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SIMULATION OF FIBRE PROCESSES, APPLICATION IN FRACTOGRAPHY

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ABSTRACT

The paper deals with applications of fibre processes in the plane. The problem of simulation of curved fibre systems with given marginal distributions of orientation, curvature and length is studied. An algorithm is proposed and the geometrical properties of the simulated fibres studied.

This problem arose in fractography when observing electron micrographs of fatigue crack surfaces. These images are modelled by thick fibres and quantified. The aim is to find geometrical parameters which correlate with crack velocity. Some results in this direction are presented using the data from fatigue tests.

Keywords: fibre process, fractography, simulation.

INTRODUCTION

Simulations of random processes of geometrical objects play an important role in applications of stochastic geometry. The theory and practice of simulations of tesselations and point processes is well developed. Very little has been done in simulations of random fibre processes since with the exception of line and segment processes a fibre process is a very complex random set (Stoyan et al., 1995). There is little hope to simulate exactly from the distribution of a general (curved) fibre process.

Starting the research in this field a less demanding challenge is investigated. The aim is to simulate a realization of a class of random processes with partial knowledge of the distribution. The mathematical model is constructed in the discrete space Z^2 (rectangular grid) eventually on a hexagonal grid. Generalizations to Z^d, $d > 2$,and other multidimensional grids are straightforward.

In the present paper fibre processes in the plane are studied to help in a practical application in fractography, when correlating the geometry of projected fracture surfaces with crack velocity. We will describe this problem in more detail. The development of information technology confronts fractography with a new possibility - extending information sources by integral statistical characteristics of greater areas. Two application directions have been studied within the research program "Computer Aided Fractography" at Czech Technical University:

1. The conservative fractographic features such as striation paths can be identified automatically. Large fracture areas can be analysed with regard to the individual crack history as

well as to the general crack growth laws. Based on this idea, the reconstitution of crack front (Lauschmann, 1997) contributed as a new method to the complex fractographic analysis.

2. The fracture surface relief can be studied as a random process. Since the three-dimensional representation of the crack surface is not generally accessible, the SEM images of crack surfaces are studied in two-dimensions. The intuitive method of "blurring" (Lauschmann and Beneš, 1997) proved that the texture of the crack surface image contains rich information about the crack growth process.

The reconstitution of fatigue crack history is traditionally based on striations. However, certain materials do not create them, and in many other cases the fine striation relief was taken off by corrosion or erosion. To find another fractographic feature correlated to the crack growth rate is of primary significance.

The magnifications dividing the areas of macro and microfractography, between 10^2 and 10^3, have not been used very often in the past. The reason is evident - no specific local geometrical features are visible in the images of the crack surface. The present idea is to interpret the crack image under magnifications mentioned above as a system of fibres (Stoyan et al., 1995). For many materials, the line-similar objects with different lengths, widths, curvatures and orientations are present in the image, see Fig.1. Although the physical interpretation of these fibres is not clear, they are an objective feature of the image. It follows that the morphological extraction of fibres from the grey-tone image is less subjective than straightforward segmentation.

In the present paper first an algorithm for the simulation of a fibre process with prescribed first-order properties is suggested. Some properties of the simulated model are studied.

In the second part of the paper real structures of fatigue crack surfaces are quantified from SEM images. Fibres are extracted using image processing and quantitatively described. Those characteristics which correlate with crack velocity are found and presented in a graphical way.

MATERIALS AND METHODS

Consider a discrete grid, either rectangular or hexagonal, in the plane. First a relation $\mathcal{N}$ of neighbourhood, independent of shifts, is defined on the grid. On a hexagonal grid we have six natural neighbours to each point, on a rectangular grid four or eight. In order to get more smooth fibres typically to each point $x \in Z^2$ we have $n = 16$ or 32 neighbours which need not to be the nearest points. We write $x \sim y \iff (x, y) \in \mathcal{N}$. The relation is symmetric and antireflexive. Each pair $(x, y) \in \mathcal{N}$ forms an oriented segment. Denote by $\mathcal{O}$ the set of orientations. $\mathcal{O}$ is finite of cardinality n, let its elements be enumerated by $i = 1, \ldots, n$. A map $\mu : \mathcal{N} \to \mathcal{O}$ is defined by $\mu(x, y) = i \in \mathcal{O}$, which assigns to each segment its orientation.

The inputs to the simulation algorithm are:

a) a stationary random point process of germs on the grid.

b) a discrete length distribution $h_i = P(m = i)$ of fibres , $i = 1, 2, \ldots$. Throughout this paper for simplicity the number of segments m of a fibre is called length.

c) a transition matrix $n \times n$ denoted $P = (p_{ij})$ on the state space $\mathcal{O}$, and an initial distribution $\pi = (\pi_i)$, $i = 1, ..., n$.

From each germ x_0 of the simulated point process a fibre is simulated as a realization of a Markov chain $C_k, k \geq 1$, with state space $\mathcal{O}$, transition matrix P and initial distribution π. The orientation $C_1 = i$ simulated from π is drawn as a segment $(x_0 x_1)$ such that $\mu(x_0 x_1) = i$. By induction the orientation $C_k = j$ is simulated using $C_{k-1} = i$ and i-th row of the matrix P. It is drawn as a segment $(x_{k-1} x_k)$ such that $\mu(x_{k-1} x_k) = j$. The simulation of a fibre is stopped when the length m simulated in advance is reached. All input quantities are stochastically independent. The union of fibres corresponding to each germ and length is called a fibre process.

In the stationary case, π is the segment orientation distribution of the process. Then by the choice of the matrix P both "isotropic" (in sense that π is uniform) and anisotropic fibre processes can be simulated.

To each transition a curvature is evaluated as the reciprocal of radius of the circle hitting three endpoints of two subsequent segments. Thus a $R = (r_{ij})$ of type $n \times n$ of curvatures is defined corresponding to the neighbourhood relation. Its elements may have both positive and negative values r_{ij} corresponding to curving clockwise or anticlockwise. Consider the matrix R^* with elements $r^*_{ij} = |r_{ij}|$. Then $I = \sum_i \pi_i \sum_j r^*_{ij} p_{ij}$ is the mean absolute curvature per transition.

The problem of existence of a transition matrix P with prescribed orientation and curvature distribution is solved in (Šimák, 1998). In that paper also formulas for probabilities of fibre overlappings are expressed. This is important when comparing theoretical and empirical characteristics of simulated and real structures.

The fractographic analyses were performed on stainless steel AISI304L from EDF-SCMI Chinon, France, which is used in nuclear power industry. The flat specimens with cross section dimensions B=3mm, W=55.8mm and a central notch were cyclically loaded with stress range $\Delta\sigma = \sigma_{max} - \sigma_{min} = 100MPa$ and stress ratio $R = \sigma_{min}/\sigma_{max} = 0.051$ at frequency $f = 1Hz$ and temperature $T = 20^oC$. The crack velocity is monitored during the fracture development being within the bounds from 10^{-2} to 1 μm per cycle. The micrograph of the fracture surface is in Fig.1. Large series of neighbouring micrographs along the crack surface are investigated.

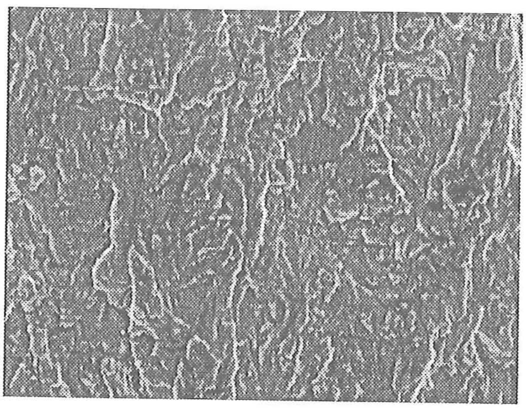


Fig.1: A micrograph of a fracture surface at a location with crack velocity about 0.07 μm per cycle. Magnification 500 × . The vertical direction corresponds to crack propagation.

There is a variety of characteristics of fibre systems as defined in stochastic geometry. The first order characteristics describe basic properties like length intensity, orientation and curvature distribution. Interactions among fibres are quantified by means of the pair correlation function and the contact distribution function. Another important quantity is the shape of fibres. The structure of the crack surface is in large scale non-stationary, the aim is to estimate parameters along given locations and plot their change with respect to crack velocity. The results from the orientation and shape analysis are presented in the following.

Fig. 2: The labelled points of fibres for a plain fibre (a), and for three endpoints (b).

It is evident already in Fig.1 that a dominant orientation φ_0 of observed fibres exists and it is parallel to the crack growth direction. When observing large series of images along the crack surface the aim is to quantify variance of orientation distribution in each image. This is done using the Steiner compact method (Rataj and Saxl, 1992) which yields estimators of the probability density $\rho(\varphi)$ of the rose of directions, $\varphi \in (0, \pi)$. The circular variance $var\mathcal{R}$ is defined as $var\mathcal{R} = \int 2[1 - \cos(\varphi - \varphi_0)]\rho(\varphi)d\varphi$ and estimated using discretization of the integral.

The shape analysis was performed using the shape space of triads of labelled planar points as described in Stoyan et al. 1995. Each fibre is described by three points in the plane, first two of them being fibre endpoints and the third fibre point which is most distant from the segment joining the endpoints, see Fig. 2a. This definition is useless when bifurcations are present. Since bifurcations are rare in the observed structure, the restriction to at most three endpoints of fibres, see Fig. 2b, is possible in this approach.

The image processing using software "ProWIm Pro" developed at the Academy of Sciences of the Czech Republic combined with own special programs has been applied. A morphological procedure has been developed which extracts thick fibres from the original grey-level image, then using the area measure restricts to most dominant ones and searches for three labelled points. Edge-effects can be neglected for large observed areas. Using the representation on the spherical blackboard (Stoyan et al., 1995) the colatitude ϑ is of interest for each fibre. The appropriate parameter is the average $\bar{\vartheta}$ of ϑ over the image.

RESULTS

First some results concerning simulations are presented. In Fig.3 there is an example of a simulated realization of fibre process on a square grid with $n = 16$ neighbours and a Bernoulli lattice point process ($BLP(p)$, cf. Stoyan et al., 1995) of germs, where p is the probability that any point of the grid becomes a germ (independently of the others).

a) b)

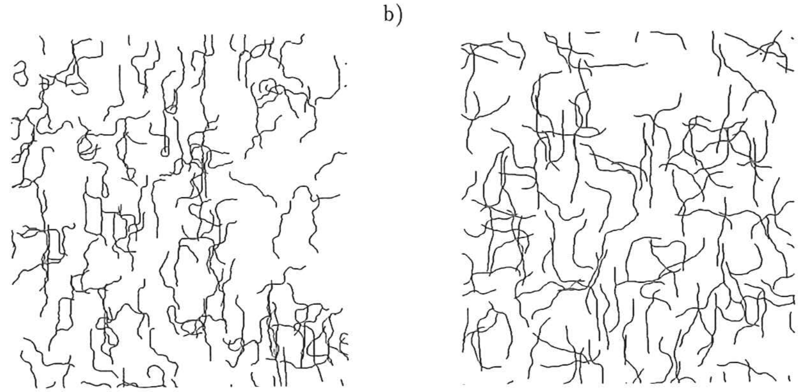


Fig.3: Simulated realization of a fibre process in eroded window $\mathcal{W} \ominus b(0,r)$, where $b(0,r)$ is a ball with radius r centered in origin. The length distribution of fibres has a finite support with largest nonzero probability corresponding to length r. The fibres in the both cases (a) and (b) have the same orientation distribution, but the curvature distribution is different.

When using the simulation algorithm on a discrete grid fibres may overlap and in this case the theoretical characteristics of the fibre process (e.g. length intensity) need not correspond to those observed. Therefore we evaluate the overlapping ratio $O_r = \frac{2Pr(A)}{p\bar{h}}$, where $\bar{h}$ is the mean fibre length and the probability $Pr(A)$, where $A =$ [there is a fibre in a given segment], depends on the orientation of the segment. Fig.4 demontrates evaluation of $Pr(A)$ using analytical techniques from (Šimák(1998)).

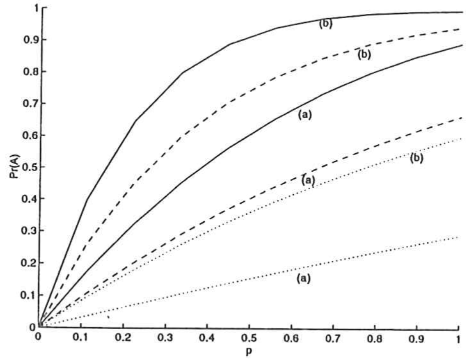
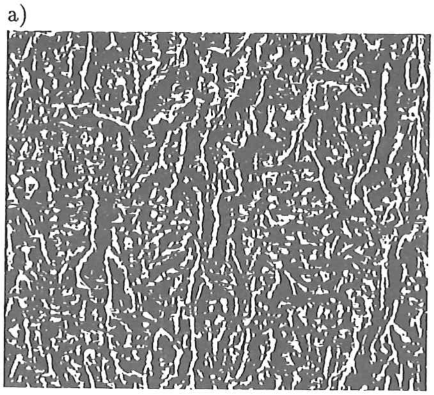


Fig.4: Probability $Pr(A)$ depending on the parameter p of $BLP(p)$ process of germs. On a hexagonal grid with six neighbours the orientation of segment is considered – (solid line), / (dashed line) and \ (dotted line). In both cases (a),(b) P is the unit matrix, $\pi = [\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{18}, \frac{1}{6}, \frac{1}{18}]$ and the length is fixed: 3 segments in (a) and 8 segments in (b).

Finally we present results of the fractographic analysis, using the above mentioned tools. Evaluation of a series of 14 neighbouring images from the beginning of crack propagation with crack velocity range $0.04 - 0.15\mu m$ per cycle is demonstrated. In Fig.5 the morphological process is demonstrated on one of these images.

a)



b)

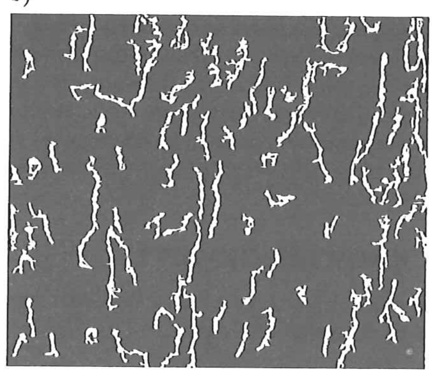


Fig.5: The image which corresponds to Fig.1 with extracted thick fibres (a) and restricted image with measured fibres (b).

The graphs on Fig.6 present results of orientation and shape analysis of this series of images.

DISCUSSION

The paper has two parts, one concerns the theoretical problem of simulation of curved fibre processes, the second is an application of planar fibre processes in fractography. The promising project of simulation will be further developed. In fact under the independence assumptions in the presented algorithm, together with the assumption of Bernoulli process of germs, the fibres are stochastically independent. This enables to get analytical results (even second-order) but for practice it is more important to have models for interactions among fibres. The degree of fit between simulated and real structure was not quantified here since in the presented fractographic images the repulsion is present. The development of different algorithms for repulsion and attraction will make further progress.

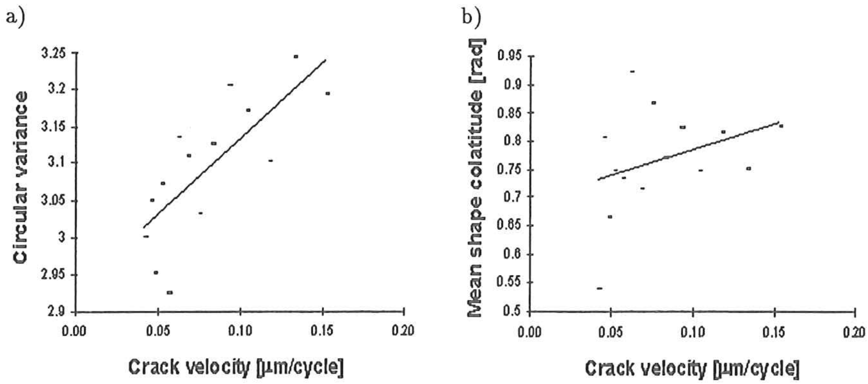


Fig.6: The dependence of circular variance $var\mathcal{R}$ (a) and mean shape colatitude $\bar{\vartheta}$ (b) on crack velocity. The regression line is drawn in both cases.

In the evaluation of fracture surfaces several known methods are used including first and second order characteristics, contact distributions and shape analysis. In fact the degree of similarity between images will be based on a combination of these parameters. For presentation two simple parameters were chosen, circular variance and the shape factor based on labelled triads. These parameters exhibit slight dependence on the crack velocity already at low velocities, while the non-stationarity of the projected surface relief is more evident at the end of cracking. Therefore these parameters are suggested for further use but still many different materials and fracture conditions have to be investigated to make proper conclusions. This is a technical task in the project "Computer Aided Fractography", where the methodology is being sucessfully developed.

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