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REVIEW ARTICLE

MORPHOLOGICAL IMAGE SEGMENTATION

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ABSTRACT

Morphological image segmentation is based on MISP (Morphological Image Segmentation Paradigm), due to S. Beucher and F. Meyer. Its central operation is the *watershed.* The paradigm itself is a complex transformation with two inputs, namely the function f to be segmented and the set M of markers, and one output, namely the (binary) contours of the segmentation.
The input functions may be scalar, or vectorial ones (e.g. colour images). They are defined in one, two or three dimensions, over Euclidean spaces, regular grids, or irregular graphs. The two inputs f and M interact via the *swamping* operation. The swamping of function f with respect to set M is the lower over-estimation of f whose minima coincide with the connected components of M. It is obtained by a procedure of reconstruction closing. Hence the techniques of morphological connected filtering intervene in the paradigm. As a matter of fact they are often also used prior to the MISP for filtering the function under study.
A first pedagogical example is given, it shows the successive steps of the paradigm. It is followed by two original algorithms applied to already known images.

Key words: MISP, image segmentation, mathematical morphology, swamping, watershed.

1. INTRODUCTION

In image processing, one says that an image is segmented when its support has been partitioned. In this sense, any thresholding creates a segmentation. By describing a segmentation as *morphological,* we mean that it basically involves min, max and inequality operations (of course, this toolbox is not exclusive: it is never forbidden to perform a subtraction or a moving average...). In this sense, the thresholding, again, turns out to be a morphological segmentation, since it is exclusively based upon inequalities.
However, historically speaking, the theoreticians and the practitioners of Mathematical Morphology oriented their efforts to segment and label images in a quite specific direction, which today delineates a certain body of operations, of know-how, and of theorems. It is this corpus that I would like to survey here, by focusing on the operators, and their meanings, rather than on their implementations or their theory on continuous spaces.

The watershed transformation appeared in our field in 1976, in the framework of a study about road surfaces. H. Digabel and C. Lantuejoul introduced the morphological notion of a catchment basin as a geometric substitute for the physical spreading of a cone of sand (in road technology this technique is a classical roughness descriptor). They presented their algorithm in the 2nd European Congress for Stereology (1978).
The next important step, due to S. Beucher and C. Lantuejoul (1979), was the idea to transpose the roughness descriptor to grey-tone functions and to try to segment them by acting on their *gradients*. Then a third and crucial moment came with the disjunction between markers and minima by the swamping operator (S. Beucher, 1982). In the meantime, F. Meyer also introduced the changing of homotopy on gradient functions, but by means of perceptual graphs (1982) or of conditional bisectors, yielding performant segmentation on biological cells (1980).
Whereas since 1982 all operations for morphological segmentation were known, and available, the technique did not expand until 1990. The ideas were probably too new and their implementation too slow. It seems that the Centre de Morphologie Mathématique was the only place where watersheds were used in practice, for 2D or 3D, still of moving, imagery. The situation changed in 1990, under the publication of two synthetic texts (S. Beucher, 1990 ; S. Beucher and F. Meyer, 1990), and the algorithmic improvements, mainly due to L. Vincent, P. Soille (1990), and F. Meyer (1991). The two synthetic papers revived the interest for the technique, with new domains of applications, such as image coding (P. Salembier, J. Serra, 1992), J. Crespo, J. Serra and R. W. Schafer (1995), also reactivated the theory, with the contributions of L. Najman, M. Schmitt (1994) and F. Meyer (1994), among others.

2. BEUCHER AND MEYER PARADIGM

What is usually understood by "Morphological Segmentation" is a technique based upon a paradigm from S.Beucher and F.Meyer, called MISP (for Morphological Image Segmentation Paradigm). This notion combines the two transformations of a swamping and of a watershed. We will first introduce here these two transformations and will continue with their combination.

2.1. Swamping

Opening by reconstruction

The swamping transformation acts on greytone images, that are modelled as numerical functions. More precisely, the definition space E of these images is supposed metric and equipped with a connectivity (in the morphological sense, see (Serra, 1988, ch. 2)). This covers Euclidean spaces, all usual digital spaces, as well as planar graphs. We will be more restrictive for the arrival space J, that we suppose discrete. J is the finite or numerable sequence $\{0, 1, ..., j ... j_m\}$. The price to pay for a continuous grey-axis would be mathematically higher, and indeed irrelevant here, since we are looking for segmentation of *digital* images.
Let $F : E \rightarrow J$ be the class of numerical functions from E into J. With every function $f \in F$, associate its cross section X_j (f) at level $j \in J$, i.e. :

$$X_j(f) = \{x : x \in E, f(x) \geq j\}$$

Given a function $g \in F$, we now introduce the *opening by reconstruction* $\gamma_{rec}(f ; g)$ of f with respect to g, via its cross sections. The section $X_j[\gamma_{rec}(f ; g)]$ is defined as the union of all those connected components of $X_j(f)$ which contain at least one point of $X_j(g)$; see (fig.1).

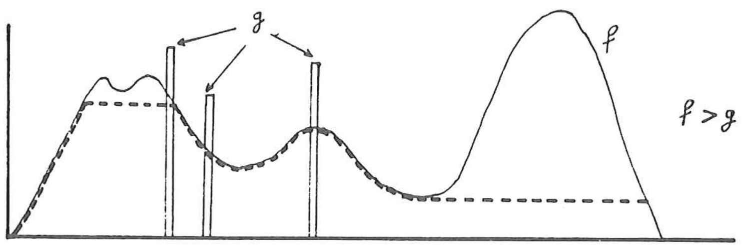

Fig. 1. Openings by reconstruction of function f according to marker g.

Proposition 1 *: Given $g \in F$, the operation $\gamma_{rec}(f ; g)$, $f \in F$, is an opening ; on the contrary, when f is fixed and g becomes variable, with $g \leq f$, the same operation turns out to be a closing.*

[Easy proof, already given in (Serra, 1982, ch. XII), among others]

The major properties of the openings by reconstruction concern the ways the maxima are treated. (A maximum of f is a connected component $C_j(f)$ of the section $X_j(f)$ such that $C_j(f) \cap X_{j+1}(f) = \emptyset$. It is a subset of E, whose associated grey level has value j).
The first property of γ_{rec} that involves maxima is related to its Domain of Invariance. When function g admits n maxima (M_i, j_i), $i = 1 \ldots n$, then $\gamma_{rec}(f ; g) = f$ for all functions $f \in F$, which have n maxima (Z_i, j_i) such that $Z_i \cap M_i \neq \emptyset$ for all i.

Similarly, we can state the two following properties :

Proposition 2 *: Given $g \in F$, for all $f \in F$, each maximum of the opening $\gamma_{rec}(f ;g)$ contains at least one maximum of f and another one of $\inf(f, g) = f \wedge g$. A maximum M of f is preserved if and only if $g(x) \geq f(x)$ for at least one $x \in M$.*

Proof : For the first part, remark that every maximum of $X_j[\gamma_{rec}(f ; g)]$ is always also a connected component $X_j(f)$; moreover, since it is a maximum, it is generated from a maximum section of $f \wedge g$. As for the second part, if $g(x) \geq f(x)$, then g marks the maximum M ; conversely, if M, of value j, is preserved, then there exists an $x \in E$ such that $x \in X_i(g)$ that marks M. But M is a maximum for f, hence we have $g(x) \geq f(x)$. Q.E.D.

In other words, under γ_{rec}, some maxima of f are grouped around those of $f \wedge g$ and the others are removed (and replaced by stairs). In this simplification process, no maximum which would not contain a former one can be generated. Finally, the only way to leave unchanged, at the same level, a maximum of f is that g (partly) covers it.

Proposition 3 *: When the set of maxima of* $f \wedge g$ *is a subset of those of f, then* $\gamma_{rec}(f\,;\,g)$ *is the largest function*

- *to be smaller than f,*
- *to have the maxima of* $f \wedge g$.

Proof : Suppose that there exists f' such that $\gamma_{rec}(f\,;\,g) \leq f' \leq f$, then $\gamma_{rec}(f\,;\,g) = \gamma_{rec}(f'\,;\,g)$, since γ_{rec} is an opening. But the maxima of $f \wedge g$ are also those of $f' \wedge g$, hence $\gamma_{rec}(f'\,;\,g) = f'$. Q.E.D.

Closing by reconstruction

The opening by reconstruction admits a dual closing, for the duality generated by the inversion $f \rightarrow j_m - f$, where j_m is the maximum grey value.*Mutatis mutandis,* the above propositions remain valid, but now concern the minima. The swamping is an operation designed to impose a set M of minima to the function f under study, so that the result be as closed as possible to the original function f. What we have seen previously suggests the following procedure :

Definition *: Let* $M = \{M_i,\ i \in I\}$ *the union of disjoint connected components* M_i. *Denote by g the function*

$$g(x) = 0 \qquad x \in M$$
$$g(x) = j_m \qquad x \notin M$$

Then the closing by reconstruction $\varphi_{rec}(f\,;\,g)$ *of f by g is called the* swamping *of f according to marker M.*

Of course, to be relevant, the technique requires that every M_i hits one minimum of f. These marked minima will then be the only ones to be kept by $\varphi_{rec}(f\,;\,g)$ (see fig. 2)

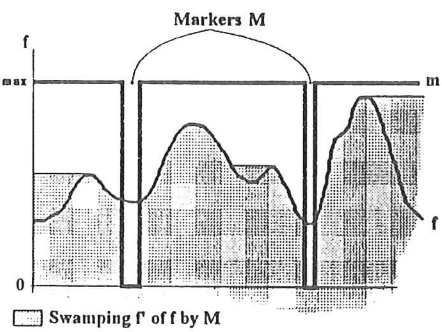

Fig. 2. Example of swamping. This operation modifies the homotopy of the function (and of its section), and fills up the zones around the minima to be suppressed until they become plateaus.

2.2. Watershed

Physical definition

The watershed transformation for greytone images was introduced a long time ago in mathematical morphology (Beucher and Lantuejoul, 1979) ; we will only describe it briefly.

Interpret the image under study as a relief, where the grey values are altitudes (the lighter, the higher). Imagine that, at each minimum, we bore a hole, down to level zero. Then we gradually immerse the relief into water so that the surface is gradually flooded from the bore holes. Since we want to access each catchment basin separately, we must avoid the confluence between floods which arise from different minima. Therefore, we build a dam as soon as two floods locally merge. The process ends when the dams are completed everywhere (see fig. 3). They form, on the relief, what geographers call *divide lines*. The projections of these lines on E are the *watersheds* and the complement of the latter is the union of the *catchment basins*.

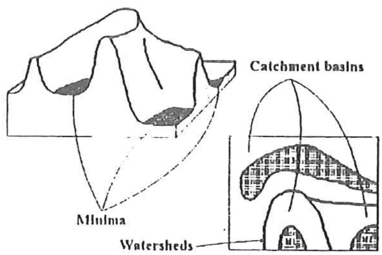

Fig. 3 : Example of regional minima, catchment basins and divide lines

The watersheds draw closed loops over E. Their interest for segmentation purposes comes, to a large extent, from this property. However, the loop may exhibit some thickness, in particular because of the ambiguous status of the flat zones. In fig. 4, for example, the watershed is the projection

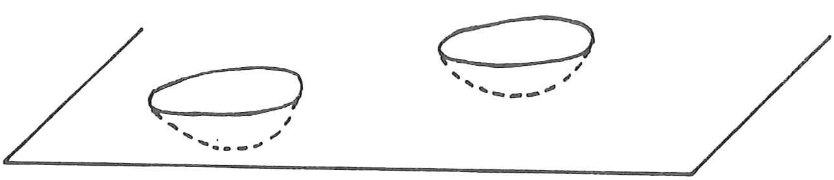

Fig. 4 : Example of an ambiguous case : is the divide line the whole plateau ?

of the whole plateau. The ambiguity will be removed if we are able to give a criterion of priority to the different holes, in the plateau which surrounds them. The simplest one, and also the most physically pertinent, is provided here by the distance function : each point x of the plateau should be directed to its closest hole.

Moreover, by so doing, we have designed the very algorithm to flood the space and to put the dam, level after level.

Flooding algorithm

The set C(f) of the catchment basins and its complement W(f), i.e. the watersheds, will be obtained recursively from the set M of the minima. Firstly, we observe that no change will occur in the result if we impose value zero to all the minima. This done, we now apply recursively the binary thickening $\tau(X,Y)$ with $X \subset Y$, which produces the complement of the geodesic skiz (skeleton by influence zones) of X in Y.The algorithm reads as follows:

initialization $C_0(f) = M = X_0^c(f)$
step n°1 $C_1(f) = \tau(C_0; X_1^c)$
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step n° i $C_i(f) = \tau(C_{i-1}\ ; X_i^c)$
final $C(f) = C_{jm}(f) = \tau(C_{jm-1}\ ; X_i^c)$ *and* $W(f) = C^c(f)$

Remark that C_i is the part of the final catchment basins where the altitudes are $\leq$ i, and similarly that $W_i = X_i/C_i$ is the part of the watershed associated to divides lower than altitude i. Hence the algorithm, just as the physical explanation, constructs the dams progressively !

MISP paradigm

In his thesis (1990) (a splendid monument erected to morphological segmentation), Serge Beucher bases a general formalism for segmentation on the two operations of swamping and of watershed. This paradigm, that we reproduce in fig. 5, comprises three moments. First,

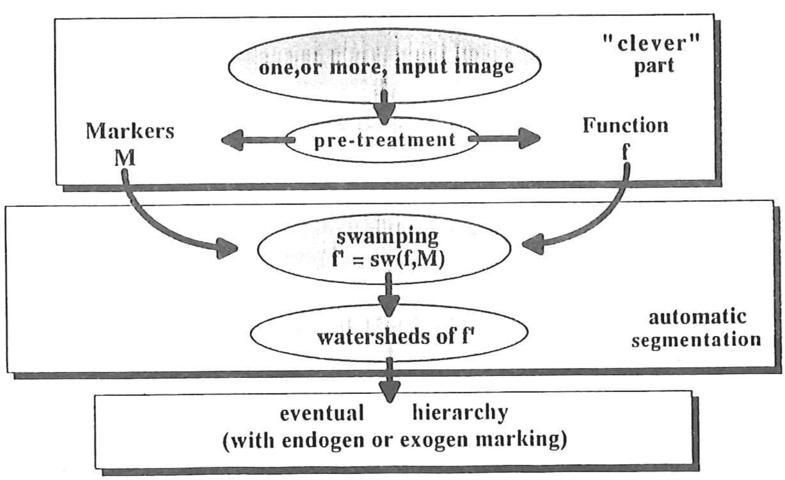

Fig. 5 : S. Beucher and Meyer Paradigm for segmentation

one has to generate one *greytone* image f and one set of markers M from the input information. This implies possible filtering, interactivity, condensation of information (e.g. for colour images), etc. Then, a second step arises, which is purely automatic. It consists in modifying the homotopy of f by swamping this function according to marker M. This results in a function f ' which is in turn transformed by watershed, which ends the second step. The segmentation may stop here. Alternatively, we can add new external information, or also come back to the first phase and use the watershed itself to produce a new marking. Such a dialectic approach generates a hierarchy of coarser and coarser watersheds, but often more and more significant.

3. HOW TO CHOOSE MARKERS FOR SEGMENTATION

In the previous section, we have explained in detail the second phase, i.e. the automatic one, of S. Beucher's Paradigm. We would like now to describe the whole process. Unfortunately, it is impossible to do it in a purely deductive way. There is no such thing as an automatic black box that would segment correctly every image. Therefore, we propose to go through a few typical examples, and to show techniques of a rather general use, by means of particular situations.

3.1. Electrophoresis gel

A basic situation is that of more or less separated objects which emerge from a background. It is typically illustrated by the spots in an electrophoresis gel (6a). In addition, it is historically the first example of the whole segmentation paradigm (Beucher, 1982). We want, here, to contour the spots and to tell which neighbours they have.Firstly, we observe, in fig. 6b, the incrdible number of minima of fig 6

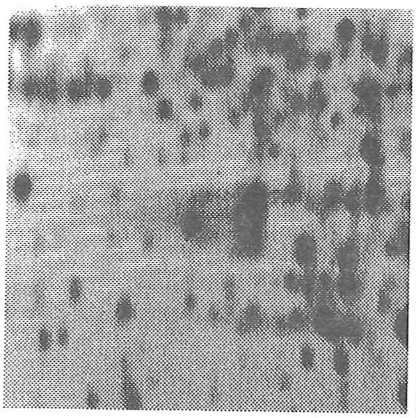

figure 6a

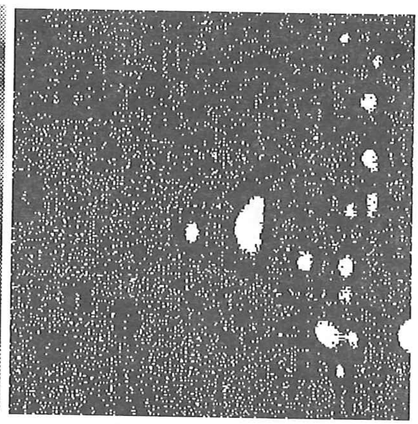

figure 6b

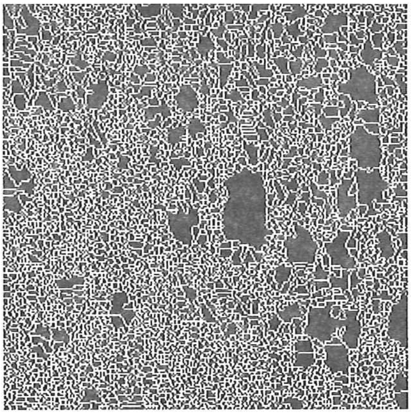

figure 6c

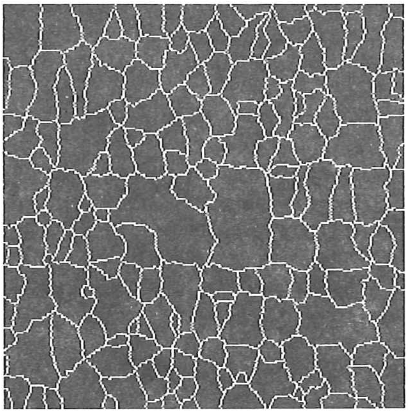

figure 6d

i/ Therefore the row watershed of the gel is just unreadable (6c). In an image there are much more minima than those detected by a human eye.

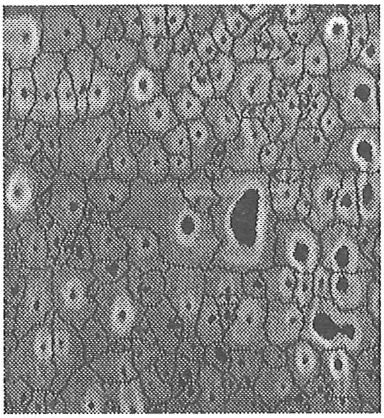

figure 6e

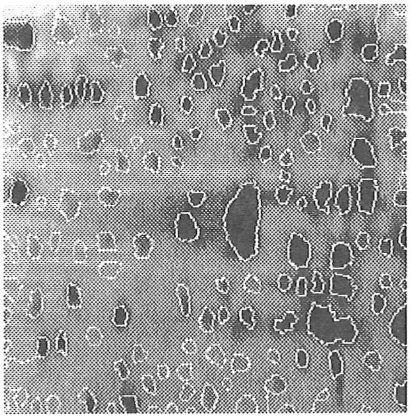

figure 6f

ii/ fig 6d is the watershed of the transform of 6a after a small alternated filter. One can control the reduction, and the pertinence, of the minima on figure 6e

iii/ the swamping of grad(f) according to the watershed 6d and the associated minima is presented in figure 6e. (Indeed, the contours of the spots correspond to the watershed of their *gradient* image, since the inflection points (i.e. the contours) turn out to appear as crest lines in the gradient image).

iv/ The watershed of the swamped gradient 6e appears in figure 6f.

Conclusion : we have seen an iterated version of the paradigm, where a first watershed is introduced as the background marker for a second watershed. We have also seen that contouring objects requires the watershed of their *gradient*, the function itself giving rather their zones of influence.

3.2. Nuclei in a smear


figure 7a


figure 7b

The example is a variant of a classical algorithm due to F. Meyer (1980), for automatic segmentation of nuclei, in cervical smears. Once more, the major problem concerns the elaboration of a pertinent marker. Starting from the initial scene of fig.7a, the gradient of which is shown in fig 7b, we will proceed by the following steps:

i/ imperfect threshold, the goal of which is to select dark zones in each nucleus, and only there. The threshold set is used to swamp the initial image, as shown in fig. 7c;

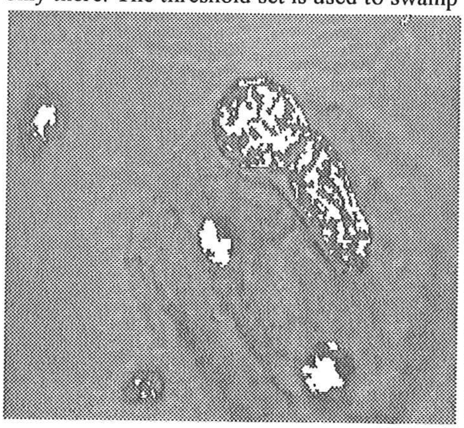
figure 7c

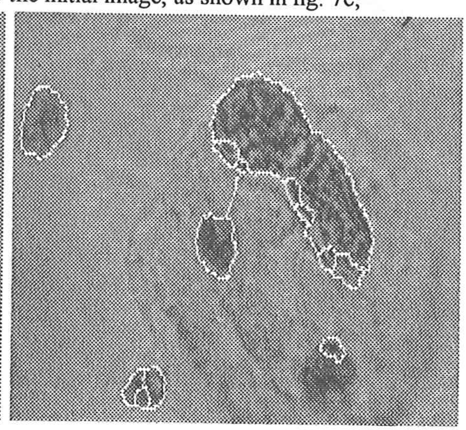
figure 7d

ii/ fig 7c, after a small dilation, serves as amarker to swamp the initial image 7a. The resultins watershed is presented in fig. 7d;

iii/ by filling the holes of fig. 7d and then slightly eroding it, we obtain fig. 7e;

iv/ the outer marker for rhe gradient image is now provided by the skiz of fig. 7e, that we see in fig. 7f, in superimposition with fig. 7e itself (alternately, we could also re-swamp and re-watershed the initial image;

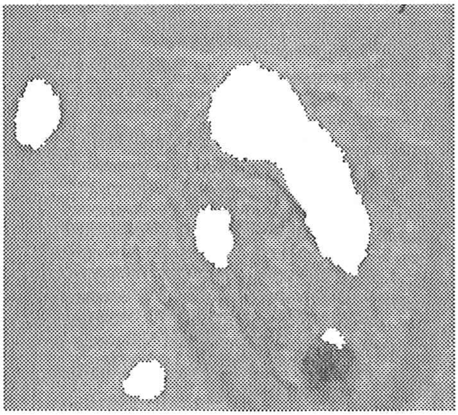

figure 7e

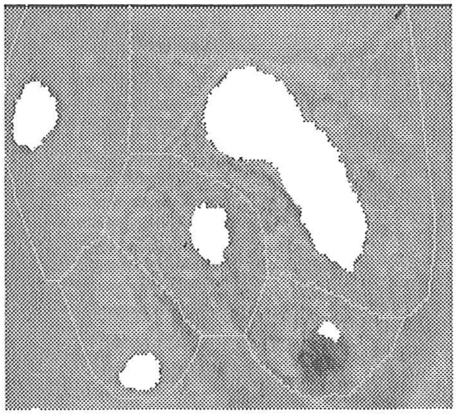

figure 7f

v/ the last steps are identical to steps iii/ to v/ of the previous example, on electrophoresis. The gradient fig.7b is swamped according to set fig. 7f , which results in fig. 7g . The watershed of the latter, shown in fig 7h, provides the required contours of the nuclei.

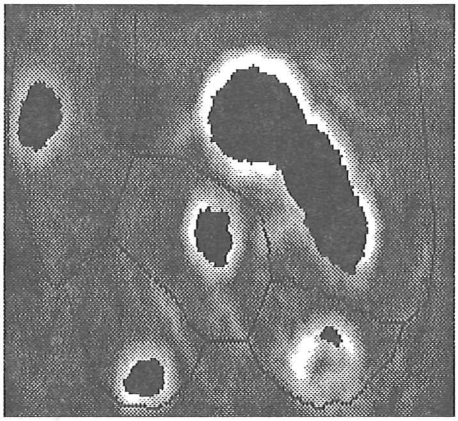

figure 7g

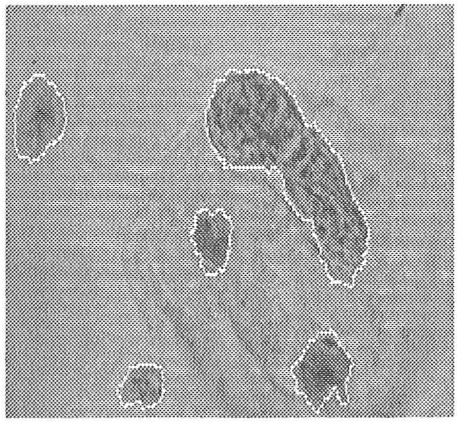

figure 7h

Conclusion : a first watershed is often used as , after simplifications, part of the marker.

3.3. WC-Co microstructures

The micrograph of fig. 8a is extracted from the PhD thesis of G. Gauthier, (1995, Laboratory of Professors Coster and Chermant, Univ. Caen, France). The segmentation we propose below is different from Gauthier's one, a bit less performing, but more pedagogic. It is based on the assumption that the *shape* of the material, i.e. the thresholding of fig.8a at a low level, already contains enough information to allow a reasonable first segmentation. The steps are the following :

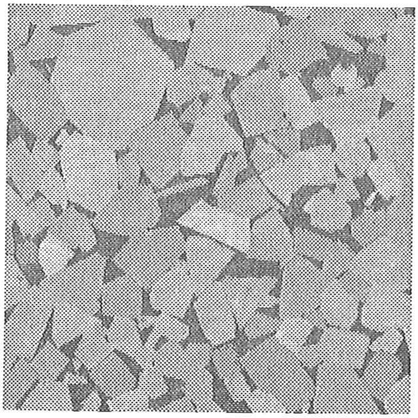

figure 8a

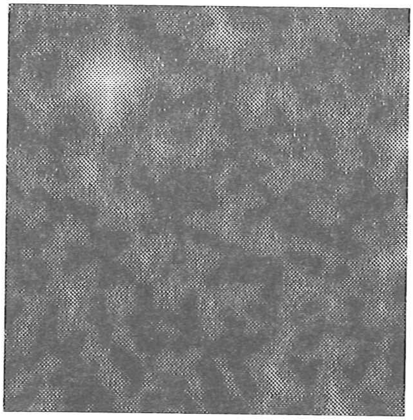

figure 8b

i/ After thresholding the initial image (see in fig. 8c), we calculate the corresponding *distance function* (fig. 8b).

ii/ the watershed of the negative of the distance function is given in fig. 8c, in superimposition with the low threshold section. The intersection of these two sets

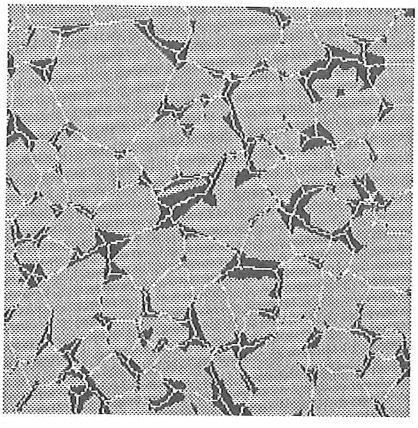

figure 8c

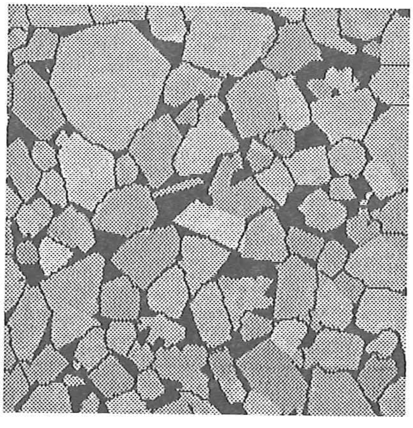

figure 8d

defines the zone to be supressed from the initial image.

iii/ the result of the segmentation is shown in figure 8d :

Improvment: Clearly, such a segmentation ignores the grey variations inside the grains.However we may introduce this additional information by binarizing the gradient of image 8a (low threshold) and substracting it from the binary version of the grains, i.e. from the greys of figure 8c. We obtain the greys of figure 8e, whose watershed of the distance function appears, in superimposition, in white. By removing from the initial image the white and the black zones of figure 8e, we yield the new segmentation 8f, which is more satisfactory.

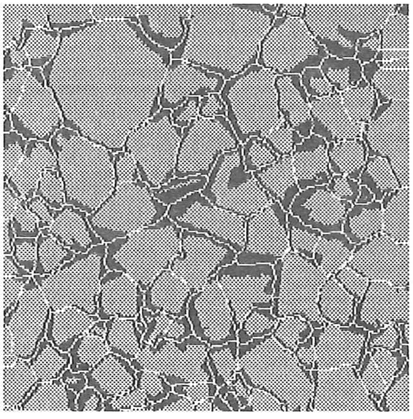

figure 8e

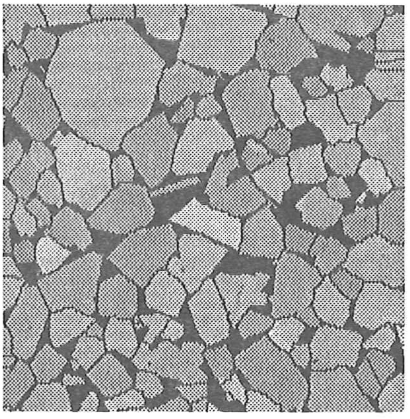

figure 8f

Conclusion : Even for grey images, the shape based segmentation is often a sufficient fist step, and it can be improved by gradient information. In addition, the distance functions particularly lend themselves to good morphological filtering.

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