ACTA STEREOL 1996; 15/1: 53-58
ORIGINAL SCIENTIFIC PAPER

ESTIMATING MEAN LINEAR INTERCEPT LENGTH USING THE TRISECTOR

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ABSTRACT

We propose a method for estimating the mean linear intercept length $\overline{L}$, of anisotropic, space-filling grains or cells. The practical sampling and estimation procedure is summarized in the following steps. (1) Take a vertical axis perpendicular to the axis of structural anisotropy. (2) Take three vertical sections mutually at an angle of 120° apart, and parallel to the vertical axis. The first vertical section is randomly oriented around the vertical axis. This probe is called a trisector. (3) Overlay a test system of cycloid arcs and points on each of the three vertical sections, and align the cycloid minor axes parallel to the vertical direction. The vertical direction must be identifiable on the vertical sections. (4) Count the total number of intersections between the cycloids and the traces of the surfaces of interest, and the total number of test points that hit the reference space. (5) Calculate an estimate of $\overline{L}$ as the ratio of total point to total intersection counts, times a constant characteristic of the test system used. We estimated the mean linear grain size of a rolled plate of an extra-low-carbon steel to 20 μm, and assessed an upper limit for the coefficient of error of this mean to about 2%. Our conclusion is, that the trisector is an unbiased and precise method for estimating the mean linear intercept length of space-filling, anisotropic grains or cells. With a small modification of the estimator of $\overline{L}$, the proposed method may also be applied to anisotropic, separated objects.

Key words: anisotropy, cycloid, ELC-steel, vertical sections, trisector, stereology.

INTRODUCTION

Mechanical properties of metallic materials are often correlated to the mean intercept grain size. Hence, material scientists may want to estimate the mean linear intercept length $\overline{L}$. For isotropic structures this is trivial, since intercept length may be measured along an arbitrary direction. The purpose of this paper is to propose an unbiased and precise method for estimating $\overline{L}$ of anisotropic, space-filling grains or cells. The estimation procedure is also demonstrated with the aid of a real material. The method is based on the following two observations. First, $\overline{L}$ is inversely proportional to the surface area per

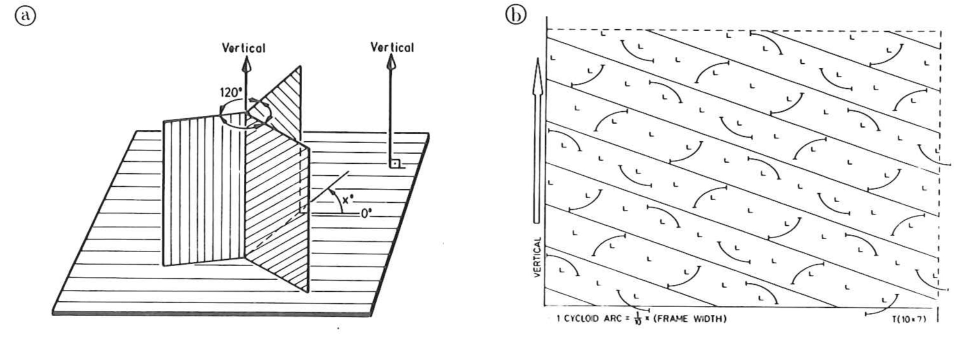

Figure 1: Schematic illustration of the trisector method for estimating mean linear intercept length. (a) Three vertical sections with a common vertical axis, taken perpendicular to the axis of structural anisotropy. (b) A test system of cycloid arcs and points is overlaid on the vertical sections in (a).

unit reference volume S_V (Underwood, 1970). Second, an efficient and reliable estimate of S_V is provided by the trisector (Gokhale and Drury, 1994). This probe consists of three vertical sections that are mutually at an angle of 120° apart, and with a common vertical axis chosen perpendicular to the axis of structural anisotropy (see Fig. 1a). To estimate S_V, intersection and point counting is carried out with a test system of cycloid arcs and points, overlaid on the three vertical sections (see Fig. 1b). An estimator of $\overline{L}$ is then proportional to the ratio of total point to total intersection counts.

We applied this outlined method to a rolled plate of an extra-low-carbon steel, and estimated its mean linear grain size to 20 μm. An assessed upper limit for the coefficient of error of this estimate was about 2%.

Our conclusion is, that the trisector is an unbiased and precise method for estimating mean linear intercept length of anisotropic, space-filling grains or cells. With a small modification of the estimator of $\overline{L}$, the proposed method may also be applied to anisotropic, separated objects.

MATERIAL AND METHODS

Material

The material used in this study was a cold rolled and annealed extra low carbon (ELC) steel with the following chemical composition (in %): 0.019C, 0.01Si, 0.18Mn, 0.003P, 0.010S, 0.0046N, 0.01Cr, 0.02Ni, 0.055Al. The structure consists of space-filling, anisotropic grains for which the ratio of the lengths of the major to the minor axis is about 3:1 (see Fig. 2).

An arbitrary sample of size 2×20 cm was available from a 0.8 mm thick plate. The 2 cm side of the plate was parallel to the rolling direction.

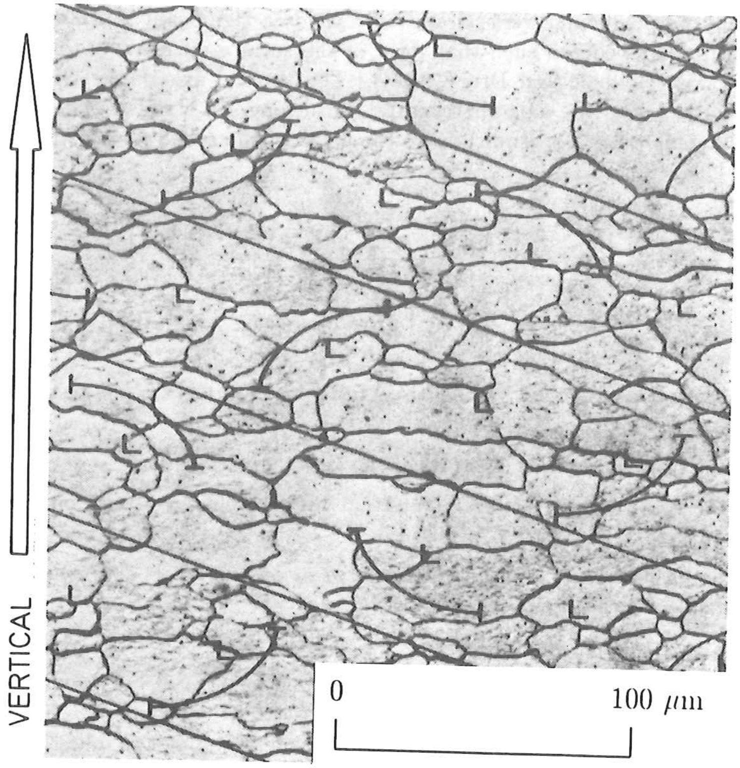

Figure 2: Vertical section from a rolled steel plate with an overlaid test system for intersection and point counting.

Stereological design

For space-filling grains or cells the mean linear intercept length $\overline{L}$ is given by the identity $\overline{L} = 2/S_V$, where S_V is the ratio of the surface area to the reference volume, see e.g. Underwood (1970, Eq. 4.4). For an anisotropic structure, S_V may be estimated on isotropic, uniform random sections by point and intersection counting via the classical stereological relation $S_V = 2 \cdot I_L$, where I_L is the number of intersections per test line length. A more convenient way to estimate S_V is to take vertical sections, and to carry out intersection and point counting with a test system of cycloid arcs and points (Baddeley et al., 1986). The vertical section design which is used in this study relies completely on the theory presented by Gokhale and Drury (1994). They optimised the vertical sampling procedure to get an unbiased estimate of S_V with a low (less than 5%) random sampling error from measurements on a few (not more than three or four) vertical sections. Provided that the vertical axis is chosen in a specified way, a sampling error less than 5% is obtained with three systematic vertical sections (Gokhale and Drury, 1994). This probe, called a trisector, consists of three vertical sections mutually at 120° apart, and with a common vertical axis chosen such that most of the surface elements are not parallel or almost parallel to the vertical axis. Two perpendicular vertical sections (which is equivalent to four mutually perpendicular vertical sections) yields a significant sampling error, even if the vertical axis is chosen such that most of the surface elements are not parallel to the vertical axis (Gokhale and Drury, 1994). The vertical axis of the trisector is thus taken perpendicular to the axis of structural anisotropy. In a rolled plate for instance, the vertical axis is taken perpendicular to the plane of the rolled plate. An estimator of $\overline{L}$ is then

$$\hat{\overline{L}} = \frac{l}{p} \cdot \frac{1}{M} \cdot \frac{\sum_{i=1}^{3} P_i}{\sum_{i=1}^{3} I_i}, \tag{1}$$

where

l/p is the ratio of cycloid arc length to test point number for the test system used,

M is the final linear magnification of the vertical sections,

P_i is the total number of test points that hit the reference space on the ith vertical section, $(i = 1, 2, 3)$,

I_i is the total number of intersections between surface traces and cycloid arcs on the ith vertical section, $(i = 1, 2, 3)$.

Sampling and preparing vertical sections for the trisector

The vertical axis was chosen perpendicular to the plane of the rolled plate. Three systematically oriented vertical sections mutually at 120° apart, and with a random start $x°$ in the interval $(0°, 120°)$ were taken from the plate. The sections were also systematically located; Fig. 1a thus only depicts the systematic orientation of the vertical sections, but not their systematic locations. We took three systematic pieces from the plate and cut these parallel to the vertical axis (i.e. perpendicular to the plane of the plate) at 56°, 176°, and 296°, respectively, where 0° corresponds to the rolling direction. Following conventional metallographical procedures, the cut pieces were embedded in a plastic cylinder (with the vertical sections as examination surface), polished and etched to reveal the grain boundaries.

Stereological analysis of the vertical sections

Four fields of view (quadrats) were systematically sampled from each of the three vertical sections, and photographed in a light microscope with a 25× objective. The longer axis of each micrograph was always parallel to the vertical axis, namely parallel to the thickness direction of the rolled plate. The final linear magnification of the vertical sections was $M = 510$.

A transparent sheet with a test system of cycloid arcs and test points was overlaid on each micrograph (see Fig. 2). The minor axes of the cycloids were always parallel to the longer axis of the micrograph, namely the vertical direction. For the test system used, the constant $l/p = 14\,\mathrm{mm}$, corresponding to about $27\,\mu\mathrm{m}$ at the specimen scale. The relevant point and intersection counts were inserted into the right-hand side of Eq. (1).

Table 1: Raw data and estimates of mean intercept length $\overline{L}$.

Sample	Φ (degrees)	I	P	$\hat{\overline{L}}$ (μm)
1	56	56	44	
	176	68	44	
	296	59	44	
		total=183	total=132	19.8 (3.4%)
2	56	59	44	
	176	55	44	
	296	66	43	
		total=180	total=131	20.0 (3.4%)
3	56	66	45	
	176	58	44	
	296	51	45	
		total=175	total=134	21.0 (3.4%)
4	56	65	44	
	176	65	45	
	296	60	45	
		total=190	total=134	19.4 (3.4%)
Pool		total=728	total=531	20.0

RESULTS

The estimates of $\bar{L}$ obtained from sampling four quadrats on each of the three vertical sections of the trisector are shown in Table 1. The observations are not independent, it is therefore a non-trivial problem to estimate the error variance of the estimator used. Cruz-Orive and Howard (1991), and Roberts et al. (1991), computed the variance among subsamples corresponding to sets of observations at systematic angles, sampled from a larger set of observations at systematic angles. Here, the error variance with respect to quadrat location for the given set of three systematic sectioning angles was computed. The data were split into four subsamples corresponding to the four quadrats in each of the three vertical sections. The coefficient of error (i.e. CEM=SEM/mean) among the four estimates, namely 3.4%, gives an idea of the accuracy of the method. Since the four subsamples are not independent, the CEM of the pooled mean cannot be estimated as 3.4%/2=1.7% . But systematic sampling is more efficient than independent random sampling, we therefore consider 1.7% as an upper limit for the CEM of the pooled estimate of the mean intercept grain size.

DISCUSSION

In this paper we proposed and demonstrated a method for estimating the mean linear intercept length $\overline{L}$, of anisotropic, space-filling grains or cells. The practical sampling and estimation procedure can be summarized in the following steps. (1) Take a vertical axis perpendicular to the axis of structural anisotropy. In this study the vertical axis was taken perpendicular to the plane of a rolled steel plate. (2) Take three vertical sections mutually at an angle of 120° apart, and parallel to the vertical axis. The first vertical section is randomly oriented around the vertical axis. This probe is called a trisector. (3) Overlay a test system consisting of cycloid arcs and points on each of the three vertical sections, and make sure that the cycloid minor axes are parallel to the vertical direction. The vertical direction must be identifiable on the vertical sections. (4) In each of the vertical sections, count the number of intersections between the cycloids and the traces of the surface of interest, and the number of test points that hit the reference space. Add these respective numbers over the three vertical sections. (5) Insert the pertinent numbers in the right-hand side of Eq. (1), and calculate an estimate of $\overline{L}$.

To assure that the random sampling error is less than 5%, the vertical axis of the trisector is taken perpendicular to the axis of structural anisotropy (Gokhale and Drury, 1994). In most practical applications, this small sampling error can be accapted since the regional to regional structural variations in homogeneous microstructures are likely to be at least 5%. A specifically chosen vertical axis contrasts the vertical sections method (Baddeley et al., 1986), where the experimenter has total freedom to choose the vertical axis. Note, however, that both the vertical section method (Baddeley et al.,1986) and the trisector method (Gokhale and Drury, 1994) are unbiased methods. The latter method is a set of three systematically oriented vertical sections designed to increse estimation precision, and thus reduce the random sampling error.

Our conclusion is, that the trisector is an unbiased and precise method for estimating the mean linear intercept length of space-filling, anisotropic grains or cells. With a small modification of the estimator for $\overline{L}$ (the right-hand side of Eq. (1) is multiplied with 2), the proposed method may also be applied to anisotropic, separated objects.

ACKNOWLEDGMENTS

We thank Anders Haglund for providing the experimental material; Connie Westman for specimen preparation; and Karl Babl and Barbara Krieger for help with the artwork. We also thank a rewiever for constructive criticism of this paper. One of the authors (AMG) acknowledges the financial support of U.S. National Science Foundation grant (DMR-9301986) for the research leading to this contribution.

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Presented at the 9th International Congress for Stereology, Copenhagen, August 20th to 25th, 1995.