ACTA STEREOL 1990; 9/1: 125-133
ORIGINAL SCIENTIFIC PAPER

EVALUATION OF CRACK ANISOTROPY IN AGRICULTURAL CLAY SOILS

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ABSTRACT

The test of Mardia (1976) was successfully used for the distinction of anisotropy of cracks in planar sections made through agricultural clay soils. Assuming a certain probability model for the directions of normals to cracks (modified von Mises distribution $g(\beta)$ for cracks in planar sections, Dimroth-Watson distribution $h(\varphi,\vartheta)$ for cracks in space), the rose of the density of intersections with testing lines of different orientations can be derived ($m(\alpha)$ for planar sections, $m(\varphi,\vartheta)$ for space). This allows to estimate the parameters of $g(\beta)$ or $h(\varphi,\vartheta)$ from the observed roses $m(\alpha)$ or $m(\varphi,\vartheta)$, respectively, as well as to assess the density of cracks in planar sections (L_A) or in volume (S_V). The procedures are illustrated by some examples of heavy clay soils in East Slovakia Lowland.

KEY WORDS: shrinkage cracks, planar sections, anisotropy.

THEORY

For cracks in agricultural soils, cylindrical symmetry of the directional distribution around the vertical axis is to be expected in all but very special cases. (Sloping lands and sloping layers in weathered sedimentary rocks can belong to these special cases, as reported by Scott et al., 1987). It suffices then to study the two-dimensional anisotropy in vertical sections. A convenient test could be that of Mardia (1976; cf. also Beneš, 1986), which is based upon a multiple correlation between a suitable linear random variable X and the corresponding directional cosines $\cos\beta$, $\cos\alpha$ in the plane of the section. Here, β is the angle between the x-axis and the direction under consideration, while α is the angle between the y-axis and the same direction. Evidently, $\cos\alpha=\sin\beta$. The linear random variable X, which depends upon the direction, and the anisotropy of which is to be quantified, must be chosen in such a manner that it satisfy as closely as possible the linear equation assumed for the alternative hypothesis, namely the equation (2) given below.

As a rule, one can recommend the intersection density m as the X-variable. It is defined as the number of intersections of cracks with testing lines of a given direction, divided by the total length of the testing lines within the sample section. If the anisotropy is perfect, i. e. if all cracks are parallel, say, to the y-axis, the intersection density is:

$$m = L_A \cdot |\cos\beta| \qquad (1)$$

where L_A ... density of cracks (cf. also the equation (5)),

β ... angle between the testing lines and the x-axis.

The equation (2) cannot be, of course, precisely identified with (1). However, the overall trends can be made similar, which is important for correlation calculations. Other examples of possible X-variables are: the total number of intersections on a testing line, the hydraulic conductivity, the electrical conductivity, etc.

Since the distribution of crack directions is axial ($X(\beta) = X(\beta+\pi)$ for all β, irrespective of the definition of X), we must replace the angle β in the original Mardia's test by its double-value 2β; otherwise the coefficients of correlation would be low and insignificant. Then, the null hypothesis states that the cracks are isotropic ($X(\beta)$=const.), while the alternative hypothesis assumes a linear dependence:

$$X = a + b.\cos 2\beta + c.\sin 2\beta \qquad (2)$$

where a, b, c ... suitable constants. This equation corresponds well to the cardioid distribution (Mardia, 1972, p.51; Marriott, 1971; Streit, 1978) or to the first terms in Kanatani's (1985) series expansion.

Let the sampling correlation coefficients between X and $\cos 2\beta$, X and $\sin 2\beta$, and $\sin 2\beta$ and $\cos 2\beta$ be r_{12}, r_{13}, and r_{23}, respectively. Then the multiple correlation coefficient $R^2_{X,2\beta}$ is defined as follows:

$$R^2_{X,2\beta} = (r^2_{12} + r^2_{13} - 2r_{12}r_{13}r_{23})/(1 - r^2_{23}) \qquad (3)$$

The product $n.R^2_{X,2\beta}$, where n ... number of observations, has an asymptotic χ^2-distribution with two degrees of freedom. Adequate comparisons with one-side χ^2_2-quantils are conceivable for $n \geqslant 20$. The test requires that the individual observations, i. e. the pairs $[X_i, \beta_i]$ for i=1..n, be independent.

A further task is to estimate the probability density of crack directions (directions of normals to cracks) $g(\beta)$ from the measured rose of the intersection density $m(\alpha)$, where $\alpha = \beta - \pi/2$ is the angle between the y-axis and the direction of the testing lines, $-\pi/2 < \alpha < \pi/2$. The y-axis is considered to be the main axis of anisotropy, with which the cracks tend to be parallel. The estimation of $g(\beta)$ from $m(\alpha)$ is, however, very sensitive to small variations of $m(\alpha)$, and many procedures fail, if the anisotropy is strongly developed. The non-parametric graphical method based upon the Steiner compact (Rataj and Saxl, 1988) brings a partial solution of this problem.

Another approach, preferred here, is to assume a certain realistic probability density function just for the expected result, i. e. for $g(\beta)$, and to look only for its parameters, by means of investigating $m(\alpha)$. The distributions of crack directions in soils are mostly unimodal and symmetrical ($g(\beta) = g(\pi-\beta)$, if the x-axis or y-axis coincides with the main axis of anisotropy). A slightly modified von Mises distribution (Mardia, 1972, p. 57; Streit, 1978):

$$g(\beta) = (L_A/I_0(k)).\exp(k.\cos 2\beta) \qquad (4)$$

where

$$\pi^{-1} . \int_0^{\pi} g(\beta)\, d\beta = L_A \qquad (5)$$

L_A ... density of cracks related to the area of the section,
k ... concentration parameter ($k \geqslant 0$),

$I_0(k)$ ... modified Bessel function of the first kind and of the order zero,

β ... angle between the x-axis and the direction of the normal to the crack, $0 < \beta < \pi$,

seems to be a sufficient approximation for most soil crack structures. It can describe, in contrast to the cardioid distribution, even the cases of extremely strong anisotropy, when k approaches infinity and $g(\beta)$ becomes close to the Dirac delta-function.

The choice of the von Mises distribution was recommended by Streit (1978) and Weibel(1980, p. 272). Mardia (1972, pp. 48-71) discusses several circular and angular distributions, among which the von Mises distribution and the wrapped normal one turn out to be the most natural, the most effective, and very close to each other. There is not yet sufficient amount of data to prove, whether the von Mises distribution does justice to the orientation of soil cracks or not. At least, our data do not contradict to the equation (8) below, which is based upon this distribution. The main axis of anisotropy is identical with the y-axis in the equation (4) and further throughout this paper.

The rose of the intersection density $m(\alpha)$ can be derived from (4), using a well-known differential equation (Hilliard, 1962; Stoyan et al., 1987, p. 241):

$$g(z) = \pi/2 \, . \left[d^2m(z)/dz^2 + m(z) \right] \qquad (6)$$

where z ... an independent variable which means α in m(z) and β in g(z).

The appropriate boundary conditions are:

$$m(z) = m(z + \pi) \; ; \quad dm/dz\big|_z = dm/dz\big|_{z+\pi} \qquad (7)$$

The solution of (6), (7), after the substitution for g(z) from (4), is:

$$m(\alpha) = \frac{L_A}{I_0(k).\sqrt{(2\pi k)}} \, . \Big[\exp(-k).\left|\cos\alpha\right|.\mathrm{erfi}\sqrt{\left[k(1+\cos 2\alpha)\right]} + \exp(k).\left|\sin\alpha\right|.\mathrm{erf}\sqrt{\left[k(1-\cos 2\alpha)\right]} \Big] \qquad (8)$$

The mean density of intersections of cracks with testing lines of arbitrary directions is:

$$\pi^{-1} \cdot \int_{-\pi/2}^{\pi/2} m(\alpha)\, d\alpha = I_L \qquad (9)$$

One can show that

$$I_L = 2 \cdot L_A / \pi \qquad (10)$$

The functions erfi and erf in (8) are defined as follows:

$$\mathrm{erfi}(x) = 2/\sqrt{\pi} \cdot \int_0^x \exp(t^2)dt \; ; \quad \mathrm{erf}(x) = 2/\sqrt{\pi} \cdot \int_0^x \exp(-t^2)dt$$

Estimates of the concentration parameter k can be obtained from the measurements of intersection density $m(\alpha)$ on testing lines in two different directions, preferentially $\alpha=0$ and $\alpha=\pi/2$. This procedure is usually referred to as the "short-cut estimate", because it reduces considerably the extent of the necessary data and calculations. The ratio of both intersection densities:

$$f = m(\pi/2) / m(0) \qquad (11)$$

equals, as a consequence of (8):

$$f = \exp(2k).\mathrm{erf}\sqrt{(2k)} / \mathrm{erfi}\sqrt{(2k)} \qquad (12)$$

If one knows f by measurement, the concentration parameter k can be estimated by inversion of (12). Estimates of L_A, either from $m(\pi/2)$ or from $m(0)$, are straightforward:

$$L_A = d_v(k) \cdot m(0) \quad \text{or} \quad L_A = d_h(k) \cdot m(\pi/2) \qquad (13)$$

where

$$d_v(k) = \sqrt{(2\pi k)}.I_0(k).\exp(k)/\mathrm{erfi}\sqrt{(2k)} \qquad (14)$$

$$d_h(k) = \sqrt{(2\pi k)}.I_0(k).\exp(-k)/\mathrm{erf}\sqrt{(2k)} \qquad (15)$$

Similar approach can be applied in the three-dimensional case, assuming rotational symmetry of the joint probability density function $h(\varphi,\vartheta)$ of the directions of normals to the cracks (φ ... altitude, ϑ ... colatitude). This assumption is corroborated by the results of the test of anisotropy given below. Accepting the Dimroth-Watson distribution (Mardia, 1972, p. 233) for $h(\varphi,\vartheta)$, we can make use of the results by Cruz-Orive et al.

(1985), where only random lines need to be replaced by testing lines and testing planes by random surfaces. Again, it is possible to obtain short-cut estimates of the concentration parameter k from the ratio:

$$f = m(\varphi,\pi/2) \;/\; m(\varphi,\ 0) \tag{16}$$

where $m(\varphi,\vartheta)$ ... the rose of the density of intersections of cracks with testing lines of the direction (φ,ϑ) (φ is arbitrary, because of the rotational symmetry),

and estimates of S_V (density of cracks per unit soil volume):

$$S_V = c_v(k)\ .\ m(\varphi,0) \quad \text{or} \quad S_V = c_h(k)\ .\ m(\varphi,\pi/2) \tag{17}$$

where the coefficients $c_v(k)$ and $c_h(k)$ can be computed numerically, by means of procedures described by Cruz-Orive et al. (1985). The concentration parameter k will be mostly negative, corresponding to the "girdle-type" distribution of normals to cracks.

MATERIALS AND METHODS

Clay agricultural soils in East Slovakia Lowland were investigated in the field by making horizontal and vertical sections (several m^2 in area). Cracks were followed visually and drawn by hand on a sheet of millimeter paper. The pictures were then investigated manually by making testing straight lines either horizontally and vertically, or in an isotropic uniform random (IUR) manner, and counting the intersections. For comparison, the cycloid test system according to Baddeley et al. (1986) was also used.

For this paper, two samples were chosen for detailed investigations, namely:

(A) Vertical section 60 x 80 cm (identification data: Milhostov, 31.8.78, SVT, 3rd part, SW wall, 1:5),

(B) Horizontal section 120 x 120 cm (identification data: Milhostov, 26.8.78, 10 cm, near to VŠP, 1:10).

These sections were similar to, but not identical, with those published by Doležal (1982) or by Stoyan et al. (1987, p. 254). Small differences in the methodology of preparation between these two sections did not allow to consider them as two sections of the same body. They were, therefore, treated separately. No distinction was made between cracks of different widths and between different soil layers in the vertical section. The variances of the measured quantities were estimated with the help of the approximate formulae developed by Doležal (1976).

The results given below in the tables 1 and 2 were compiled from a wider set of data.

RESULTS AND DISCUSSION

1. Test of anisotropy:

Section A: $n.R^2_{X,2\beta} = 12.246$; $\chi^2_2(\alpha=0.995) = 10.597$

Section B: $n.R^2_{X,2\beta} = 5.133$; $\chi^2_2(\alpha=0.900) = 4.605$

There is not sufficient warranty for anisotropy in section B.

2. Analysis of anisotropy and crack density:

(section A only; results are rounded, standard deviations are given in parentheses)

Horizontal testing lines: $m(\varphi,\pi/2)=0.121(0.009)$ cm^{-1}; n=13

Vertical testing lines: $m(\varphi,0)=0.043(0.008)$ cm^{-1}; n=17

$f = m(\varphi,\pi/2) / m(\varphi,0) = 2.809(0.541)$

2.1. Two-dimensional analysis: k = 1.887(0.512)

$d_v = 3.111(0.483)$; $L_A = d_v \, . \, m(\varphi,0) = 0.134(0.032)$ cm^{-1}

$d_h = 1.109(0.047)$; $L_A = d_h \, . \, m(\varphi,\pi/2) = 0.134(0.012)$ cm^{-1}

From isotropic uniform random testing lines:

n=42; $I_L=0.079(0.007)$ cm^{-1}; $L_A=\pi.I_L/2=0.124(0.011)$ cm^{-1}

2.2. Three-dimensional analysis: k = -3.353(1.202)

$c_v = 4.594(0.347)$; $S_V = c_v \, . \, m(\varphi,0) = 0.198(0.038)$ cm^{-1}

$c_h = 1.636(0.011)$; $S_V = c_h \, . \, m(\varphi,\pi/2) = 0.198(0.016)$ cm^{-1}

From the cycloid test system: $I_L = 0.109$ cm^{-1}

$S_V = 2.I_L = 0.218$ cm^{-1} (variance not estimated).

3. Differences in anisotropy of cracks with various widths and in various depths in the soil:

Only demonstrative examples are given here, without intention to give a representative picture. The concentration parameters k relate to the Dimroth-Watson distribution. There is a tendency for greater anisotropy of larger cracks and in deeper soil layers, which has natural genetic reasons not yet fully explored.

Table 1. The influence of the crack width upon the concentration parameter k (Plešany, 1973, depth 50-70 cm, vertical sections)

widths of cracks (mm)	k
0.5 - 1	0
2 - 3	-1.39
4 - 5	-3.59
6 - 8	-4.99

Table 2. The influence of the soil depth upon the concentration parameter k (Milhostov)

Depth (cm)	1978		1979	
	k for crack widths (mm):			
	1 - 2	2	0.5	1 - 2
0 - 20	-0.93	-5.28	-0.68	-6.43
20 - 40	-4.50	-2.45	-2.91	-2.97
40 - 60	-4.84	-	-4.15	-9.74
60 - 80	-	-	-5.56	-2.71

ACKNOWLEDGEMENT

I thank sincerely Dr. Viktor Beneš for his kind help during the preparation of this paper.

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Received: 1989-02-24
Accepted: 1989-09-19