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THREE DIMENSIONAL HOMOTOPIC THICKENING OF DIGITIZED SETS

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ABSTRACT

The thickening and its dual operation, the thinning, are fundamental transformations in image analysis. These operations are defined in any dimensional space. A fast algorithm is presented here, allowing a three dimensional thickening of a digitized image preserving its connectivity.

Key Words : Mathematical morphology, Euler-Poincaré characteristic, thickening.

INTRODUCTION

Three dimensional images can be obtained from serial sectioning. If these images are digitized and stored in a computer, they can be transformed and analysed automatically. For this purpose, the Hit or Miss transformation (HMT) of mathematical morphology offers powerful tools like erosion, dilation, thinning and thickening (Serra, 1982).

Up to now, these transformations have been implemented mainly in two dimensional space. Nevertheless some references are dealing with three dimensional operations on digitized images (see, for example Lobregt et al., 1980; Tsao and Fu, 1981; Hafford and Preston, 1984; Bhanu Prasad et al., 1988; Kong and Rosenfeld, 1989; Mukherjee and Chatterji, 1989).

In a recent work, three dimensional erosions and dilations have been implemented on structures digitized on a grid corresponding to the cubic face centered network of crystallography - more compact and more symmetrical than the cubic network - (Bhanu Prasad et al., 1988). The thickening presented in this paper, is the natural continuation of this work.

THICKENING BY HIT OR MISS TRANSFORMATIONS

The hit or miss transformations of mathematical morphology can be used to perform thickenings on digital pictures (Serra, 1982). The neighbourhood of each pixel is compared to a structuring element and a point is added to the digitized set when the two configurations are identical. Depending upon the choice of the structuring element, the connectivity of the image may be the choice of the structuring element, the connectivity of the image may be modified. In this paper, we deal only with homotopic transformations i.e., preserving the connectivity.

TWO DIMENSIONAL THICKENING

A two dimensional homotopic thickening can be performed with the structuring elements L, M or D of the Golay alphabet (Serra,1982). An illustration of this process is given in Fig.1 where a set X is thickened by the structuring element L* (dual of L) on an hexagonal grid.

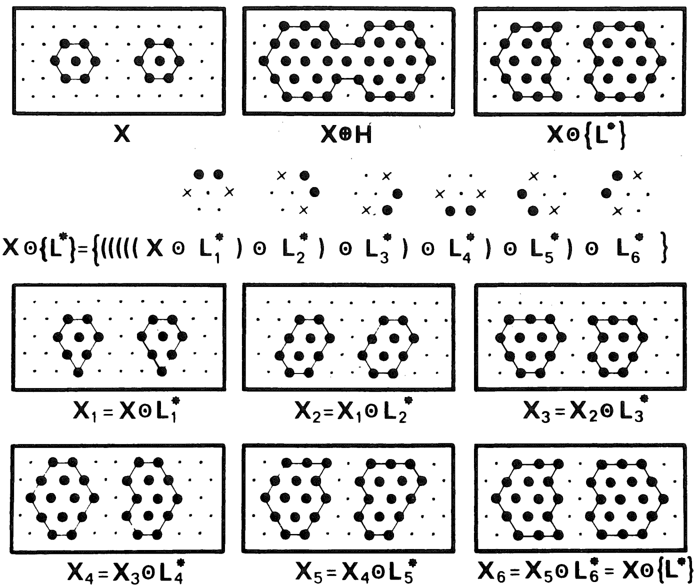

Fig.1 : 2-D Thickening (⊙) with the structuring element {L*}.

$L_1^* = \begin{pmatrix} \bullet\ \bullet \\ x\ \cdot\ x \\ \cdot\ \cdot \end{pmatrix}$

- • corresponds to 0
- ● corresponds to 1
- x can be either 0 or 1

The center point is added when this configuration is verified. The five other directions $L_2^* \ldots L_6^*$ are obtained by rotation.

It must be noticed that one complete step of thickening is obtained by the successive rotations of L* in the six directions of the hexagonal grid. The thickening of X is compared with the dilation of X by an elementary hexagon H. Roughly speaking, the homotopic thickening can thus be seen as a dilation preserving the connectivity.

THREE DIMENSIONAL THICKENING

The same operation can be applied in three dimensional space. For the digitization, a face centered cubic grid has been chosen : it is the highest symmetrical one and each pixel has 12 neighbours at the same distance (Bhanu Prasad et al.,1988). The natural structuring element is thus a cuboctahedron with six square faces and eight triangular faces. The direct transposition of the 2D structuring element L* on this 3D cuboctahedron can be seen in Fig.2.

Two families of configurations must be tested : eight directions for the triangular faces and six directions for the square faces. Thus for each step of thickening, fourteen configurations must be analysed. This process is too slow for three dimensional transformations where a huge number of points must be tested.

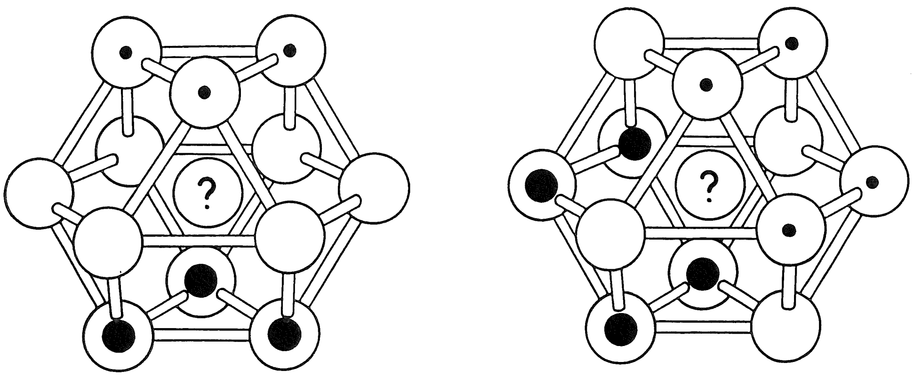

Fig.2 : Two configurations of structuring element L* in a three dimensional face centered cubic grid

FAST 3D THICKENING

By construction, the preceeding structuring element preserves the connectivity of the structure : for example a point is added above one triangular face of the lower plane only if the corresponding triangular face of the plane above the point is empty (Fig.2). But there exists another way to realize homotopic thickenings precisely because we want to preserve the homotopy of the image : instead of testing if the neighbourhood of a point is identical to the structuring element, a point is added to the structure if the connectivity of the structure is not modified.

For this purpose, each point of the grid is analysed with its twelve neighbours (elementary cuboctahedron).

a) If the central point is 1, it is left as it is.

b) If the central point is 0, the Euler-Poincaré characteristic, N_3 is calculated (Bhanu Prasad et al., 1988). All the possible values for N_3 are : -1, 0, 1, 2, 3, 4

with (-1) : two loops
(0) : no point or one loop
(1,2,3,4) : one or several disconnected components.

Four examples of the neighbourhoods giving N_3= -1 (eight possible configurations), N_3 = 0 (377 configurations) and N_3= 4 (nine configurations) are given in Fig.3.

c) The value 1 is given to the central point (thickening) if the preceeding value of N_3, obtained in step b) is 1. It is the only case for which a thickening can be performed without modifying the connectivity.

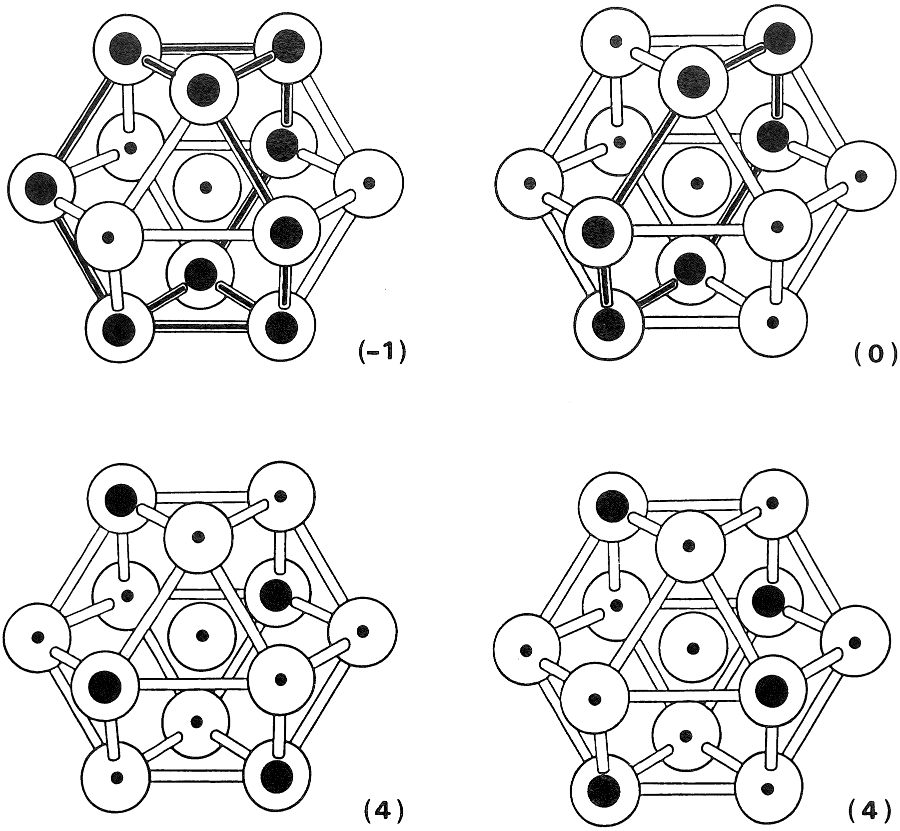

Fig.3 : Four particular configurations leading to N_3= -1,0 and 4.

PRACTICAL IMPLEMENTATION

The basis of the thickening has been given but a practical problem has yet to be solved : the thickening must not be done in the scanning direction! Each time a point is added, the neighbourhood of the next pixel will be affected by this modification. To avoid this, three images must be used as follows :

a) Copy of the input image (I) into the output image (O) and complementation of the output image (O^c).
b) For a given pixel, the test for thickening (N_3= 1) is made on its neighbourhoods in the images (I), (O) and (O^c).
c) If the answer is positive in the three cases, the corresponding pixel is modified in output images (O) and (O^c).
d) The procedures b) and c) are repeated for all the pixels.

It must be noticed that, instead of using the complementary image (O^c), the neighbourhood of each pixel being read from (O) is only complemented before testing.

By using this procedure, a thickening of size 1 of the input image (I) is stored in the output image (O). A thickening of size n can be obtained if this process is repeated with the output image of size (n-1) as input image. A complete sequence of thickening up to stability is given in Fig. 4 on a simple image.

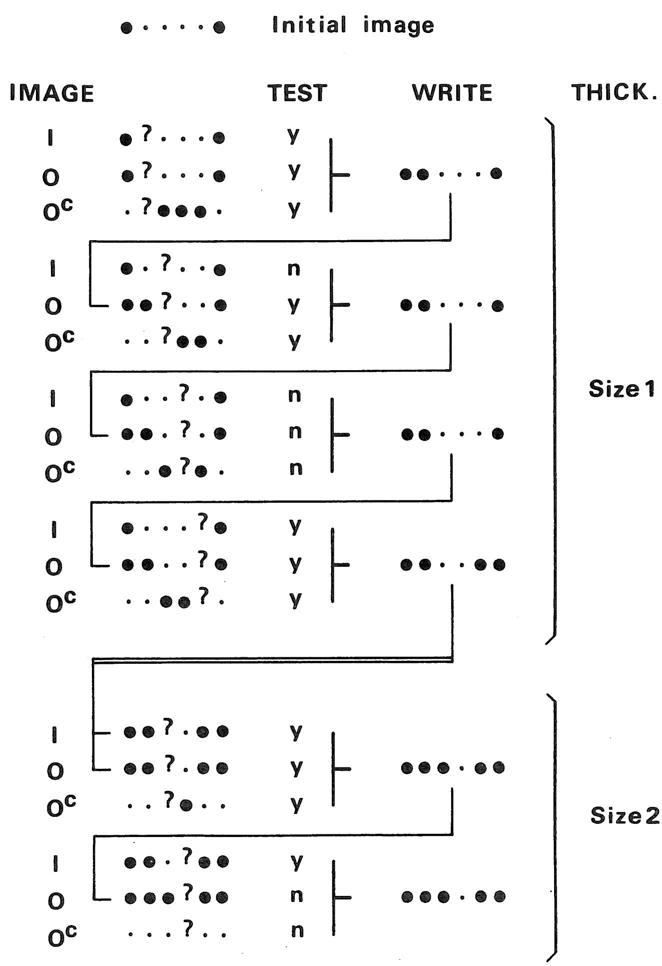

Fig. 4 : Example of homotopic thickening in 1-D space. (•) and (●) correspond respectively to 0 and 1 for the points. The test point (is it possible to add a point (●) without modifying the connectivity ?) is indicated as (?). The result of the test can be yes (y) or no (n). If the result is (y) for the three images, the pixel is modified in the output image (O).

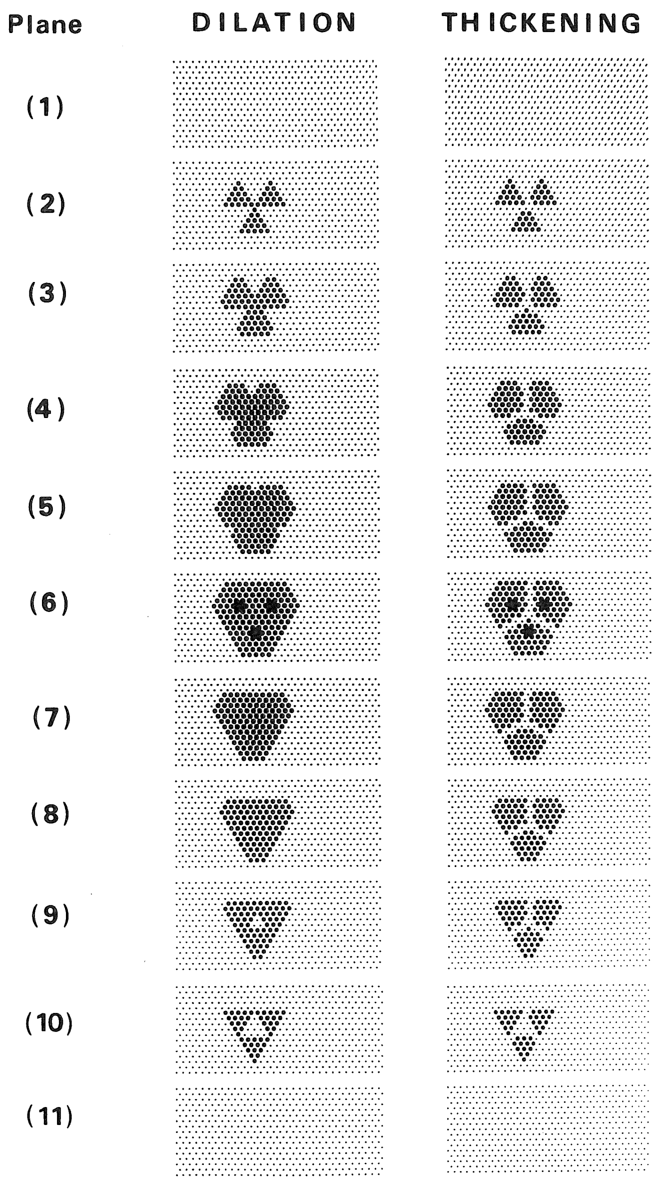

Fig.5 : Three dimensional dilation and thickening of size four.

It must be noticed that for 3D transformations, instead of calculating N_3 for each pixel, a table is prepared for all possible neighbourhoods of a point. In the case of the fcc grid, 4096 (2^{12}) possible configurations with their corresponding N_3 values are stored in this table.

Finally, a three dimensional thickening is illustrated in fig.5. Three points have been placed at the vertices of an equilateral triangle (edge size 5) contained in the plane of altitude z = 6. An homotopic 3-D thickening of size 4 is shown and compared with a corresponding 3-D dilation of same size.

CONCLUSION

The homotopic thickening of a digitized set can be obtained by comparison of the neighbourhood of each pixel with all the possible configurations and orientations of an adequate structuring element. But this procedure is very long when a large number of points must be tested (3D analysis). A fast algorithm based on the conservation of the connectivity of the neighbourhood has been presented here. This process allows to save a big amount of time. For example, if a structure containing about fifty percent of pixels 1 is contained in a FCC grid of size (131x154x164), the time necessary for a thickening of size 1 is roughly fifteen minutes, as for an elementary dilation. Thus, this operation is fourteen times faster than the same transformation with a structuring element. This fast algorithm was presented in the case of a FCC grid. It can also be used as it is on any digitization grid but the 26 possible neighbours of a point in a cubic grid need the use of a table containing 2^{26} values!

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