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ON THE USE OF COMPLEMENTARY SET FRACTAL DIMENSION IN IMAGE ANALYSIS

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ABSTRACT

The distribution of objects viewed in the same plane is often observed in materials science. When the structure of objects, or that of their distribution remains invariant, whatever the observation range may be, using fractal metrology can be worthwhile.

For this purpose, a "distribution index", independant of the regularity in shape of each object can be proposed.

This distribution index is defined as the convergence exponent of a family of adjacent geometrical elements (disks, squares, losanges...) used to describe the Apollonian packing of the residual set E of the objects.

This characterization has been applied with success to metal, mineral or biological materials for different applications working energy transfers at the interfaces.

Keywords: apollonian packing, Besicovitch-Taylor index, complementary set fractal dimension, corrosion pits, residual set of objects.

INTRODUCTION

A family of objects, whether regularly or irregularly shaped, dispersed in a planar domain is common in Materials Sciences.

These objects are linked, for example, to solid phases (precipitates, inclusions, aggregates, third bodies...) or voids (pores, corrosion pits...).

For their classification, an attempt can be made to these phases according to classical morphological criteria, which describe either the objects themselves (shape, size, diameter...) or their distribution (surface density, reciprocal distance statistics...)

When the structure of objects, or the structure of their distribution remains invariant, according to the observation range, it would be a significant advantage to introduce an unified notion of fractal metrology.

What is required is a "distribution index" that would be independant of shape, whether regular or irregular.

A family of objects, dispersed on a plane, is characterized by its complementary set, referred to a residual set.

3D maps of *Np* x *Np* points are represented with a perspective angle, operator selected between 10 and 80 degrees. In addition, to better render the surface geometry, hidden line removal is performed for the selected perspective angle.

For each representation, an additional view can be associated by inverting the "z-signal", so that holes can be better visualized.

Zooming on selected fields is also possible.

The acquired data can be represented as contour maps. In this case, the total roughness (z_{max} - z_{min}= Rt) is subdivided in 16 equal levels. The set of identical color points represents data points at the same level.

In this way, selective contour maps are an attractive way to represent surface asperities such as corrosion pits at a well-defined z_{ij} level (see Fig. 2)

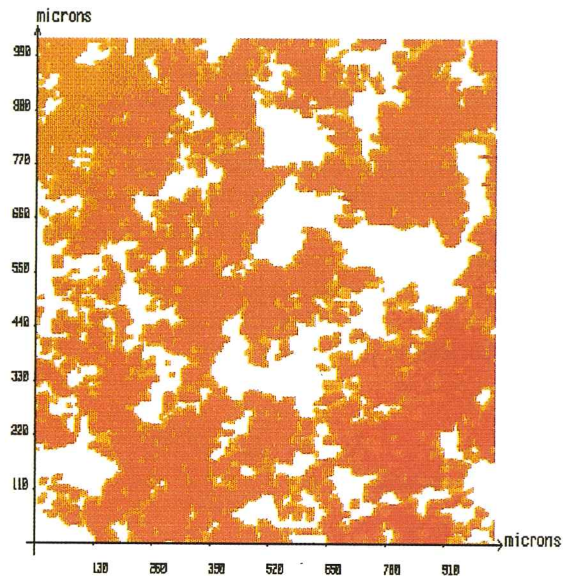

Fig. 2. Contour plot showing the planar distribution of corrosion pits in an Aluminium alloy AlCuMg (2024).

The determination of fractal dimension was applied on level contour maps, whose level cut is such as :

- $z_{ij} \leq h$ corresponding to points belonging to the object,
- $z_{ij} > h$ corresponding to points located in the residual set.

In practice, the level cut value, will be :

$$Z = \frac{h - z_{min}}{z_{max} - z_{min}} \times 100 \qquad (5)$$

This is the case particularly when this set is fractal, that is, when it takes an invariant form relative to certain types of transformation on the plane itself.

This study has two main purposes :

(i) to define the concept of the residual set dimension of the apollonian packing,

(ii) to describe some pratical applications in corrosion science.

Data was obtained on binary images provided by a "home-made" image analyser.

APOLLONIAN PACKING

In an empty domain D, in the shape of a curvilinear triangle for example, it is possible to construct an "apollonian packing" of disks as follows :
the disk of maximal size inscribed in D is determined. The same operation is repeated inside the three empty curvilinear triangles that are left, and so on (see Fig. 1).

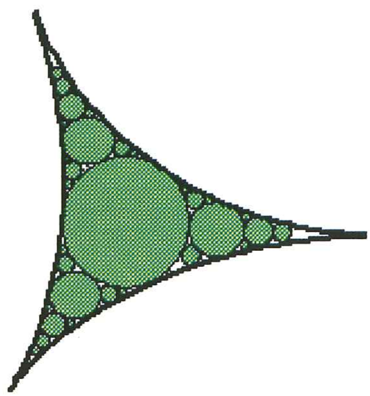

Fig.1. Schematic representation of the construction of an "Apollonian Packing" of disks.

The family of interior disjoints disks, of diameter C_n ($C_1 \geq C_2 \geq C_3 \ldots C_n$) thus constructed, does not cover the curvilinear triangle D completely. Let E be the uncovered complementary space residual set. It may be shown that this set is null area (Tricot,1984) (Tricot, 1986) with fractal dimension $\Delta(E)$ such as :

$$\Delta(E) = \lim_{\varepsilon \to 0} \left(2 - \frac{\log|E(\varepsilon) - E|}{\log(\varepsilon)} \right) \tag{1}$$

Where $E(\varepsilon)$ in the set of all points located at a distance less than or equal to E. The quantity $|E(\varepsilon) - E|$, defined as the area of the "half Minkowski-Bouligand sausage" (Bouligand,1929), may be estimated as the following:

- the contribution of each disk of diameter $C_n \geq 2\varepsilon(C_1, C_2, C_3, C_4, \ldots)$ is approximately $\pi(\varepsilon C_n - \varepsilon^2)$, which is the area of the interior ring of width E,

- the contribution of each disk of diameter $C_n < 2\varepsilon$ ($C_5, C_6, \ldots$) is equal to the area of the disk = $\pi C^2_n/4$.

Therefore we have:

$$|E(\varepsilon) - E| \cong a\varepsilon \sum_{C_n \geq 2\varepsilon} C_n + b \sum_{C_n \leq 2\varepsilon} C_n^2 \qquad (2)$$

where a, b are constants.

We can demonstrate that the series ΣC_n diverges and that conversely, the serie ΣC^2_n converges. There exists a critical value α_0 called the "convergence exponent" of the series which is :

$$\alpha_0 = \inf\left\{\alpha : \sum C_n^{\alpha} \text{ converges } \right\} \qquad (3)$$

This exponent has already been used by Besicovitch and Taylor (Besicovitch & Taylor, 1954) in their study of the complementary intervals of an open set define on a straight line.

Our analysis here is a planar generalization of this index, where the complementary intervals are replaced by disks or other regulary-shaped objects.

The exponent α_0 is commonly referred to as the Besicovitch-Taylor Index.

The selection of squares shapes or losanges shapes is more convenient for applications in digital space of dimension n.

Let us suppose that the size of the structuring elements, now numbered in increasing order $C_1 \leq C_2 \ldots C_n$ if $N(i)$ is the number of elements of size (diameter) C_i we can define a distribution index in Z^2, identicaly to the indice α in R^2 :

$$\alpha = \lim_{k \to 1} \left(1 + \frac{\log\left(\sum_{i=k}^{n} N(i) C(i) \right)}{\log C_k} \right) \qquad (4)$$

where C_k is an arbitrary value selected for taking into account the all structuring elements of size greater than this value.

The experimental determination of α is then defined in a log-log plot representation.

EXPERIMENTAL RESULTS

Main applications deal with corrosion pits, whose shapes and surface distribution can be visualized by means of a 3D roughness analysis using Mechanical Scanning Microscope (Wehbi, 1988).

Each point on the sampled surface is z in the (x,y) plane with its coordinates (x_i, y_j) (i = 1 to N, y = 1 to N) and in the direction with its altitude z_{ij}.

Analog data z_{ij} are digitized on 4.096 levels (12 bits) and stored for analysis. Vertical resolution is such that $\Delta z/z = 10^{-3}$, lateral resolution is 1 μm.

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For each representation, an additional view can be associated by inverting the "z-signal", so that holes can be better visualized.

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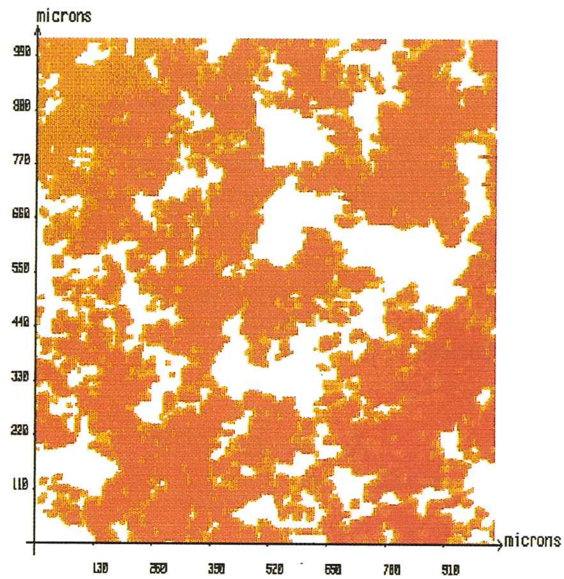

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In practice, the level cut value, will be :

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The selected structuring elements are either squares (see Fig. 3) or losanges (see Fig. 4).

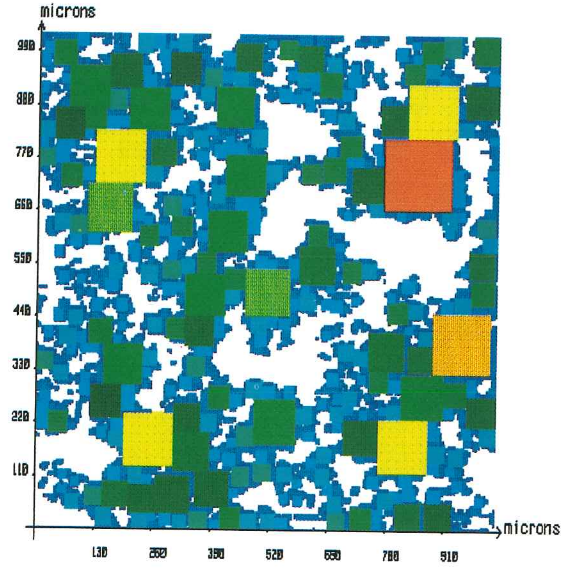

Fig. 3. Packing Apollonian of squares in order to determine fractal dimension of the complementary set.

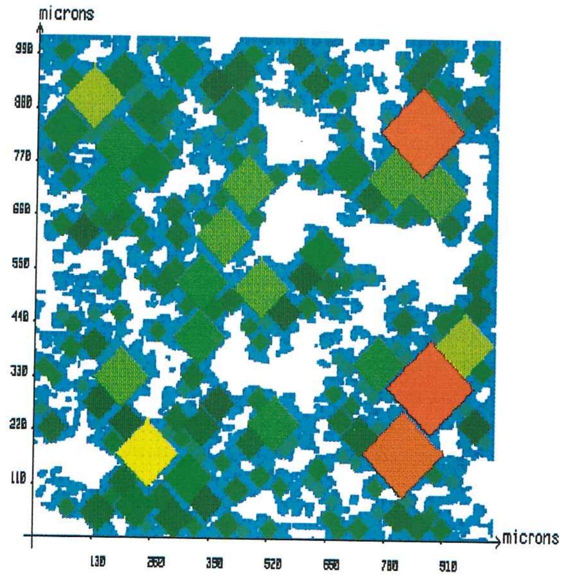

Fig. 4. Packing Apollonian of losanges in order to determine fractal dimension of the complementary set.

Table 1 shows experimental data concerning corrosion of picture distribution dispersed on an aluminium-copper (4%) Magnesium (1%) alloy (2 024) under galvanostatic conditions.

Table 1. Repartition index of corrosion features.

Repartition index	Squares	Standard deviation	Losanges	Standard deviation
80%	1.63	0.01	1.68	0.01
85%	1.62	0.01	1.60	0.01
90%	1.63	0.01	1.58	0.01

Data plotted in Table 1 has been obtained by means of log-log plot representation (see Fig.5).

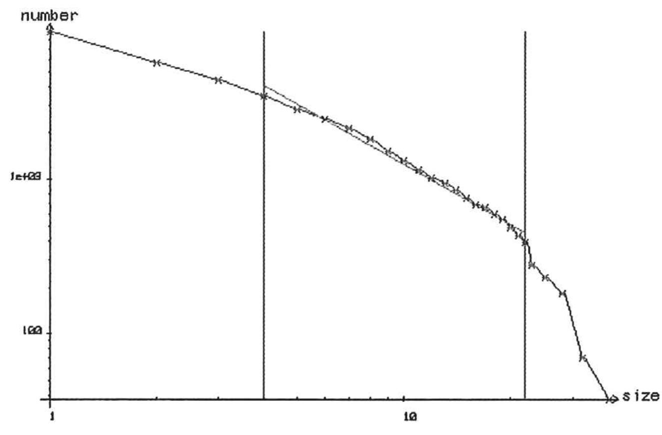

Fig. 5. Log-log plot for the Besicovitch-Taylor index determination.

The measurements are relative to the description of the 150 corrosion pits.

Others structuring elements (triangles, hexagons...) have also been selected for verification of the validity of the results.

For that purpose we have used a "home-made" image analyser equipped with a Matrox image analyser cards installed on a HP Workstation (Serie 9 000).

The main comments are :

a) the accuracy of the measurements depends on the shapes of the choosen structuring element,
b) the complementary set fractal dimension is greater than the contour fractal dimension $\Delta(E) = 1.13$,

c) the experimental values seem to be independant of the selected experimental conditions (current density, interfacial polarization...).

The results thus obtained have been compared to those given by specific electrochemical data (PSD, C(β)...).

CONCLUSIONS

Using the Besicovitch-Taylor Index, it was possible to describe the planar distribution of specific objects.

The concept classicaly applied to a planar family of regular shaped objects was extended to any form of object.

Experimental data proved that the same considerations can be applied to irregularly shaped disjointed objects, provided that the following two conditions are respected :

(i) their border is a curve of finite length, of the order of their diameter C_n.

(ii) their interior diameter also remains in the order of C_n.

It may happen that condition (i) is not fulfilled when the borders are rough and may themselves be considered as fractal curves.

We have concluded that the residual set Index characterizes only the coplanar "distribution" of objects, independently of their shape. It represents then their "Distribution Index".

Applications concerning the distribution of corrosion pits can be extrapolated (for example to the studies of wear debris).

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