ACTA STEREOL 1995; 14/1: 17-22
STERMAT'94
ORIGINAL SCIENTIFIC PAPER

AN ENHANCED MODEL FOR THE FIBRE PULL-OUT AND FAILURE IN THERMOPLASTIC COMPOSITES

Viktor Beneš*, Zdeněk Kořínek, Marek Postler

*Dept. of Mathematics, FSI, Czech Technical University,
Karlovo Nám. 13, 12135 Prague 2, Czech Republic

Dept. of Material Sciences, FSI, Czech Technical University,
Karlovo Nám. 13, 12135 Prague 2, Czech Republic

ABSTRACT

An enhanced method of evaluation of fibre-matrix adhesion in short fibre composites is developed. A parametric model of the fibre pull-out and failure is proposed. The parameters are estimated by means of the degree-of-fit criterion of the empirical and theoretical probability distribution function of fibre pull-out length. This length is experimentally measured on the fracture surface, while the theoretical probabilities are obtained by means of simulations. The practical results obtained are based on the known fibre length distribution in injection-moulded composites made from polymers and glass fibres.

Keywords: composite material, fibre pull-out and failure, length distribution.

INTRODUCTION

The presented research is motivated by the fact that the level of adhesion between fibres and matrix affects substantially the mechanical properties of composites. In Kořínek and Beneš(1994) a method for the fibre-matrix adhesion was developed for parallel fibres based on the comparison of experimental and model probability distribution functions of fibre pull-out length. Analytical formulas were proposed containing two unknown parameters corresponding to the pull-out intensity and the probability of the failure. The random position of the failure was described by a fixed triangular distribution with the mode corresponding to maximal stress location.

These parameters were estimated by means of a maximum degree-of-fit criterion between the theoretical and experimental distribution functions of fibre pull-out on the fracture surface. Theoretical probabilities were obtained by means of simulations based on the known initial distribution of the fibre length. Practical measurement was performed for five composite materials. The results obtained encountered a very good fit of the proposed model with experimental data for those three materials in which the intensity of pull-out was relatively large and the probability of failure low. However, the degree-of-fit was less satisfactory for materials of type MOSTEN and PROPATHENE, where on the contrary the intensity of pull-out was small and the probability of failure larger.

This leads to the need of revision of the model of failure, the new one is presented here. We let the model of pull-out unchanged, but for the probability of failure a two-parametric model is proposed, reflecting separately both the influence of the length of fibre and the location of the fracture surface. It is assumed that each fibre failures at most once. The distribution of the failure position is again triangular but it includes the fourth unknown parameter, the critical length, which determines the zone of zero probability of failure on a fibre. The fibre-matrix bond strength can be calculated from the critical length by the Kelly and Tyson (1965) expression.

MATERIALS

The experimental material measured is the same as in Kořínek and Beneš(1994), Table 1, therefore the microstructural parameters and experimental conditions are only shortly repeated here.

Injection-moulded specimens for a Charpy impact tester were made from commercially available polymers and glass fibres. Fracture surfaces of the tested specimens were examined by a scanning electron microscope (SEM). The experimental fibre pull-out length distribution function F_e was derived from micrographs of the under-surface regions with unidirectionally aligned fibres (in direction α) near the notch tip. The original fibre length distribution function F was evaluated by a direct measurement of the length of each fibre after separation of fibres from under-surface layers of the composite after burning off the matrix. The basic assumption is that fibres are parallel which was fulfilled by taking the fracture surface from the appropriate part of the test specimen, see Fig.1.

The theoretical model of spatial distribution of parallel fibres in the matrix assumes that centres of segments projected on a line of orientation α form a stationary point process. The fibre lengths L are well described by a Weibull distribution $F(x) = 1 - e^{-(\frac{x}{\theta})^{\beta}}$, $x \geq 0$, β, θ are nonnegative real parameters. We denote $f(x)$

the corresponding probability density.

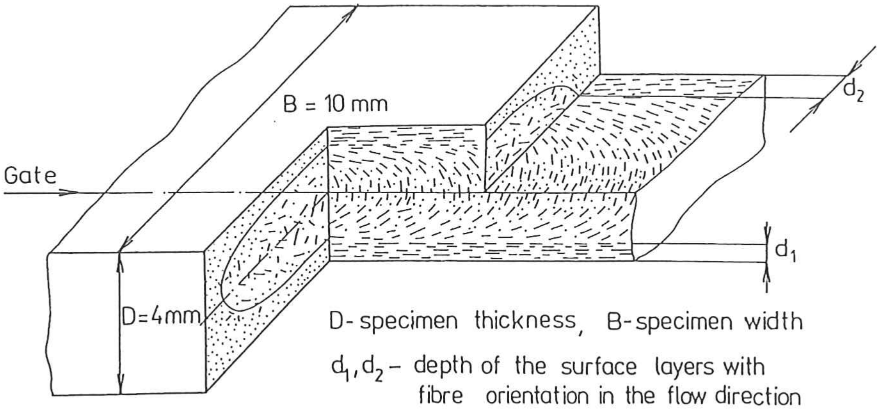

Fig. 1. Scheme of fibre orientation through the longitudinal end transverse cross-sections of the moulded tensile specimen.

METHODS

Let us first consider the situation when a brittle tranverse crack of the matrix propagates across the composite and no fibre failure occurs. The result is represented by model based on the assumption that fibres are only pulled out on their shorter side.

Lemma 1 *The theoretical distribution function G of the pull-out length l_p of fibres with random length L with distribution function $F(x)$ is equal to*

$$G(x) = F(2x) + 2\int_{2x}^{\infty} \frac{f(t)}{t} dt, \; x \geq 0. \tag{1}$$

Proof: For fixed fibre length L the random pull-out has conditional distribution function

$$\begin{aligned} P(l_p \leq x | L) &= \tfrac{2x}{L}, \quad x \in \langle 0, \tfrac{L}{2} \rangle \\ &= 1, \quad x > \tfrac{L}{2}. \end{aligned}$$

Integrating with respect to the density $f(x)$ of L we get the result. q.e.d.

However, for commercially available thermoplastic compounds with low fibre length and a strong fibre-matrix bond the model has to be modified to include:
(1) partial sliding fibres in matrix at their longer part,
(2) tension failure of fibres.

In Kořínek and Beneš(1994) a model for partial slide l_v of the fibre was proposed:

$$l_v = l_p \exp\left[-\frac{L - l_p}{c}\right],\ c > 0 \tag{2}$$

where l_p is pull-out length. Equation (2) gives the protruding length of fibre l_t

$$l_t = l_p + l_v = l_p\left[1 + \exp\left(-\frac{L - l_p}{c}\right)\right].$$

Because there is a positive failure probability P of the longest fibre bridging the brittle crack of matrix, we try to model next two quantities: the probability of failure and the position B of failure, see Fig.2.

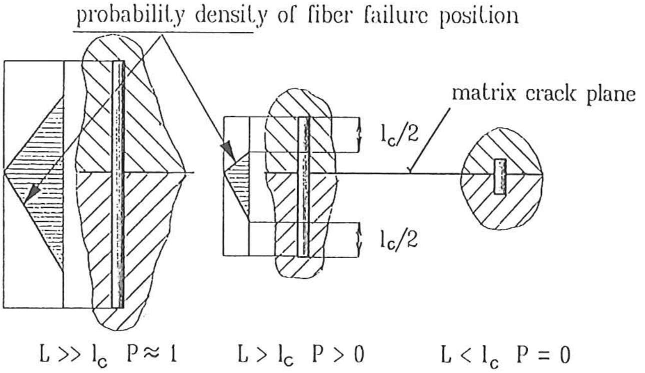

Fig.2. The probability of failure P and the triangular probability denstity of the position of the failure depending on the critical length l_c.

From the Kelly and Tyson(1965) theory there exists a material constant l_c called the critical length such that fibres shorter than l_c do not fail. For $L > l_c$ it is assumed that fibres do not fail on intervals $I_1 = \langle 0, \frac{l_c}{2}\rangle$ and $I_2 = \langle L - \frac{l_c}{2}, L\rangle$ and the random location B of the failure has a triangular probability density function on the interval $I = \langle \frac{l_c}{2}, L - \frac{l_c}{2}\rangle$, where the vertex of this triangle is in the point l_p. This corresponds to the fact of largest probability of failure near to the fracture surface represented by the coordinate l_p (maximal stress location). When $l_p \in I_1$ or $l_p \in I_2$, then again $P = 0$ independently of L.

Under these assumptions a suitable model for the probability of failure is suggested:

$$P = \left(\frac{2l_p - l_c}{L - l_c}\right)^a \left(\frac{L - l_c}{L_m - l_c}\right)^b,\ l_p \in I, \tag{3}$$

where a, b are unknown positive parameters, L_m is a given maximal length of fibres. It is expected that $b < a$, $b < 1$.

Assuming not more then one failure of the fibre we obtain the total length of protruding

$$\begin{aligned} l_t &= (l_p - B)(1 + exp[-(l - l_p)/c]), && B < l_p \\ &= (B - l_p)(1 + exp[-l_p/c]), && l_p \leq B < 2l_p \\ &= l_p(1 + exp[-(B - l_p)/c]), && B \leq 2l_p. \end{aligned} \tag{4}$$

The formula for the distribution function of l_t given by (4) is untractable. However, a Monte-Carlo algorithm using (2) - (4) enables us to obtain the value of l_t. Thus for given l_c, c, a, b and parameters θ, β of the random length L the empirical distribution function F_s of l_t is obtained by a computer simulation. Assuming that θ, β are known, the desired optimal values l_c, c, a, b are solutions of the problem

$$\min_{a,b,c,l_c} Q, \text{ where } Q = \max_{l} |F_e(l) - F_s(l)|, \tag{5}$$

F_e being the experimental distribution function of pull-out length in the specimen.

RESULTS AND DISCUSSION

Among the five composite materials studied in Kořínek and Beneš(1994) we present results of the new model application on PROPATHENE, for which the worst degree-of-fit was obtained using the older model.

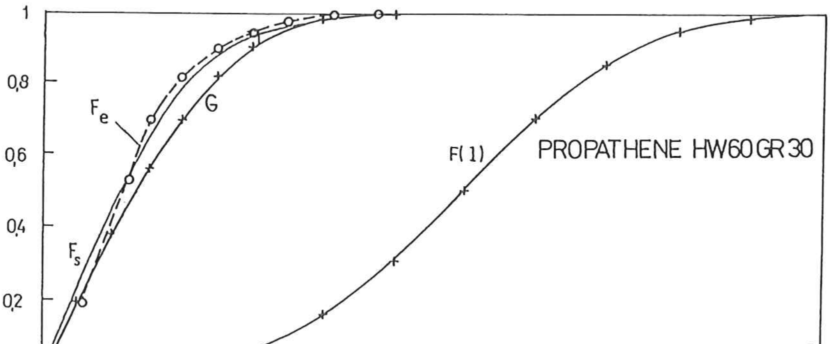

Fig.3. Probability distribution function of fibre length F, experimental pull-out length F_e, theoretical pull-out length G without failure model and computer simulated optimal distribution F_s in the new model.

The curves of theoretical distribution function G and experimental cumulative frequency F_e of pull-out lengths, original length distribution F, and computer simulated distribution F_s with optimal parameters are plotted in Fig.3. They correspond to parameters $\theta = 0.663, \beta = 3.467$ measured after burning off and $a = 0.42$, $b = 0.08$, $c = 0.05$, $l_c = 0.4$ obtained by optimization (5). For this material the pull-out is low, but simultaneously we obtain that about 19% of fibres fail.

The presented four-parametric model of the fibre pull-out and failure leads to a better fit (expressed by Q in (5)) of the simulated and experimental data also for materials with a high rate of fibre failure. The parameters a, b in (3) reflect the influence of fracture surface position along the fibre and the length of the fibre, respectively, on the probability of failure. Moreover the critical length l_c being an unknown parameter enables comparison of its estimation here and that using standard methods (Kořínek and Beneš, 1994). The better results are obtained by more computational effort when optimizing (5) with respect to four parameters.

ACKNOWLEGDEMENTS

The research work of the first author was supported by the Grant Agency of the Czech Republic, project no. 201/93/2172.

REFERENCES

Kelly A, Tyson WR. Tensile properties of fibre reinforced metals: copper/tungsten and copper/molybdenum. J Mech Phys Solids 1965; 13: 329-350.

Kořínek Z, Beneš V. Probabilistic aspects of evaluation of fibre-matrix adhesion from pull-out fibre lengths on fracture surfaces of short fibre composites. Proc 6ECS Prague, 1993. Acta Stereologica 1994; 13/2: 389-394.