STIT Tessellations – A Reference Model for Random Division Processes, Fragmentation and Crack Structures

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Keywords

Stochastic geometry, random tessellation, fracture system, random division process, fragmentation

Introduction

The STIT tessellation (**ST**able with respect to **IT**erations of tessellations) is a mathematical model that describes the geometry of a random division process, such as cell division, fractures in stone, in coatings etc. This model can be treated theoretically very well, as the results shows which were published in the last decade since the definition appeared for the first time (*Nagel and Weiss, 2005)*. This makes the STIT tessellation a reference model for random division which can be adapted to real structures for applications.

Materials and Methods

The STIT tessellation model can briefly be described as follows: Consider a bounded (e.g. rectangular) window W which is a 'cell' at time 0. After a random time (exponentially distributed with a parameter depending on W) it is divided by a random line (with a certain law). This division results in two new cells $C1, C2$ say. Now these two cells have independent random life times (again exponentially distributed, but the parameters adapted to $C1, C2$ respectively). At the end of its life time, a cell is divided by a random line; and so on. The key to obtain a tractable model is the tuning of the parameters of the life time distributions and of the choice of laws of dividing lines. E.g. for a stationary and isotropic model, the life time parameter of a cell C is its perimeter, and the random dividing line is an IUR (isotropic uniform random) line.

This model can analogously be defined in 3D (and arbitrary higher dimension), then starting with a spatial window W , e.g. a cube, and sequential divisions by random planes.

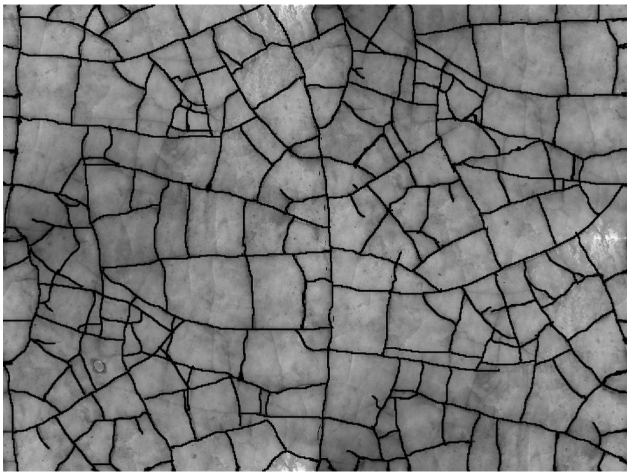

Figure 1. Craquelée on a ceramic surface

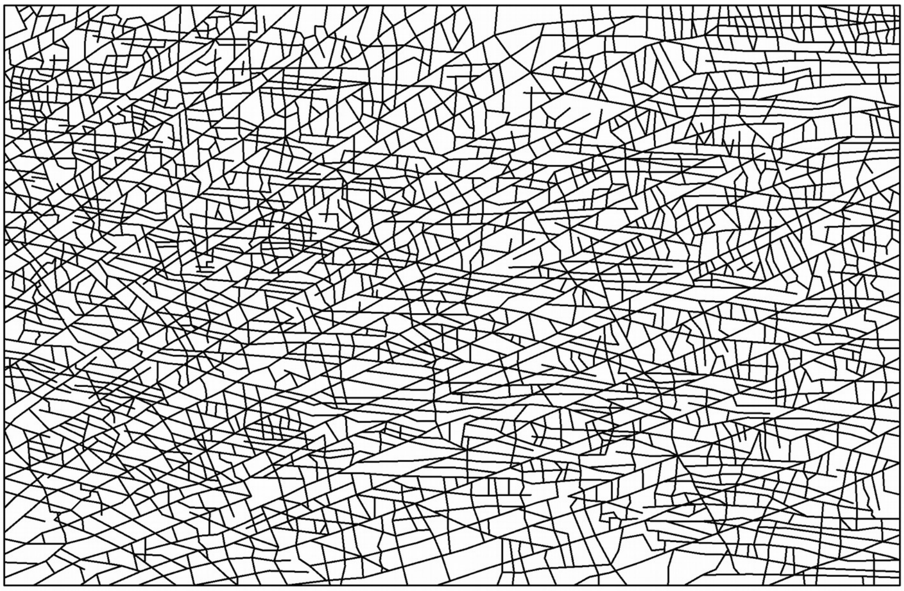

Figure 2. Arches fracture trace map. (from: Mosser/Matthäi)

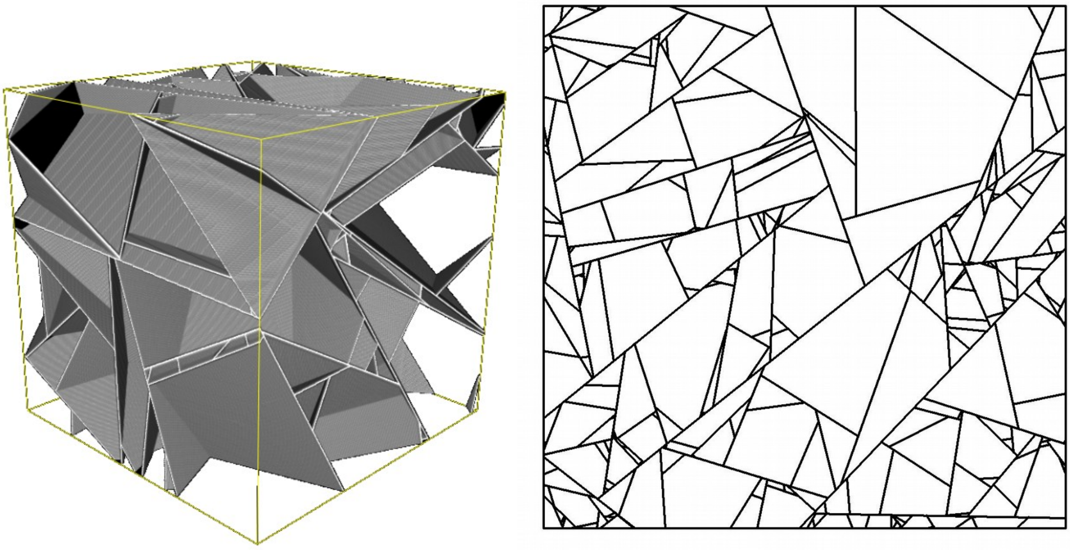

Figure 3. Simulation of a 3D (left) and 2D (right) STIT tessellation

Results and Discussion

There is already a series of theoretical results in the literature, featuring qualitative and quantitative properties of STIT tessellations (see e.g. *Mecke et al. (2007, 2011), Nagel and Weiss (2005, 2008), Redenbach and Thäle (2011), Schreiber and Thäle (2010,2011), Thäle and Weiss (2010)*). For applications, this model has to be tuned. The theoretical base for such an adaption is given in *Georgii et al. (2013)*. Some examples, where STIT tessellations were considered for applications, are *Emery and Ortiz (2011), Mosser and Matthäi (2014), Yu et al. (2014).*

Conclusion

The STIT tessellation model has a sound mathematical definition and can be treated theoretically very well. Thus it can serve as a reference model for applications where random division processes or fracture systems are studied.

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