beta}/(\hat{R}_\alpha \hat{R}_\beta),$$

$$\hat{\Lambda}^* = 1 - \hat{R}^*_{\alpha\beta}/(\hat{R}_\alpha \hat{R}_\beta).$$

The question arises as to whether the correction factor 2 conceals an invariant of the system, whether we have exposed an unknown general result of stereology. The answer is that there is no new theorem. This can be demonstrated by using as our population of particles one that can be studied entirely theoretically. It is not appropriate for representing the results of comminution however.

We take as before a population of identical spheres, with proportion α liberated of phase α, proportion β liberated of phase β, and proportion $\gamma = 1-\alpha-\beta$ composite. But now the spheres have unit radius and the composite particles have a spherical core of radius r of phase α, inside a concentric shell of thickness $1-r$ of phase β; r is the same for each particle. The particle volume V is $4\pi/3$, and the relative volume of α phase in a composite particle is $\xi = r^3$.

In Table 3 we use the abbreviations

$$\nu = \alpha + \gamma\xi, \qquad \phi = 8\pi^2/15,$$

$$\eta = \gamma\xi(1-r^2)/4. \qquad \rho = \tfrac{1}{2}\,\gamma\{r + \xi - \tfrac{1}{2}(1-r^2)\ln\left(\frac{1+r}{1-r}\right)\},$$

There is no factor relating the values of R_α for d = 1, 2, and 3; nor for $R_{\alpha\beta}$. Certainly we do not have the factors 1 and 2 that we saw in Table 2 when the composite spheres had a planar phase interface.

Table 3. The components of the Davy index of liberation for spherical particles of radius one in which the composite particles have a spherical core of radius r of α phase inside a spherical shell of thickness $1-r$ of β phase.

	Volumes $d = 3$	Plane sections $d = 2$	Line sections $d = 1$
EX	V	$\frac{1}{2}V$	$4/3$
EX_{α}	νV	$\frac{1}{2}\nu V$	$4\nu/3$
EX^2	V^2	ϕ	2
$EX_{\alpha}X$	νV^2	$(\nu+\eta)\phi$	$2(\alpha+\rho)$
$EX_{\alpha}X_{\beta}$	$\xi(1-\xi)\gamma V^2$	$5\eta\phi$	$2(\rho-\gamma r\xi)$
R_{α}	ν	$\nu+\eta$	$\alpha+\rho$
$R_{\alpha\beta}$	$\xi(1-\xi)\gamma$	5η	$\rho-\gamma r\xi$
T	$\frac{\xi(1-\xi)\gamma}{\nu(1-\nu)}$	$\frac{5\eta}{(\nu+\eta)\{1-(\nu+\eta)\}}$	$\frac{\rho-\gamma r\xi}{(\alpha+\rho)\{1-(\alpha+\rho)\}}$

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