

CURVATURE OF LINES IN DIGITIZED IMAGES

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ABSTRACT

A shape descriptor has been developed for linear objects, which may be used to describe the deviation of a curved line from a straight one. The features required for this parameter are high invariance for orientation, translation, and scale, particularly in the discrete 2-dimensional space of digitized images. To that end, the impact of the single raster points of the image (pixels) on the computed shape parameter must be reduced. Only a robust but sensitive measurement of curvature can be successfully used to correlate this parameter with other properties belonging to linear objects. As a rule, commercially available image analyzers offer some formfactors, which do not accomplish this function well enough. A possible field of application is the study of cortical infolding (gyration) in relation to cytoarchitectonics in neuroanatomy.

Keywords: cerebral cortex, curvature, gyration, image analysis, linear objects, shape

INTRODUCTION

In modern image analyzers pictures are obtained using e.g. a tv-camera and stored in an image memory board with a certain resolution in x- and y direction. This limited geometrical resolution significantly influences the measured geometrical data. The degree of deviation of such data from theoretically expected values depends on hardware (resolution, quality of AD-converter), on object characteristics (size, shape, orientation, position), on the kind of measured parameters (e.g. area, formfactor) and on the special software being used for calculating these parameters (Rink, 1978). Most of the problems mentioned above are caused by the fact that in digitized images the object boundary is no longer a smooth line as in Euclidian geometry, but a structure composed of mostly quadratic raster points (pixels) (Bookstein, 1978; Russ, 1989). Accordingly, the deviation of measured data from theoretically expected values increases if the particle borderline is more dominant for that parameter. This is more likely for small objects and for length measurements than for large objects and for area measurements.

Simple linear objects and the analysis of their shape or curvature may serve as an example for such difficulties. Many shape parameters normally used do not fulfil the requirements

of invariance for changes of position, orientation and scale in raster images. Therefore, for curved linear objects a new shape parameter has been developed which is highly invariant for orientation and position and which is not available on commercial image analyzers.

MATERIAL AND METHODS

In this methodological investigation we examined artificial binary objects which were obtained either by skeletonization of real objects or by graphical drawings using the cursor of an image analysis computer. The length of the single line element was about 30 pixels in this study. For image analysis an IBAS^R-system (Kontron, Eching, Germany) was available, equipped with the IBAS^R-software release 2.0. The system was controlled by an AT-386 with co-processor running under a DOS-4.01 operating system.

RESULTS

Two of the built-in parameters of our image analysis system, the perimeter (PERIM) and the convex perimeter (CPERIM), allow us to calculate the following dimensionless shape parameter for isolated linear objects:

$$LCURV = 1 - \frac{CPERIM}{PERIM} \quad (1)$$

Theoretically, values of 0 for straight lines and values approximating 1 for multiply folded lines are expected, thus giving a clear idea of the degree of bending of a single line

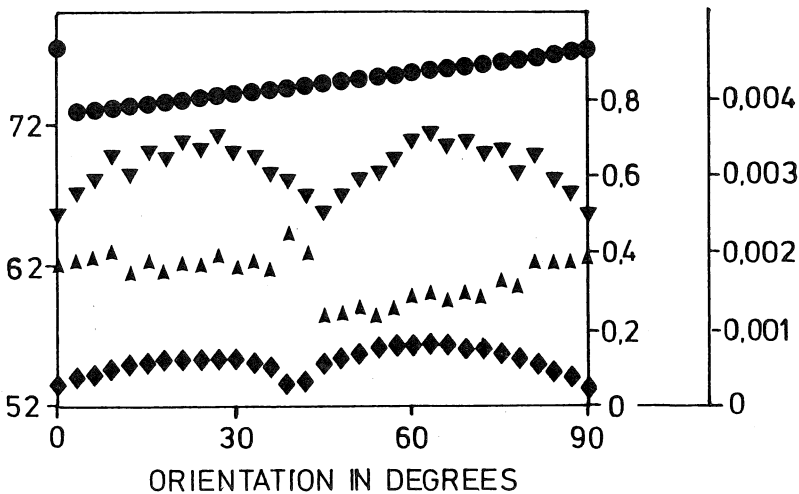


Fig.1: Primary and calculated shape parameters for straight lines, 31 pixels long, depending on orientation. Apply the left scale [pixel] for perimeter (▼) and convex perimeter (▲). Symbol ◆ and the inner right scale belong to the shape parameter calculated according to formula (1). Symbol ● and the outer right scale belong to the shape parameter calculated according to formula (2).

element. But both primary parameters react very sensitively to digitization, resulting in huge deviations from the theoretically expected values. Fig.1 demonstrates the rotational behaviour of the primary parameters (perimeter and convex perimeter) as well as of the combined parameter, calculated according to equation (1) for rotated straight lines, 31 pixels long. Although only one of the invariancy characteristics required was tested, the result is unacceptable. When the orientation is changed, the measured data vary by more than 10%. Similar results were obtained with other available shape parameters (circularity, elongation) or with new ones derived from these formfactors in combination with other primary parameters.

Using the following approximations (fig.2)

$$CPERIM = b + s = s + 2 * \arcsin\left(\frac{s}{2 * ROC}\right)$$

$$PERIM = 2 * s = 4 * r * \arcsin\left(\frac{s}{2 * ROC}\right)$$

equation (1) may be rewritten as:

$$LCURV = 0.5 - \left(\frac{s}{4 * r * \arcsin(s/(2 * ROC))}\right). \tag{2}$$

$$\left(0 < \frac{s}{2 * ROC} \leq 1\right)$$

In these formulae s means the step size for subdividing a given line L (fig.2, 3) into shorter line segments, b is the segmented part of the original line which is approximated by the arc of a circle with radius ROC. Using elementary mathema-

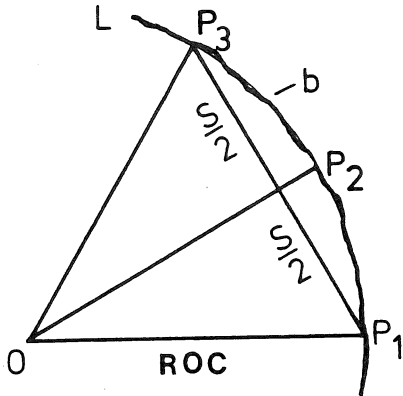


Fig.2: Schematic drawing for derivation of formula (2): a continuous line (L) is subdivided into shorter elements (b) between the points P₁ and P₃. Their distance equals s. OP₂ is normal to P₁P₃ for moderate curvature of b.

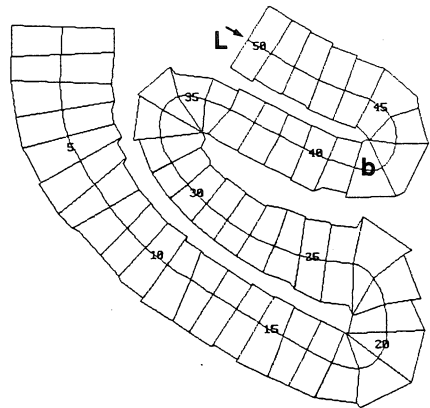


Fig.3: Trace L of a defined lamina in a folded cerebral cortex of a monkey together with a sampling scheme for cortex data along both sides of it. Local curvature of b may be correlated with data of cytoarchitecture in the corresponding frames.

tical methods the circle radius may be calculated using the coordinates of at least 3 points belonging to b , as it would be performed in constructing the circle transform of a curve (Shapiro and Lipkin, 1977). These 3 points can easily be obtained by image analysis operations (P_1 and P_3 as endpoints of b , and P_2 as the logical AND result of the binary images of b and a circle around P_1 with radius $s/2$). All structures are slightly dilated with an octagon as a structuring element before measuring the coordinates of the P_i ($i=1,2,3$). This ensures that a common pixel is found by the mentioned AND-operation and it reduces the influence of single outstanding pixels when the coordinates of the P_i are calculated as the centers of gravity of the dilated points.

On applying the orientation test again as with the other form-factor (fig.1), a very stable result (cf. the different scales in fig.1) is obtained owing to the position of the variable ROC in the denominator and as a reciprocal argument of the arcsin function.

As an application the curvature of the line shown in fig.3 was analyzed using the method described. Originally, this line demonstrates the folding (gyration) in a brain cortex of a monkey, which we wanted to correlate with neuroanatomical data (Mißler et al., in prep.). The maxima of the derived shape parameter coincide with the positional numbers (19, 22, 33, 43) of the heavily curved line segments (fig.4).

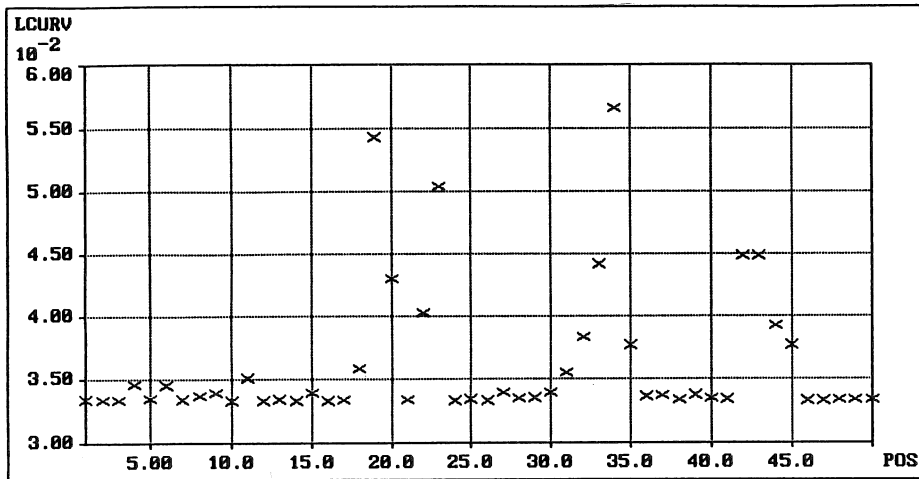


Fig.4: The shape parameter LCURV for the 50 line segments b of fig.3. High values of the shape parameter correspond with those positions (POS) along the tracing line where the line segments appear heavily curved.

DISCUSSION

Formfactors have been closely studied in particle morphometry (Oberholzer, 1983). The different approaches document that there is not a generally applicable parameter for all situations (Underwood, 1982; Flook, 1984; Gschwind et al., 1986). Although image analysis computers allow the extraction of more

shape information from an image than earlier visual methods could, special problems arise with the raster format of such images (Russ, 1989).

Shape parameters are used to differentiate particles of varying form. The approach of this study is to analyze 2-dimensional objects of very similar shape (all linear, differing only in curvature), which seems to be a sub-problem of the overall shape discussion. A theoretically powerful shape parameter, like that computed by equation (1) from primary parameters available on an image analysis system, is strongly influenced by the interrelation with the raster structure of the digitizing system, as shown in fig. 1 for different orientations of linear objects. It should be mentioned here that in a more general sense the parameter computed according to equation (1) is a measure of concavity with theoretical values of 0 for non-concave (i.e. completely convex) objects and values between 0 and 1 for rising concavity.

For linear objects of limited length in relation to curvature (cf. formula (2)) and curvatures with the same sign (expressed as $d^2\alpha/db^2$, α : angle of orientation) a new shape parameter has been developed, which is not available on commercial image analyzers. Its suitability as a parametric descriptor for the degree of bending of line segments is based on the reduction of the role of single pixels for parameter computation. Remember that digitized straight lines are smooth lines only if their orientation is a multiple of $\pi/2$ in a quadratic raster. Only these orientations result in line thickness of 1 pixel. Furthermore, depending on length, a line cannot have any direction one chooses, two crossing lines with intermediate orientations must not have a common point, and so forth (Eins, 1987). Therefore, one single pixel may play an important role in calculating object parameters (Russ, 1989). This influence is reduced when, in our calculation of the shape factor, the coordinates of significant points P_i ($i=1,2,3$) are obtain-

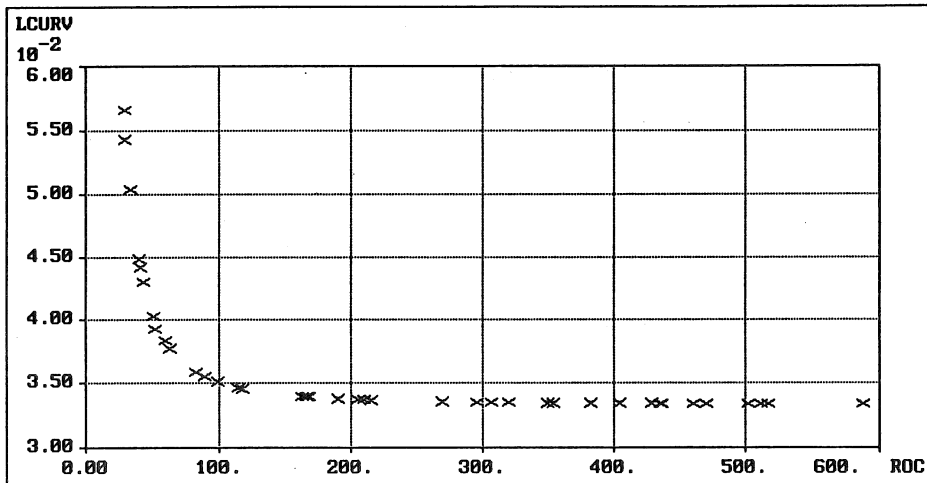


Fig.5: Correlation of the shape parameter LCURV with the radius of curvature ROC [pixel] of the line segments b (fig.3) according to equation (2).

ed as the centers of dilated structures. Already, the radius ROC of the circle passing through the P_i is an appropriate measure of curvature (by definition curvature is the reciprocal value of the radius of curvature). Its disadvantages as an analytical parameter (not as a transform for reconstruction of the original curve) are its large range of possible values and its dimensionality. These were the reasons for developing a different measure of local curvature as given by equation (2). The correlation of both parameters by equation (2) is graphically shown in fig.5. For a wide range of large radii of curvature (i.e. nearly straight line segments), the curvature parameter itself remains nearly constant at a low value. The more the radius decreases, the more sensitively the curvature parameter reacts, thus combining the properties of high robustness and sensitivity which satisfy, along with the mentioned invariance criteria and lack of dimensionality, the requirements for an efficient numeric shape descriptor (Russ, 1989). The measuring points of fig.5 were taken from an experiment which is shown in fig.3.

In this experiment the linear trace represents a neuro-anatomical layer in the cortex of a monkey (Mißler et al., in prep.). Let us regard in this context only the measured data for the curvature parameter which possibly correlate to structural parameters in the corresponding sampling frames. Depending on position (1 to 50) on line L the measured values of the local curvature for single line segments clearly demonstrate the suitability of that parameter (fig.4) especially, when they are compared with other shape factors in use.

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