Comparison of several surface area estimators for 3D binary images

Legland, David
INRA, URBIA Biopolymères-Interactions-Assemblages, Nantes, France
david.legland@nantes.inra.fr

Keywords
Surface area estimation, Crofton formula, discretisation, resolution

Introduction

The analysis of 3D images often requires the determination of morphometric features from binary images obtained after a segmentation procedure. The surface area is a quantity well defined in continuous space, which can easily be related to physical phenomenon. Its practical determination from digital binary images can be performed by different methods. An isosurface mesh can be computed, from which a surface area can be obtained. An alternative is to use a discretised version of the Crofton formula that consists in counting the number of intersection with a set of lines within the image. While theoretical results on the convergence of estimators exist, the practical choice for a practitioner is not always easy. Also, the case of discretisation grids with non cubic voxels is rarely taken into account. The objective of this study is to compare the results obtained by different methods, in order to provide concrete guidelines for the choice of a method, as well as some hints on the relative error that can be expected in practice. The experiments are performed on a variety of synthetic shapes exhibiting different curvatures and elongations. Binary discretisations are performed on different resolutions and with different relative orientations with the shapes.

Materials and Methods

Several 3D shapes with various geometries are considered: cube, ball, ellipsoid, torus, cylinder, "capsule" (a cylinder with half-balls glued at each side), portions of ball... Each shape was transformed into a 3D binary image by Gauss discretisation: a voxel is set to "1" if its centre belongs to the shape and to "0" otherwise. Examples of such discretisations are shown on Figure 1.

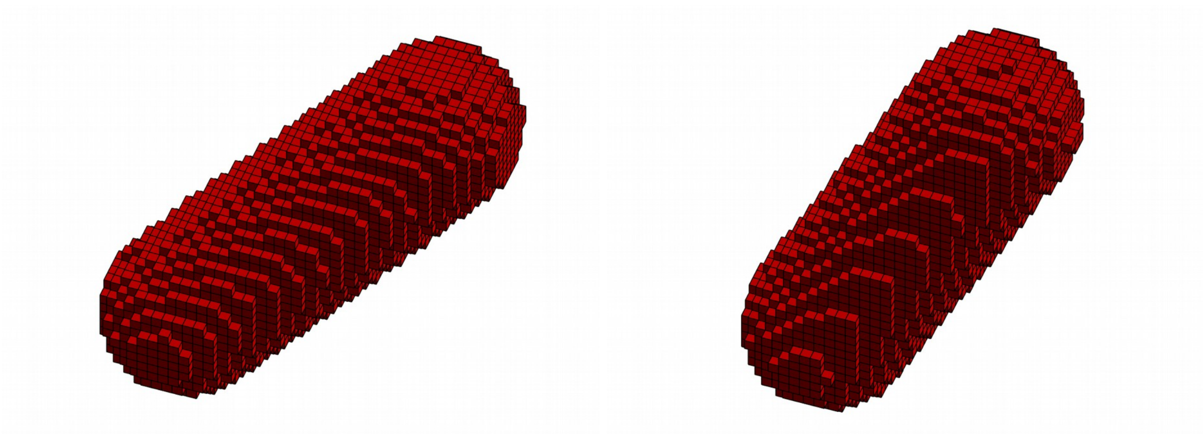


Figure 1. Two different Gauss discretisations of the same 3D binary particle.

The binary structure was transformed into a polygonal mesh by using the marching cube algorithm (Lorensen and Cline, 1987), resulting in a triangular mesh. The mesh surface area was computed by summing the individual area of each triangular facet. Crofton surface area was computed following methods presented in (Legland et al., 2007) and (Ohser and Mücklich, 2000), which consists in counting the number of intersection of the binary structure with a collection of lines. In practices two cases were considered, corresponding to lines oriented with the three main directions in images, and to lines oriented to 13 directions, including also diagonals. The set of directions was weighted according to the relative spherical area of Voronoi domains obtained after projection of direction vectors on the unit sphere.

The methods were compared based on two criteria. The first criterion was the bias, measured as the relative error of the estimates with the (known) theoretical value (Howard and Reed, 2010). The second criterion was the precision, assessed from the variability of the estimates obtained on series of shapes with random positions and orientations.

The impact of the resolution was investigated by comparing measurements obtained with different ratios of grid sizes over shape size. The position and the orientation of the shapes were adequately randomised.

The influence of relative orientation of the shape with the grid was investigated by measuring surface area for different orientation of shapes presenting an axis of symmetry, and randomizing the position of the shape centre.

Results and Discussion

Average relative errors of surface area estimates for the different synthetic shapes are given in Table 1. It results that whatever the resolution and the type of shape, the surface areas obtained with Crofton estimators are on average slightly underestimated (by a relative error of few percents), whereas the surface areas obtained with isosurface method are strongly overestimated (by a relative error of around 7-8 percents). Not surprisingly, the average error obtained with 13 directions is better than the average error obtained with three directions, due to the increase in the number of considered directions.

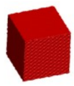
	Ball	Cube	Torus	Cylinder	Capsule	Octant
						
Surface	20106.19	15000.00	11844.47	10053.10	8482.30	6283.26
Isosurface	8.37 *(0.03)*	7.39 *(1.79)*	8.38 *(0.19)*	7.84 *(0.95)*	8.40 *(0.42)*	7.00 *(1.04)*
Crofton (3 dirs.)	-0.01 *(0.05)*	-1.64 *(7.46)*	-0.20 *(1.31)*	-1.06 *(4.91)*	-0.26 *(2.41)*	-1.52 *(4.46)*
Crofton (13 dirs.)	-0.01 *(0.03)*	-1.09 *(1.13)*	-0.05 *(0.11)*	-0.65 *(0.56)*	0.01 *(0.24)*	-1.43 *(0.66)*

Table 1. Relative errors (in percents) obtained with different estimators on several shapes after integration of shape orientation. The variability of the estimates is indicated in brackets.

The lowest variability is obtained with Crofton estimator computed with 13 directions. The strongest variability in the estimates is observed for Crofton estimator with three directions. The variability obtained with isosurface is greater than the variability obtained with Crofton with 13 directions, but the values are comparable. It can be noticed that whatever the method, the variability increases with the proportion of flat regions in shapes.

For the shapes that present an axis of symmetry, it is possible to represent the relative error in surface area estimates depending on relative orientation of this axis with the image main axes (Fig. 2). Surface areas obtained with isosurface are over-estimated whatever the orientation of the structures. When using Crofton estimators, surface areas are underestimated when the structure is aligned with one of the directions used for estimation.

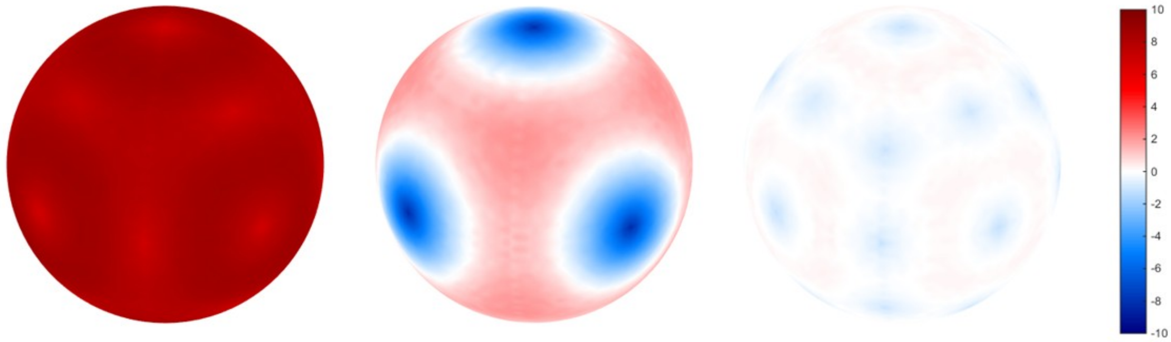


Figure 2. Representation of relative error in surface area measure depending on relative orientation of the shape with image axes (the whiter, the better). From left to right: isosurface, Crofton with three directions, Crofton with 13 directions.

Conclusion

Current results show that surface area obtained with isosurface is systematically over-estimated, but estimates present little variability. Surface area obtained with Crofton estimator computed with three directions is on average unbiased, but is less precise. The best option is the Crofton estimator computed with 13 directions, as its produces less bias due to the discretisation of the direction, and less variability than the isosurface method.

Further work will focus on the study of discretisation grids with non uniform scaling, a situation commonly arising with confocal microscopy or with Magnetic Resonance Imaging. Also, the comparison with other types of estimators based on local configurations would be of interest.

References

Howard C.V. and Reed, M.G. (2010) '*Unbiased Stereology*', second edition, QTP Publications, Liverpool, London.

Legland D. and Kiêu K. and Devaux M.-F. (2007) 'Computation of Minkowski measures on 2D and 3D binary images'. *Image Anal. Stereol.*, V.26, pp.83-92

Lorensen W.E. and Cline H.E. (1987) 'Marching cubes: a high-resolution 3D surface construction algorithm' *Proceedings of the 14th ACM SIGGRAPH on Computer Graphics*, V.21, pp.163-169

Ohser J. and Mücklich F. (2000) '*Statistical Analysis of Microstructures in Materials Sciences*'. J. Wiley & Sons

Pirard E. and Dislaire G. (2011) 'Sensitivity of particle size and shape parameters with respect to digitization' 13th International Congress for Stereology, Beijing.