ACTA STEREOL 1995; 14/2: 185-190
ORIGINAL SCIENTIFIC PAPER

NON PLANAR SURFACE ANALYSIS BY INUNDATION FUNCTIONS

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ABSTRACT

The analysis of non planar surfaces, modelled by $\mathbb{R}^2\times\mathbb{R}$ functions, has been made in previous works using stereological parameters and functions depending on the size of structuring element applied for image transformations like the grey level granulometries and roughness function.
This study has been carried on a topographical point of view. At first, different models of inundation for non planar surfaces are defined : one floods the relief by successive threshold and others flood simultaneously all catchment basins. Some parameters are chosen to follow the evolution of appeared relief in function of the level of inundation so as to characterize the topography of the non planar surface. Finally, this analysis by flooding transformation for SEM images and simulated images is tested and criticized.

Keywords : fractography, inundation, mathematical morphology, non planar surfaces, topography, topology.

INTRODUCTION

The morphological studies of non planar surfaces are very useful in many domains. The most important is surely the quantitative fractography. The confocal microscope (CM) and the scanning electron microscope (SEM) are the most convenient apparata to analyse the non planar surfaces. The CM gives grey level images corresponding to the true relief (the radiometric value of each pixel is proportional to the altitude). The SEM gives a grey level image which does not give the altitude but can be described in a perspective sense. With these apparata, it is impossible to see the unfolded parts of the surface. In this paper, one considers that the non planar surfaces have no overlapping and so they can be modelled by $\mathbb{R}^2\times\mathbb{R}$ functions.
Some morphological tools to study these surfaces have been proposed in previous papers. The « stereological parameters » were presented by Coster (1992), Hénault and Chermant (1992). The « local » parameters, in the case of $\mathbb{R}^2\times\mathbb{R}$ function f, are the volume per unit area of

support, $\mathcal{V}_A(f)$, the surface roughness, $\mathcal{S}_A(f)$, and the vertical roughness for surface, $\mathcal{N}_A(f)$, (so called the integral of connectivity number per unit area).

These parameters are not sufficient to describe the morphology of non planar surfaces. This is the reason why we have proposed to use morphological functions to complete the description (Gauthier et *al.* 1994 ; Coster et *al.* 1994). The proposed functions are the granulometric functions by opening and closing and the surface roughness function obtained from the Steiner method. The aim of this paper is to describe a new way of investigation of non planar surfaces : the transformation by inundation (Gauthier 1995).

The « physical » model of inundation is too complicated. It is impossible to translate this model in term of simple computer algorithms. In addition, the result depends strongly on the unknown topography. So, three simple models are proposed and presented : the immersion model, the lower inundation model and the upper inundation model.

THE MODELS OF INUNDATION

The immersion model

The immersion model is the simplest one. Let describe a dry relief by a function f. To each local minimum corresponds a catchment basin (CB). The relief is filled up by the water which comes from the holes associated with each minimum. This method is used in the classical watershed segmentation proposed by Beucher (1990). Fig. 1 illustrates this method. The level z (equal to h+min(h)) of water is the same for all the relief, but the CBs are not filled up at the same time (the deepest CB at the first time). In the transformed function by immersion, the parts of relief with an altitude lower than z are replaced by the surface of the lakes. The immersion transformation can be written in terms of morphological operators according to the following relationship :

$$\mathrm{imm}(f,h) = \sup(f, h + \min(f)) \quad [1]$$

In addition, the filling function is defined by :

$$\mathrm{R}_{imm}(f,h) = \mathrm{imm}(f,h) - f \quad [2]$$

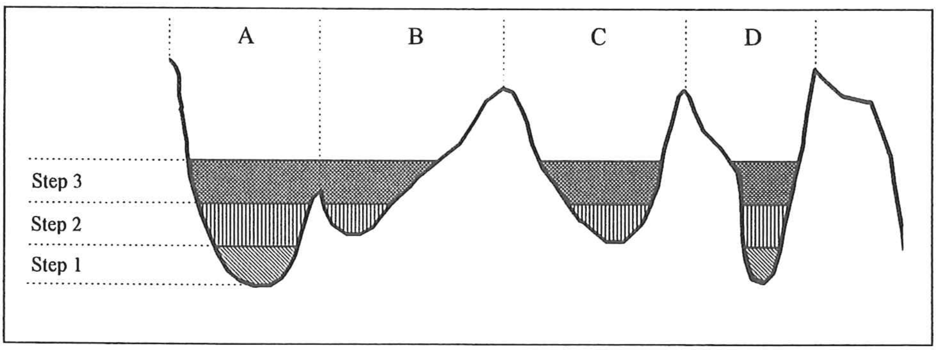

Fig. 1. Illustration of the immersion of a relief.

The lower inundation model

To build the lower inundation model, three rules are used and illustrated in fig. 2.

Rule 1 : The CBs are filled up at the same time and the depth of each lake is h (fig. 2 step 1).

Rule 2 : The filling of each lake have reached the watershed stops (fig.2, CB B at the step 2).
Rule 3 : If two lakes reach the common watershed, then the filling continues according to the first rule (fig. 2 step 3).
The lower inundation transformation can be written in terms of morphological operators according to the following relationship :

$$\mathrm{L\_inond}(f,h) = h\text{-}\min(f) \qquad [3]$$

The corresponding filling function is given by :

$$R_{L\_inund}(f,h) = h\text{-}\mathrm{concave}(f) = h\text{-}\min(f) - f \qquad [4]$$

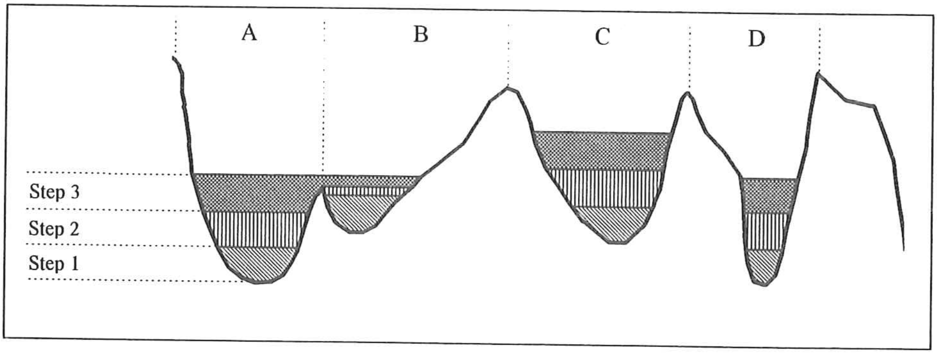

Fig. 2. Illustration of the lower inundation of a relief.

The upper inundation model

The upper inundation model is described in fig. 3. The first rule is the same like the previous model (fig. 3 step 1). The second and the third rules are replaced by the following rules :
Rule 2 : If a lake reaches the watershed, then the neighbouring CB is filled up at the same level (fig. 3, CB A and B at the step 2).
Rule 3 : Each lake is filled up as the height between the maximum of its local minima and the level of water is h (fig. 3 step 3).
This model of inundation cannot be obtained from simple morphological operators, but its computation is possible. The associated filling function can be also performed by difference with the initial function.

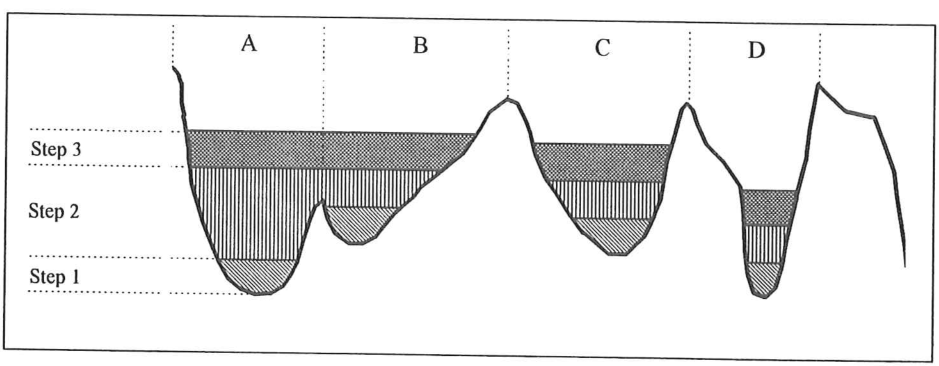

Fig. 3. Illustration of the upper inondation of a relief.

THE MEASUREMENTS ON INUNDATION PROCESSES

The measurements on inundation processes have been performed on ductile fracture surfaces of steel, brittle fractures of alumina and boolean function models.

Parameters

Before the inundation, the system is composed of two phases : the solid phase (**S**) and its complement, the gas phase (**G**). A third phase appears during the inundation, the liquid phase (**L**). The union of solid and liquid phases is also called condensed phase. These phases are separated by the liquid-solid, ∂(**L/S**), gas-solid, ∂(**G/S**) and gas-liquid, ∂(**G/L**), interfaces. Only the liquid phase is modified when the inundation process is performed. Table I resumes the different parameters which can be used.

Table 1. Chosen parameters to follow the inundation process.

Local parameters	*Analysed sets*
Volume per unit area, $\mathcal{V}_A$	liquid phase, **L**
Integral of connectivity number per unit area, $\mathcal{N}_A$	Condensed phase, (**L$\cup$S**)
Surface roughness, $\mathcal{S}_A$	∂(**L$\cup$S/G**)
Surface area fraction, A_A	∂(**G/L**)
Specific connectivity number, N_A	∂(**G/L**)
Specific perimeter, L_A	∂(**G/L**) $\cap$ ∂(**L/S**)

Functions linked to the immersion model

The three last measures, $A_A(h)$, $N_A(h)$ and $L_A(h)$ for an immersion of height h are equivalent to the measures obtained from thresholding at $h + \min(f)$. The three first measures, $\mathcal{V}_A$, $\mathcal{N}_A$ and $\mathcal{S}_A$, can be obtained by integration from $A_A(h)$, $N_A(h)$ and $L_A(h)$ according to the following relationships :

$$\mathcal{V}_A(L,h) = \int_0^h A_A\left(\partial(G/L)_z\right)dz \qquad [5]$$

$$\mathcal{N}_A(L\cup S,h) = \mathcal{N}_A(L\cup S) + \int_0^h N_A\left(\partial(G/L)_z\right)dz \qquad [6]$$

$$\mathcal{S}_A(L\cup S,h) = A_A\left(\partial(G/L)_h\right) + \int_0^h L_A\left(\partial(G/L)_z \cap \partial(L/S)_z\right)dz \qquad [7]$$

The knowledge of $A_A(h)$, $N_A(h)$ and $L_A(h)$ are sufficient to study the topography of non planar surfaces by the immersion process because these relations exist.

$A_A(h)$ varies between 0 to 1. The corresponding graph is sigmoidal like cumulated size distribution curves (fig. 4) and can be summarized by the same parameters (quartiles, median,...). From the quartiles, for example, a coefficient of symmetry and a coefficient of spreading can be calculated. The typical evolution of $N_A(h)$ is given in fig. 5. The first part corresponds to an increasing number of lakes and reaches a maximum. After that, some islands appear and the number decreases to go through the zero value and reach a minimum (negative) value corresponding to a maximum of islands (peaks). The difference between the maximum and the minimum is function of the roughness. The evolution of $L_A(h)$ is given in fig. 6. The curves increase to reach a maximum when $N_A(h)$ reaches the zero value.

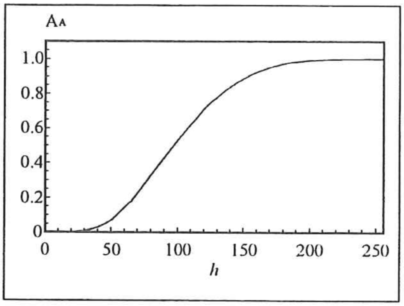

Fig. 4. Surface area fraction of the liquid-gas interface according to the level h of immersion applied to SEM images of fracture surface of steel.

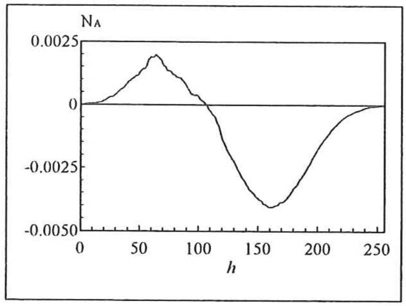

Fig. 5. Specific connectivity number of the liquid-gas interface according to the level h of immersion applied to SEM images of fracture surface of steel.

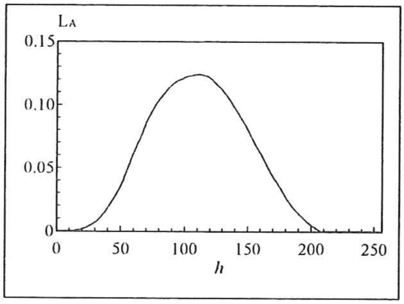

Fig. 6. Specific perimeter of the liquid-gas interface according to the level h of immersion applied to SEM images of fracture surface of steel.

Functions linked to the lower inundation model

It is impossible to obtain equivalent relationships between the parameters in the case of the lower inundation model. In addition, this model is not symmetrical like the previous one (equivalence between peaks and basins by negation of the function). The $A_A(h)$ function varies between 0 and 1, but the slope of the curve at the origin is greater than in the case of immersion (positive slope at $h = 0$) because all catchment basins are filled up simultaneously (fig. 7). Therefore, the behaviour at the origin is very sensitive to the noise. Figures 8 and 9 exhibit the behaviour of $N_A(h)$ and $L_A(h)$ functions for the lower inundation model. The behaviour at $h = 0$ is explained by the same way than the behaviour of $A_A(h)$. The maximum of $N_A(h)$ can be obtained very near to the origin and corresponds to the number of CB per unit area. The $N_A(h)$ function is also very sensitive to the

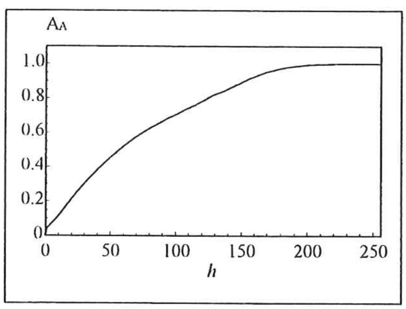

Fig. 7. Surface area fraction of the liquid-gas interface according to the level h of lower inundation applied to SEM images of fracture surface of steel.

noise. The slope of $L_A(h)$ function at the origin is function of the curvature of the concave parts.

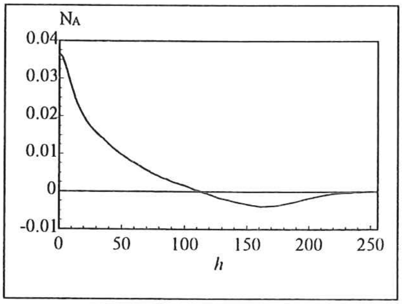

Fig. 8. Specific connectivity number of the liquid-gas interface according to the level *h* of lower inundation applied to SEM images of fracture surface of steel.

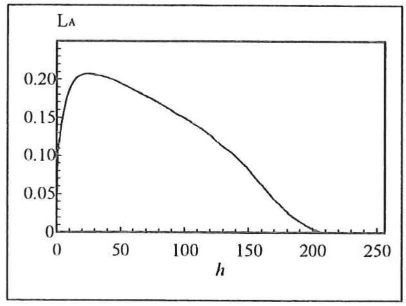

Fig. 9. Specific perimeter of the liquid-gas interface according to the level *h* of lower inundation applied to SEM images of fracture surface of steel.

The upper inundation model has been eliminated because the process of this inundation is not continuous.

CONCLUSION

Three models of inundation have been investigated experimentally on simulated and fractured surfaces. The model of immersion seems be the most interesting because it is symmetrical and can be performed by a simple thresholding. The « stereological » parameters for functions can be estimated from several relationships by integration. The lower inundation model can be use with an *a priori* knowledge on the nature of the relief (peaks or basins). This behaviour is equivalent to opening and closing for granulometric investigations. The lower model of inundation is very interesting when the relief has some microroughnesses in addition to the basins and peaks.

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Presented at the 9th International Congress for Stereology, Copenhagen, August 20th to 25th, 1995.