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2D AND 3D TOPOLOGICAL ANALYSIS OF MODELED POROUS MEDIA

Eric Moreau [1], Stéphane Sammartino [1,2], Gérard Touchard [1]

[1] Laboratoire d' Etudes Aérodynamiques, UMR 6609 CNRS, Université de Poitiers, 40 avenue du Recteur Pineau, 86022 Poitiers cedex, FRANCE
[2] Laboratoire Hydrogéologie, Argile, Sols et Altérations, UMR 6532 CNRS, 40 avenue du Recteur Pineau, 86022 Poitiers cedex, FRANCE

ABSTRACT

The ultimate goal of our work is a better understanding of hydraulic and electrical phenomena in geological porous media. In this paper, the topology of different 3D homogeneous porous media modeled by random packing of spherical particles is characterized using the notion of *critical neck* defined by Moreau *et al.* (1996). Briefly, this consists in performing successive openings and geodesic propagations in order to determine the proportion of pores which are connected by narrowings of increasing thickness.

This topological analysis is performed in two and three dimensions on different types of media: periodical packing, random packing of equal spheres and random packing of unequal spheres. Then the relation between the 2D and 3D critical necks in function of the medium properties is discussed.

Keywords: 3D topology, stereology, neck, opening, geodesic propagation, porous medium.

INTRODUCTION

For several years, our laboratory has been working on electrokinetic phenomena in porous media. These ones might be used in various purposes as clay consolidation (Lo *et al*., 1992), decontamination of fine-grained soils (Probstein and Hicks, 1992 ; Moreau *et al.,* 1997a), etc.

It is a well-known fact that the electrical and hydraulic behaviors of a porous soil depend highly on the geometry of its fluid flow paths. However until now, most studies of this type have been performed on soil thin sections without knowing whether the 3D morphological and topological properties of the medium could be accurately obtained from 2D representations. That the reason why a part of the work performed recently in the laboratory on image processing consists of comparing the 2D and 3D morphology and topology of porous media (Moreau *et al.,* 1996, 1997b, 1997c).

We performed a recent study on streaming current generated by water flow through glass porous media of different pore sizes. This work needed the knowledge of the neck radius of the fluid flow paths (Moreau *et al*., 1998). It was determined experimentally but it would have been interesting to obtain it from two-dimensional microscopical image analysis.

Consequently, the goal of this work is to characterize the relation between the 2D and 3D critical necks in terms of the medium properties.

CRITICAL NECK

In order to be able to use the theoretical knowledge on electrokinetic phenomena in a porous medium, this one has to be modeled. In the case of frittered glass, it seems that the fluid flow path network may be represented by N parallel channels. Each channel presents then a radius r_0 which corresponds to its critical neck, *i.e.* its main narrowing. In this case, the laminar fluid flow rate D through the porous medium may be defined as follows :

$$D = \frac{\pi \, \Delta P \, N \, r_0^4}{8 \, \mu \, l} \tag{1}$$

ΔP being the pressure gradient applied between the both sides of the medium, N the number of channels, μ the fluid dynamic viscosity and l the channel length close to one because the low tortuosity of the fluid flow paths.

An approach to characterize the critical neck set of a porous medium has been defined. It consists of performing successive erosions, geodesic propagations and dilations in order to determine the proportion of pores which are connected by narrowings of increasing thickness. This processing allows to plot the evolution of the normalized connected porosity $F(r)$ in terms of the opening size r performed on the porous phasis :

$$F(r) = \frac{A\left(D^{rB}(P_G(E^{rB}(X)))\right)}{A(X)} \tag{2}$$

X being the initial porous phasis, $A(X)$ the surface area of X (volume in three dimensions), $E^{rB}(X)$ an euclidean erosion of X by a structuring element B of size r (in pixels), $D^{rB}(X)$ an euclidean dilation and P_G a geodesic propagation from the upper to the lower edge of the image in order to remove the porous regions which do not belong to the percolant network. Thus $F(r)$ corresponds to the ratio of the connected porosity by the initial porosity and it is the result of a transformation of size r. Fig. 1 shows an example on a two-dimensional cross-section obtained from a three-dimensional random packing of equal size spheres. Fig. 1a represents the initial porous medium. Porosity is close to 40%. A geodesic propagation is realized. Some parts of the porous network become consequently isolated. The connected porosity is then 39%. Thus the initial normalized connected porosity $F(0) = 39/40 \approx 98\%$. Then three morphological operations are performed successively in order to remove the porous regions which are connected to the main percolant network by narrowings smaller than two pixels : an erosion of size one, a geodesic propagation and a dilation of size one. Porosity corresponds now to 38%. Thus $F(1)$ = 95%. This operation is performed with erosions and dilations of increasing size. Fig. 1b represents the porous phasis for r equal to 4. When r reaches 5, the porous phasis is not connected any more. Thus in this case the critical neck r_0 corresponds to 5 pixels. Fig. 1c represents the $F(r)$ behavior. Fig. 2 shows the $F(r)$ behavior for a fracture network in clay soil.

3D images are digitized in cubic grid. Thus, erosions and dilations are performed in cubic graph, *i.e.* 6-neighbourhood and geodesic propagations are realized in 26-neighborhood. 2D images are represented in square grid. Then square graph is used for erosions and dilations even through propagations are performed in octagonal graph. These 2D and 3D grids induce an error of distance computation in the both cases (Moreau, 1997c). It is not very important for this study because its purpose is to evaluate the ratio of the 3D values by the 2D ones. Furthermore, we will use hexagonal and cuboctahedronal grid in the future (Serra, 1982 ; Jouannot and Jernot, 1996).

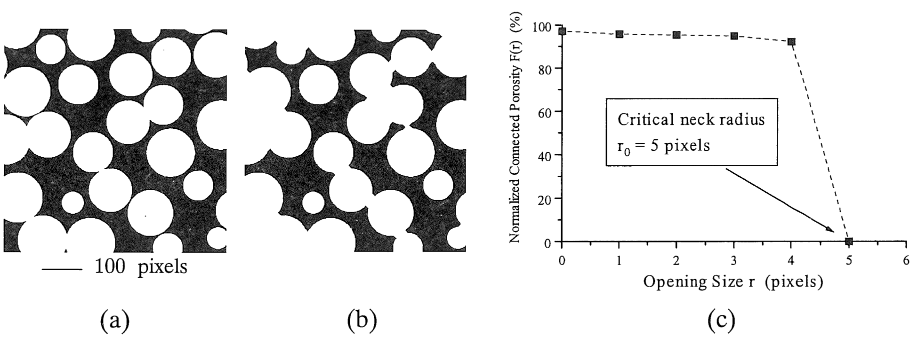


Fig. 1. Different steps of the critical neck algorithm: initial porous network (a), porous network after an erosion of size 4, a geodesic propagation and a dilation of size 4 (b) and F(r) versus r (c).

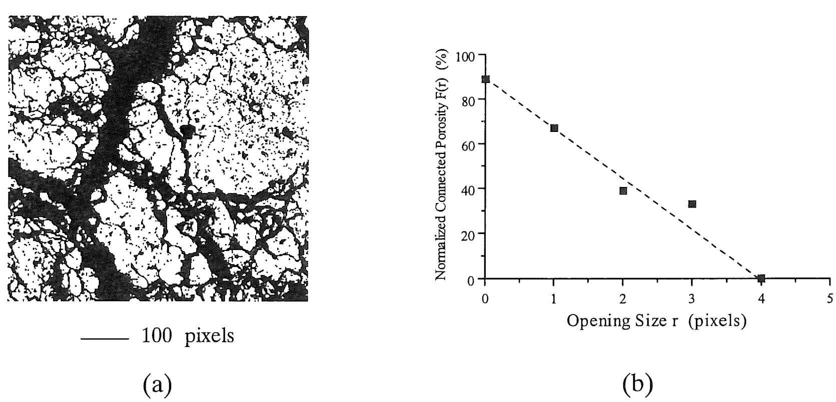


Fig. 2. Binary image of a fracture network in clay soil (a) and F(r) versus r (b).

MODELED POROUS MEDIA

We use randomly packed arrangements of spheres in order to model glass ball porous media (Moreau *et al.*, 1998). Physicists and geologists interested in physical properties of matter or fluid flow through porous media produced a large number of sphere packing algorithm. A review has been published by Allen (1985) and more recently by Rogers *et al.* (1994).

For our study, processings have been performed with a *Silicon Graphics O^2* workstation. Different types of three-dimensional media have been constructed and digitized in cubic grid : periodical packings, randomly packed distributions of equal size spheres and randomly packed of unequal size spheres. Each final 3D image is taken in the medium region of an initial larger constructed packing in order to avoid the border effects. Processings are performed on images of size 600×600×600 voxels. The spheres correspond to the solid phasis of the porous medium. Twelve three-dimensional porous media with different morphological properties have been constructed and analyzed. All the results go to the same way. The results concerning five of these ones will be presented. They are sorted from the more to the less porous, and named as follows :

- M1 : cubic close-packed spheres of radius equal to 40 voxels $V_V = 46.7\%$
- M2 : random packing of equal spheres of radius equal to 40 voxels $V_V = 45\%$
- M3 : random packing of unequal spheres of radius between 10 and 70 voxels $V_V = 40\%$
- M4 : random packing of unequal spheres of radius between 10 and 70 voxels $V_V = 36\%$
- M5 : hexagonal close-packed spheres of radius equal to 40 voxels $V_V = 28.2\%$

RESULTS & DISCUSSION

The first analysis consists of plotting the normalized connected porosity $F(r)$ in terms of the opening size r for the 5 three-dimensional porous networks. As shown in Fig. 3, the $F(r)$ behavior depends on the morphological properties of the medium. The first remark is that the critical neck r_0 depends on the sphere size. More, the denser the medium is, the faster the curve decreases and the smaller r_0 is. Indeed the curve characterizing the periodical cubic packing M1 presents a low decay for the small values of r and decreases fast when r reaches the value r_0 equal to 22. This means that all the channels of the porous medium have the same radius equal to r_0. The others samples which are less porous present a faster decrease and a smaller value of r_0. In fact, as shown in Fig. 2, when the porous network is composed of several paths of different radii, the $F(r)$ plot may decrease linearly and present another type of behavior.

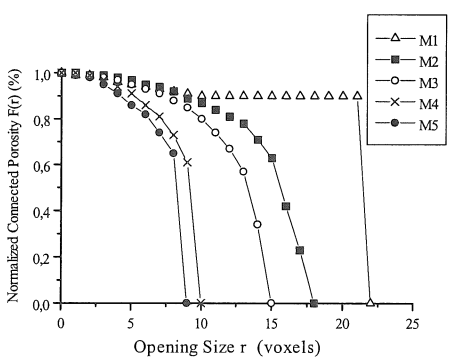
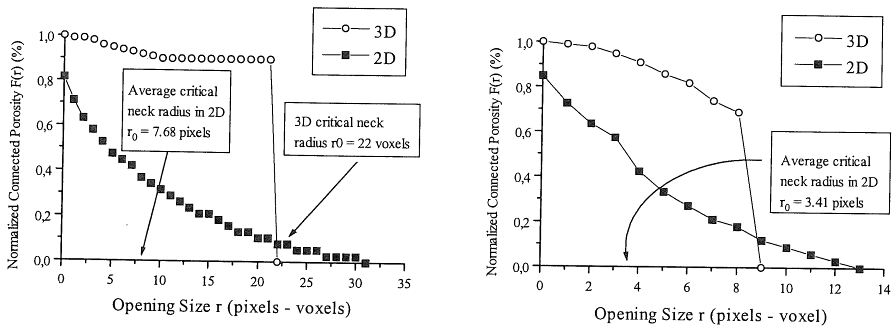


Fig. 3. F(r) versus r for 5 porous media : from the more porous (M1) to the less one (M5).

$F(r)$ and r_0 are now compared in two and three dimensions in order to determine if a relation could exist between the porous networks defined in R^2 et R^3.

Fig. 4 shows that the 2D and 3D behaviors are completely different. Indeed the two-dimensional porous media are less connected than the three-dimensional ones even through some cross-sections present a r_0 value greater than the 3D one. Furthermore, Table 1 shows that the ratio of the 3D r_0 by the 2D one seems to be contained between 2.5 and 3 whatever the type of sphere packing. We precise one more time that this result is true for the twelve analyzed media. This information is very interesting because it means that the critical neck of a three-dimensional medium composed of spherical particles might be deduced from a two-dimensional analysis.



(a) (b)

Fig. 4. F(r) versus r in 2D and 3D for M1 and M5. The 3D behavior corresponds to the analysis of the three-dimensional porous network. The 2D one is an average of all the curves obtained from the processing of a large number of 2D cross-sections.

Table 1. Analysis of 5 porous networks : initial porosity V_V (%), critical neck r_0 in 3D (voxels), average critical neck r_0 in 2D (pixels) and ratio of 3D r_0 by 2D r_0 (R).

Medium	**M1**	**M2**	**M3**	**M4**	**M5**
V_V (%)	46.7	45	40	36	28.2
3D r_0 (voxels)	22	18	15	10	9
2D r_0 (pixels)	7.68	6.6	5.7	3.65	3.41
R	2.86	2.72	2.63	2.74	2.64

CONCLUSION

Analysis of twelve periodical and random packings of spherical particles has allowed to determine a statistical relationship between the three-dimensional critical neck radius and the two-dimensional one obtained from the processing of cross-sections. It seems that the ratio is nearly constant and through the range of 2.5 to 3. This result is interesting because it would allow to determine the critical neck of a three-dimensional porous medium composed of spherical particles by the analysis of two-dimensional cross-sections.

Furthermore, we have to take into account the limits of this study which may be improved. First, we should confirm this analysis with larger three-dimensional images digitized in cuboctahedronal grid in order to decrease the errors of distance computations. Second, we have to analyze many more types of porous media as random packings of deformed spheres or Boolean media (Matheron, 1967).

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