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Arend Heyting and Phenomenology: Is the Meeting Feasible?

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Abstract In the literature one can see the increasing trend of supporting intuitionism through phenomenology. Brouwer's pupil, Arend Heyting, is said to be a forerunner of this trend, as he used a phenomenological terminology in order to define intuitionist negation, by elaborating the first intuitionist logic. In this paper, the author tries to explore—with reference to the unpublished material stored in the Heyting archive—how much of Heyting's general thought is compatible with phenomenology. In the conclusion she suggests that Heyting and Husserl, insofar as they both think consciousness must be the very beginning of knowledge, share a same anti-psychologistic attitude which coexists with an attempt to overcome solipsism. Yet, the phenomenological concept of degree of evidence cannot be applied to Heyting's scale of evidence (including small natural numbers, large natural numbers, infinitely proceeding sequences, the universal quantifier), on the one side because it is not clear if the latter is common and shared by all intuitionists, and, on the other side, because the former presupposes a revisable evidence that does not fit to Heyting's viewpoint. Furthermore, Husserl's and Heyting's conceptions of the nature of mathematics and logic and of their relationship are essentially different. From an intuitionist viewpoint mathematics is the domain of evidence, while logic transcribes its regularities. From a phenomenological viewpoint, mathematics remains outside the domain of evidence. Apophantic logic coincides with mathematics (without either of them absorbing the other), but transcendental logic lies at a higher level.

Introduction

Recently, it has been proposed to use phenomenological methods and concepts in order to solve some problems arisen from the intuitionistic foundations of mathematics.[1] Some authors have claimed that Brouwer's pupil, Arend Heyting, was a forerunner of this trend, as—in a certain period of his scientific production—he used a phenomenological terminology to express the intuitionistic interpretation of logical constants [see Tieszen 1984, p. 404 and Gethmann 2002b]. On the contrary, Tomasz Placek has raised some doubts about the similarity between Heyting's perspective and phenomenology in his 1999 volume *Mathematical intuitionism and inter-subjectivity*, where he affirmed that Heyting's thought—unlike Brouwer's, which could easily be expressed in terms of a transcendental ego—was intrinsically psychologist. Faced with such different viewpoints, we now want to ask whether there really was (or can be) an agreement between Heyting's perspective and phenomenology.

In order to evaluate pros and contras about the question, it is useful to consider not only Heyting's published writings, which have mainly to do with mathematics and contains only a few remarks relating to philosophical issues, but also his unpublished writings stored in the Noord-Hollands Rijksarchief Haarlem and presented by Miriam Franchella in her 1995 "Like a bee on the window-pane."

1. An overview

We recall here that Heyting's aim in his unpublished writings was not to build a systematic philosophy. Namely, he affirmed: "In what follows some philosophical questions will be discussed without need of a general definition of philosophy" (F8.7).[2] But he only gave the amount of philosophical considerations required for treating and overcoming solipsism. This was a peculiar aspect of Brouwer's personality and mystical attitude, which could easily be used against intuitionism. So, it is clear how this topic was

[1] See Tieszen 1984, 1988, 1995. and van Atten 2002, 2003, 2004, 2006.

[2] A.S. Troelstra made the inventory of the Heyting Nachlass. The material we refer to was found in a red wrapper with the inscription "filosofie." Its content has been divided into groups F1/F21. The F1 and F2 groups are the older ones and presumably date from the period 1930-1940. The others seem to date from 1978-1980.

important for Heyting both from a personal viewpoint and in relation to his activity of promoting intuitionism.

In his "local," restricted philosophy, Heyting affirmed first of all that the best way to avoid dogmatism is to start with data of consciousness, which he described as follows in the oldest part of his notes:

> It is as if I were a fine cloth, but covered by a thick coat of paint so that only coarse folds remain visible. If I manage to remove the coat of paint, the design comes to light, and it is an indescribable design. You cannot define anything finer; its lines have no importance, its development is in itself nothing: still there is in it all the force of the world, all beauty and all emotion. Only its surface is touched by the paint: under the latter it preserves all its beauty. But why cannot it exist out of its covered state? As soon as it is removed, contact with the outer world makes it change its shape and loose its colours so that I speedily cover it again in order to save it from vanishing. (F1.4)

In the recent part of his notes, he stressed again that the life of spirit is "full indeterminacy," it is not splittable, it does not consist of separable units related to each other. It is "reason," the faculty of isolating, that the man has. It performs the individuation, i.e., it distinguishes among different impressions and sensations of our Self and then links them with each other. The most frequent link is the temporal one, between remembrances, which play an important role in the work of reason. Heyting supplied us with an example:

> Such impressions have been followed by tactile impressions, some of which reason has attached to the concept "body." It follows that also in this case, after some movements, we can have the tactile sensation that there are bodies. And again on the basis of preceding experiences, it arrives at representations with the type and the form of the bodies; representation consists of visual and tactile imagined sensations. (F1.5)

Spatial schema intervenes to help us in this operation. Such schema does not come from experience, because "experience does not give us anything determined," but it is "built by reason by its own strength and then it is applied to experience" according to a process of induction, after noting that different tactile and visual sensations can be classified within the tri-dimensional space and that this is a fruitful tool for surviving.

Heyting used the Popperian terminology of world 1-2-3, in a permuted order, to present his hypothesis of the path of consciousness towards the

world: world 1 is the content of consciousness; world 2 is that of other minds; world 3 is the external world (to which even the own body belongs), and then he adds a world 4, which is the world of abstraction (F5 p. 24).

We have said that Heyting did not aim at building either a philosophy or an ethics (F1.4). In his unpublished writings, which are very rich of reflections about the content of consciousness as starting point for philosophy, his final aim was to overcome Brouwer's solipsism. He added also some hints for reflections about how philosophical trends spread, arguing that the cause of it is the lack of a common philosophical language. Such a language is difficult to obtain as philosophy must start from daily language and then refine it so that it can express all conceptual subtleties:

> Two philosophers do not agree. How can this happen? Both are clever and have reflected deeply on the specific question.
> It must be that they use words with different meanings, and since the meaning of a word can be determined only by the use made of it, the difference in meaning must be determined by the individual's work. This is an important task of secondary philosophy. Instead of two philosophers one can also study the work of one in this way! (F8.19)

Heyting's general attitude towards philosophy explains his remarks about the Cartesian cogito, which allows us to begin our comparison between Heyting and Husserl.

On this purpose, it is useful to recall that Heyting, although referring to many twenty-century philosophers in his unpublished paper (Russell, Popper, Eccles, Gallie, Krech, Nuchelmans, Olivers, Wisdom, Austin), did not mention Husserl, except for two titles of his works: *Ideen* and *Krisis*, yet without further explanation.

We can only compare them from a theoretical viewpoint, without historical support.

The appreciation of the Cartesian "cogito" is shared by both authors. Heyting more specifically suggests that it is better to express it as "*cogitatur*" (in order to stress both its intersubjective and objective character), emphasizing the fact that for him "thought" means the whole life of soul (F1.3) and that, "in order to follow these descriptions, we must free ourselves from the daily habits of thinking" (F1.4). All of this presents astonishing similarities with Husserl's epoché. There is still in Husserl also a *criticism* to the limits of the Cartesian cogito, which he considers is a piece of the natural world, a remainder of an incompletely performed epoché, which we maintain to later re-obtain the world we started from:

> Hier machen wir nun, ganz Descartes folgend, die große Wendung, die, recht vollzogen, zur transzendentalen Subjektivität führt: die Wendung zum *ego cogito* als dem apodiktisch gewissen und letzten Urteilsboden, auf den jede radikale Philosophie zu gründen ist. (Hua 1, p. 7)

According to Husserl, the Cartesian cogito contains a danger: if we save a particle of the world inside our Self, we can prove the rest of the world by means of syllogisms. Furthermore, Descartes did not discover the infinite field of transcendental self-consciousness, of the ego, neither he passed to "cogitations": he did not consider the *cogitatum*, that is the intentional correlate of the cogito.

2. Solipsism

We have seen that solipsism was the main aim of Heyting's philosophical reflections, hence a further comparison comes out immediately as Husserl put the question whether phenomenology should be stigmatized as transcendental solipsism, and solved the problem by meeting the other men in an immanent way.

We first need to take a closer look at the details of Heyting's analysis of solipsism.

Heyting shared Mithoff's idea that "solipsism gives prolegomena to every philosophy" (F8.7)—and, for this reason, he said that he rehabilitated solipsism, which scares so many philosophers. Philosophy has to start with the data of consciousness: "According to my viewpoint, philosophy should start with a solipsistic viewpoint as only my own content of consciousness is given to me directly." Yet, he added, "no philosophy should stop at this point" (F7.7). In particular, this does not mean either to believe in solipsism or to make a theory out of it. Because solipsism, as a belief or as a theory, should be however incommunicable (otherwise it would be self-contradictory). On this purpose, he quoted Wittgenstein: "Namely, when the solipsist indeed affirms that only he exists, and nothing besides, then it is effectively possible to carry his theory ad absurdum" (F8.16). As he was convinced that the contact with other human beings is an unquestionable datum, he affirmed (F5 p. 1) that the solipsist

> is like a bee closed inside a room, that cannot find the way outwards. It sees the outer world, but, when it thinks it is reaching it, it flies against the window-pane separating it from the other (namely, the thought that even it is true is real thought).

Against solipsism, Heyting used also a comparison with autism, to affirm that who is isolated from the other, is isolated from himself (F8.3).

So Heyting and Husserl both think we find the other egos inside our own immanence, or, better, that we find them first as immanent in ourselves. Yet, when encountering the others in this way, i.e., by observing the data of consciousness, Heyting firstly experiences a spiritual contact, i.e., perceives himself as positively modified, as enriched, by other people. On this basis, he hypothesised the existence of other people, and, secondly, he attached them a body, by analogy with himself. Body, hence, remains in the background. For Heyting, other minds absolutely come before the outer world:

> In my opinion the notion of other mind is more primitive than that of an outside world. As long as he does not physically attack me (there are other exceptions) another man's mind is to me more important than his body (even if he threatens to attack me I try to change his mind, not his body). (F19.1)

Furthermore, he emphasized the social context. "The individual, he said, cannot be separated from the culture where he lives" (F8.3), and he specified that the attachment to other people with sentiments analogous to ours

> held only for men of our own group. Not long ago workers were considered by the rich as completely different beings, and nowadays many people still think and feel the same about black people. The mass media continuously make groups larger. (F8.10)

According to Heyting, intersubjectivity has a further role to play: some beliefs—like that about the permanence of objects in our absence—are reinforced by the social community where we live:

> Why am I convinced of the existence of Japan? Well, because I have been taught so at school and imagine that some men there perceive things as I myself perceive my environment. Here intersubjectivity is going to play a big role. (F7.11-12)

For Husserl, finding other people inside ourselves requires that we start with our body as a *Leib*, i.e., not merely as a physical body, but as an organic body which can be dominated by our will: "Meinen Leib, das einzige Objekt […], in dem ich unmittelbar *schalte und walte* und in der Sonderheit walte in jedem seiner *Organe*" (Hua I, p. 128). Our body is recognized as being linked to our spirit (our ego), as a body-spirit. The other egos occur because our ego and other egos (when coming into our field of perception) form an

original pairing, where we perceive the other's body as similar to ours but also as distinct from it, as we cannot organically dominate it. Hence the other's body is recognized to be linked to a spirit just like our own body. The natural body belonging to my sphere appresents all the other egos—which appresent each other and ourselves—, i.e., as a community of monads which share an objective nature and an objective world:

> Zu beachten ist dabei, dass es im Sinne gelingender Fremdapperzeption liegt, dass eben ohne weiteres die Welt der Anderen als die ihrer Erscheinungssysteme als dieselbe erfahren sein muss wie die meiner Erscheinungssysteme, was eine Identität des Erscheinungssystems in sich schließt. (Hua I p. 154)

Husserl considers the existence of anomalies, for instance in the case of blind or deaf persons, and explains that the objective world has existence by virtue of a harmonious confirmation of apperceptive constitution, a confirmation performed by continuously experiencing a consistent harmoniousness, which always needs to be restored and extended through corrections:

> Die Einstimmigkeit erhält sich nun auch vermöge einer Umbildung der Apperzeptionen durch Unterscheidung zwischen Normalität und Anomalitäten als ihre intentionalen Modifikationen, bzw. der Konstitution neuer Einheiten im Wechsel dieser Anomalitäten. (Hua I, p. 154)

> Dadurch urgestiftet ist die Koexistenz meines Ich (und meines konkreten ego überhaupt) und des fremden Ich, meines und seines intentionalen Lebens, meiner und seiner *Realitäten*, kurzum eine gemeinsame Zeitform, wobei von selbst jede primordinale Zeitlichkeit die bloße Bedeutung einer einzelsubjektiven, originalen Erscheinungsweise der objektiven gewinnt. (Hua I p. 156)

The body is essential to our ego, it is deeply linked to it and submerges it in space-time. Husserl set himself as a goal a phenomenology of time that only consider the essence of time, by clarifying the ways in which time reveals itself to consciousness. It came out that two inseparably united intentionalities are present in the unique stream of consciousness, in the instant actuality of the stream of consciousness. Through the former, the immanent one, objective time is constituted, where there is duration and alteration of what endures; in the other intentionality, it is the quasi-temporal arrangement of the phases of the flow that becomes constituted.

On the contrary, Heyting did not devote a deep analysis to time. He only stressed that it comes in only after individuation, as a linking among

objects. The datum of consciousness is a mass without form (and with a fantastic beauty) on which identification happens. Spatialization and temporalization come in only after the constitution of objects, as a linking among objects. Time does not have for Heyting the same relevance as for Husserl: namely, even if he stemmed from the intuitionistic tradition, which has Kant as an ancestor, Heyting did not put the intuition of time even at the beginning of arithmetic, but the human faculty of individuating, as time seemed to him too closely linked to the psychological side of man. This is a relevant point for our aim.

Brouwer used "intuition" with reference to two-ity (as abstraction of temporal schema), to infinitely proceeding sequences and to species; then, he specified a notion of mathematical truth in terms of fully performed mental construction (without using the expression "evidence"): the belief that we can trust the applicability of logical laws "is based on the certainty that we consider systems that have been built mathematically" (CW I p. 75), and "at the point where you enounce the contradiction, I simply perceive that the construction no longer goes, that the required structure cannot be imbedded in the given basic structure" (CW I p. 73). For Heyting, on the contrary, the reference to temporal intuition disappears, and natural numbers, infinitely proceedings sequences and species are based on the acknowledgment that we have a faculty of individuating:

> We can count all sorts of things but they have one property in common, namely that they can be isolated. Isolating an object, focusing our attention on it is a fundamental function of our mind. No thinking is possible without it. In isolating objects the mind is active. Our perception at a given moment is not given as a collection of entities, it is a whole in which we isolate entities by a more or less conscious mental act. […] In reality what we isolate mentally are not objects, but perceptions. I can fix my attention on a certain impression, in most cases visual. In practice that impression is immediately associated with innumerable memories, impressions and images to form the notion of an object in the general sense of the word. But for counting it is inessential what there is isolated, it is the mental act of isolating that matters. The entity conceived in the human minds is the starting point of all thinking, and in particular of mathematics. When we think, we think in entities. This does not mean that all our mental life consists of thinking entities. On the contrary, the more intensely we live, the less we think in isolated entities. Under the influence of strong emotions the world seems a whole, loaded with emotion. Only after the emotions are soothed we map out aims and ways to attain them. (1974, p. 4)

This reminds us of Heyting's description of consciousness in his unpublished papers, which can better reveal his attitude. There, he specified: "My only weapon here is introspection. Psychology as a science is useless for this research because it continuously changes the concept of truth" (F6.2), by showing that he meant introspection as something not linked to psychological attitude (at least in his intentions). His attitude was not psychologistic. He introduced the concept of evidence as self-evidence and avoided the word "intuition," as this seemed to him as connoted in a psychologistic way. Furthermore, he spoke of self-evidence of mathematical statements as a unique criterion for their truth (1958b, p. 103), but he specified that a mathematical theorem "expresses the success of a certain mathematical construction" (1958b, p. 107), i.e., that the truth of the theorem consists in a mathematical construction fully performed. Hence, his concept of self-evidence was however the Brouwerian one as "performed mental construction," expressed in a way that did not seem psychologistic to him. Therefore, Heyting's "psychologism" cannot be opposed (as Placek did) to the phenomenological interpretation of his thought. On the contrary, Heyting wanted to avoid any charge of psychology, as also phenomenology wanted.

3. Abstract objects

In his theory of knowledge, Husserl distinguished between intuition as immediate perception and intuition as categorical intuition. The former takes place in a direct way, but any focussing on some part of the perceived is excluded. The perceived is caught as a whole. Categorial intuition is the source of what is called an object, and is based on given intuitions. The "object" is never given in its entirety but it is seen as the ideal end of a series of approximations, which are explained in terms of intentions and their fulfilment. We can first consider medium-sized objects of daily experience. They are only given in a perspectival manner: there can be indefinitely many percepts of the same object, all differing in content. Some parts of the object are given and some are not, so this suggests the limiting case of an adequate perception in which the object is not given imperfectly. That is why the relation of fulfilment admits degrees in which epistemic value steadily increases. In case of fulfilment, a synthesis of identity takes place:

> Immerhin deutet uns die relative Rede von "mehr oder minder direkt" und vom "selbst" die Hauptsache einigermaßen an: dass die Erfüllungssynthesis eine Ungleichwertigkeit der verknüpften Glieder zeigt, derart, dass der

> erfüllende Akt einen Vorzug herbeibringt, welcher der bloßen Intention mangelt, nämlich dass er ihr die Fülle des *"selbst"* erteilt, sie mindestens *direkter* an die Sache selbst heranführt. Und die Relativität dieses *direkt* und *selbst* deutet wieder darauf hin, dass die Erfüllungsrelation etwas vom Charakter einer Steigerungsrelation an sich hat. Eine Verkettung solcher Relationen erscheint darnach als möglich, in denen sich der Vorzug schrittweise steigert; wobei aber jede solche Steigerungsreihe auf eine ideale Grenze hinweist oder sie schon in ihrem Endglied realisiert, welche aller Steigerung ein unüberschreitbares Ziel setzt: das Ziel der absoluten Erkenntnis, der adäquaten Selbstdarstellung des Erkenntnisobjekts. (Hua 19/2, pp. 597-598)

Of course, it is also possible for the intention to be disappointed: a "frustration," that however presupposes a partial fulfilment. Also the frustration is a synthesis, a synthesis of distinction:

> Der Übereinstimmung entspricht aber als korrelate Möglichkeit die "Nichtübereinstimmung", der "Widerstreit". Die Anschauung "stimmt" zur Bedeutungsintention nicht, sie "streitet" mit ihr. Widerstreit "trennt", aber das Erlebnis des Widerstreites setzt in Beziehung und Einheit, es ist eine Form der Synthesis […] von der Art der Unterscheidung. (Hua 19/2, p. 575)

The same activity of knowledge allows us to get to ideal objects:

> Die Evidenz irrealer, im weitesten Sinne idealer Gegenstände ist in ihrer Leistung völlig analog derjenigen der gewöhnlichen, sogenannten inneren und äußeren Erfahrung, der man allein — ohne einen anderen Grund als den eines Vorurteils — die Leistung einer ursprünglichen Objektivierung zutraut. (Hua 17, p. 163)

As identity synthesis presupposes a temporal structure, all objects have a temporal being: ideal objects are so in the sense of being at all times: they are supertemporal as they are omnitemporal, because they are freely reproducible at all times (Hua 1 p. 155).[1]

As in their case we do not refer to perceptual stuff but to the data of categorial intuition, in order to consider the possible different perspectives, we have to use a specific method: the free variation in imagination. What persists through this is some invariant, the essence common to all variants, the eidos:

[1] On this purpose, see Lohmar 1993.

> Die hierbei zu vollziehende Variation des (als Ausgang notwendigen) Exempels ist es, in der sich das "Eidos" ergeben soll und mittels deren auch die Evidenz der unzerbrechlichen eidetischen Korrelation von Konstitution und Konstituiertem. Soll sie das leisten, so ist sie nicht zu verstehen als eine empirische Variation, sondern al seine Variation die in der Freiheit der reinen Phantasie und im reinen Bewusstsein der Beliebigkeit — des "reinen" Überhaupt — vollzogen wird. […] Eben in dieser Deckung tritt aber das in dieser freien und immer wieder neu zu gestaltenden Variation notwendig Verharrende, das Invariante hervor, das unzerbrechlich Selbige im Anders und Immer-wieder-anders, das allgemeinsame Wesen. (Hua 17, p. 255)

Such variations are intentions that can be fulfilled or not, as in the case of medium-sized objects. The evidence is given in the (ideal) case of an adequate intuition of the object, and it should be distinguished from the feeling that can accompany it. Perfect adequacy is possible only when the "object" is the transcendental ego. Hence, in general, there are degrees of evidence.

In his unpublished writings, Heyting presented the levels of a knowledge starting with self-consciousness: after individualization, spatialization and temporalization, we have a so-called "abstraction," developing along these steps (F11.5):

> direct experience (people are included);
> representation of the space around me where I can move;
> rememberings of neighbourhoods where I was before;
> communications by other people;
> spatial relationships among the represented neighbourhoods;
> systematization of those relationships through maps and globes;
> insertion of all structures inside a generalization towards infinity;
> astronomy;
> small microscopic objects;
> theoretical physics. Particles existing a fraction of a second.

He specified that most people reach only the fifth step. Furthermore, he added:

> Each of these abstract concepts begins with something simple and evident. So also "existence": firstly, there are the objects of my direct neighbourhood, which exist; finally stars and mesons. How many steps are there between them, and how does the concept of existence change by passing from one to

> another? Gods, natural numbers and large cardinals are at the top of those steps: at which step does the rule for the existential quantifier hold? (F5 p. 20)

Heyting's theory of our knowledge of abstract objects is not further elaborated. Only mathematical objects are carefully examined in his intuitionist writings, where he collected the doubts raised by some authors and presented a scale of evidence for intuitionistic notions, mirroring the disagreement among intuitionists on the question of which Brouwerian notions are evident. In 1962 he summed up the situation as follows:

> asserts like 2 + 2 = 4,
> general asserts on natural numbers,
> the notion of order type ω,
> the notion of negation,
> the universal quantification,
> free choice sequences,
> the notion of species.

Can we interpret this scale in the sense of Husserl's "degrees of evidence"?

We should first ask whether the scale of evidence alleged by Heyting is something "objective," that is, whether either everybody agree with seeing the same difference between the levels of evidence, or there is a difference of opinion among intuitionists about mathematical objects. In his presentation, Heyting oscillated between wondering *himself* if, for instance, all species of all natural numbers form a species (1962, p. 195), and giving a scale in terms of a mere "increase of hypotheticity" (by specifying,[1] however, that his exposition of them does not follow a linear order). It is clear that, *sic rebus stantibus*, the phenomenological model accepting degrees of evidence is not applicable, as the differences between degrees should be intended with respect to the transcendental ego and not with respect to an individual ego.

Let us try to suppose (even if this does not fully square with Heyting's presentation) that the scale of evidences be "common and shared." We should notice that, in order to use the phenomenological degrees of evidence inside intuitionism, one should give up that idea of "fixedness" of evidence, after it has been experienced: namely, phenomenology admits that what is evident is revisable.[2] "The possibility of deception is inherent in the evidence

[1] 1962, p. 195.
[2] See on this purpose Tieszen 1997, p. 455.

of experience and does not annul either its fundamental character or its effect" (Hua 17, p. 156, translated by D. Cairns). However, "the evidence of a new experience is what makes the previously uncontested experience undergo a modification of believing" (Hua 17, p. 164). This is "the living truth from the living source" (Hua 17, p. 246) and not that kind of truth that comes from sciences and that is falsely absolutized.

Surely, such a viewpoint was not in Brouwer's spirit. We recall here this quote: "[…] truths which, just like mathematical truths, anybody who has once understood will forever affirm" (CW I, p. 106). So, if we want to support intuitionism through phenomenology, we should at least modify Brouwerian intuitionism under this respect. Yet, we can hypothesize that Heyting, on his side, was favourable to revisable evidences, as in 1958b (p. 103) he affirmed:

> It can be asked whether in intuitionistic mathematics absolute rigour and absolute certainty are realized. The obvious answer seems to be that absolute certainty for human thought is impossible and even makes no sense.

Furthermore, we find in some of his unpublished writings an explanation for his statement: in consciousness, i.e., at the starting point of all our knowledge,

> there is nothing definite, distinct to be found out; sensations follow each other and have no proper individuality. When we, nonetheless, assume definiteness, we speak of distinct "things" and say that all of them take place in space and time, it is necessary for us to substitute our content of consciousness consciously or unconsciously through something else; in other words, we begin with a falsity, with a lie (the lie of the discrete). (F2)

But, when confronted with his own scale of evidences, at the very beginning, in 1958, he stressed (1958a, pp. 337-338) that the difference between the degrees of evidence was only a question of nuances, and did not imply a dangerous jump as it would be required to allow clearly non-constructive notions (like the actual infinite). However, he later realized that, at its very basis, a degree of evidence challenges evidence as a criterion of truth, as a definitive certainty: "What is intuitively clear in mathematics has been proved not to be intuitively clear" (1962, p. 195). That is, while phenomenology holds that evidence, even in its being self-corrigible, is a guarantee of truth, for Heyting the realization that evidence is not forever fixed is a ground for rejecting intuitionism as *foundational* school in mathematics (1953, p. 197). This is a proof that he considered a revisable evidence not a

suitable criterion of truth. His refusal to consider intuitionism a foundational school was extended also to the other foundational schools: in their absoluteness, each of them was lacking of something the other possessed. For instance, intuitionism undervalued the role of language in mathematics, while formalism did not stress enough that formal systems in themselves are not the real objects of mathematics, but represent "some ideas left vague on purpose" (1953, p. 198). Consequently, he invited philosophers of mathematics to change their aim and to look for the constructivist, Platonist, formalist elements inside the practice of mathematics. He himself went on by doing intuitionist mathematics in order to explore its potentialities, but not with the belief that it was the only way for doing mathematics.

Furthermore, we have to stress that phenomenologists talk about degrees of evidence also with respect to a same object (the "determinable X") whose evidence increases with time. On the contrary, in Heyting's scale we have different objects which have different degrees of evidence with respect to each other. Hence, we have to establish if, from a phenomenological point of view, it is possible to charge some entities with the characteristic of being *intrinsically* less evident than others. Is the property of being "less evident" a transitory characteristic, or can it be a definitive one?

Some authors believe that it is possible to hold that some entities are less evident than others forever. Richard Tieszen affirmed (1989, p. 136):

> The degree of evidence we have for the existence of large numbers must obviously differ from that we have in the case of quite small natural numbers. We can actually complete constructions for small numbers, but not for large numbers. The evidence would not be "adequate" and perhaps it would also not count as "apodictic."

Tieszen states here that, in the case of large natural numbers, the intuition of the number is founded on the intuition of its "parts", just in the same way as the intuition of a medium-sized physical object is founded on the intuition of its parts (Tieszen 1989, p. 136): "The insight into the possibility of continuing the construction for natural numbers is analogous to that insight involved in seeing that we could continue ordinary perception of an object or objects."[1]

Also van Atten proposed (2004, p. 84) to differentiate inside phenomenology between classical mathematical objects and intuitionist mathematical objects, according to their level of evidence: intuitionism would be "the

[1] Yet, we can notice that among medium-sized objects we do not find special ones that are more evident than others, i.e., that are analogous to small natural numbers.

mathematics of a class of objects that are given to us with a particularly high degree of evidence." In this way he admitted the possibility that some objects might be intrinsically less evident than other ones.[1] As we recalled above, the latest Heyting thought that the only task of the philosopher of mathematics is to search for the constructivist, Platonist, formalist elements inside the practice of mathematics, so he was open to accept different kinds of mathematical objects coexisting with each other. Yet, we have to stress once more that he supported both this view and the abandonment of intuitionism as foundational school, because he believed that the former is not compatible with considering evidence a criterion of mathematical truth.

4. A last question: negation inside logic

We can come back to the reasons put forward by some authors in support of the contention that Heyting's attitude towards phenomenology was generally positive. The main reason for this hypothesis is the fact that Heyting defines negation in terms of the disappointment of an intention. Now, we have to recall here that the father of intuitionism, L.E.J. Brouwer, set logic the creative task of transcribing the linguistic regularities present in the language of mathematics, so the performance of a mental construction might become a criterion of truth. Although this criterion required a reinterpretation of logical constants (with respect to the classical interpretation), Brouwer did not engage himself in a systematic work: he only expressed the new meaning of the law of excluded middle, in order to show that it was no longer valid. The task of reinterpreting all logical constants remained open. It was fulfilled by his pupil, Arend Heyting, also stimulated by a prize established by the Amsterdam Mathematical Society. Heyting tried to exploit the meaning of logical constants within a framework where the notion of assertion was specified. At the very beginning Heyting defined a proposition as "a problem or, better, a certain wait" (1930b, p. 958), while in 1931, he accepted (1931, p. 113) Oskar Becker's remark that a proposition could be seen as "an intention of alleging proofs." Yet, in 1934, Heyting came back (1934, pp. 16-17) to his initial definition of proposition as "posing problems" and in 1956 he definitively stated (1956, p. 98) that "a mathematical proposition *p* can be asserted as soon as a mathematical construction with certain given properties has been carried out." Throughout these changes of "proposition," the

[1] About the admissibility of intuitionist mathematical entities from a Husserlian viewpoint see also van Atten 2002 and 2007.

intended meaning of negation remained the Brouwerian one. It is worthwhile to stress that in 1931, negation was described (1931, pp. 113-114) as the "disappointment of an intention" by referring to Oskar Becker's *Mathematische Existenz* (in particular on pp. 54-69). In partial agreement with Husserl's *Sixth Logical Investigation*, Becker had stressed how "disappointment" and "non-fulfilment" of an intention could not coincide: while disappointment entails a partial fulfilment of the intention (for instance, the disappointment of the intention of "a book on the table" entails at least the presence of the table, in order to ascertain that the book is not there), mere non-fulfilment is a total negativity (as for the above example: even the table is lacking). Heyting took note of the fact that the disappointment (and not the mere non-fulfilment) of an intention seems to describe well the concept of negation as designed by Brouwer. In his 1907 dissertation, Brouwer had written: "At the point where you enounce the contradiction, I simply perceive that the construction no longer goes, that the required structure cannot be imbedded in the given basic structure" (CW I, p. 73). It was a way of explaining the fact that there was at issue a construction such that only a part of it could be performed.

In a letter of September 1934 [see van Atten 2005], Becker let Heyting notice how important and useful the meaning of intuitionistic logic as "task calculus" was, and how it could be generalized through the phenomenological concept of intention ("task" could be seen as a particular case of "intention"), used in the Husserlian "objective-noematic" sense. Nevertheless, from 1934 on, Heyting came back to the initial definition of proposition as "putting problems." In 1955 (p. 17) he gave a cryptic explanation for this change, by referring to the fact that "Kolmogoroff has proposed a conception which is close to the previous one, but overcomes it insofar as it gives sense to Heyting's calculus independently of intuitionistic hypotheses." In fact, Kolmogoroff's paper [1932, p. 58] specifies that this conception "did not require any particular epistemological premise, for instance an intuitionistic one," although it coincides "in its form" with intuitionistic logic. We can guess that Heyting's aim here was to avoid awkward questions (like the epistemological ones), in order to meet "the skeptics" and to let them try to perform mental mathematical constructions.

As for Heyting's definition of negation, it was Glivenko, in a letter of 24.10.33 (Troelstra 1988, p. 16), that showed him the analogy between negation and implication: "We take your axioms and then the following one: 'an element 0 exists such that, for any given element b, $0 \to b$'. Formally, this system will be equivalent to yours. Only, at the place of $\neg a$, there will be the operation $a \to 0$." Heyting got the message and definitely stated (1956)

his interpretation of negation as an assertion $\neg p$ that can be affirmed if and only if there is a construction which leads the supposition that a construction *p* be performed to contradiction. Hence any reference to a phenomenological terminology disappeared.

We see that Heyting's use of a phenomenological terminology was limited to an early stage of his reflection about logical constants. Furthermore, his definition of negation, although expressing Brouwer's thought, was criticised, during the years 1946-1951, by G.F.C. Griss as intuitionistically unacceptable, for the reason that intuitionism should rather start from (forever fixed) evidences in order to reach other (forever fixed) evidences. Griss' criticism was at the origin of Heyting's formulation of the scale of evidences, which led him to abandon the epistemological perspective that considers intuitionism a suitable foundation of mathematics. Hence, his definition of negation cannot be used as an argument to say that phenomenology can support intuitionism.

Furthermore, there is a difference between Husserl and Heyting: about the definition of logic.

Heyting gave various definitions of logic. In 1930, he simply repeated the Brouwerian definition: logic is a collection of linguistic regularities present in mathematics. On the contrary, in 1954 he affirmed that logic was *a part of mathematics*, consisting of its most elementary theorems. In 1955, he recalled (p. 16) that in intuitionistic mathematics conclusions are not derived from logical rules forever fixed, but each conclusion is directly validated through its own evidence. Logic belongs to the applications of mathematics. Nonetheless he states again that there exist rules according to which it is possible, in an intuitively clear way, to generate new theorems on the basis of given mathematical theorems; The theory of this connection is a "mathematical logic," which thus becomes "a part of mathematics and whose application outside mathematics would be senseless" (p. 16).

Finally, in 1956 he wrote: "The word 'logic' has many different meanings. I shall not try to give a definition of intuitionist logic, any more than I have begun this course with a definition of mathematics. Yet, a preliminary remark will be useful. Our logic has only to do with mathematical propositions." (p. 97) In other words, in spite of some fluctuations, the idea remains that logic has to do with mathematical assertions, i.e., that it describes mental constructions.

In his *Formale und transzendentale Logik*, at the end of a long and tortuous path of reflections and reassessments about the question, Husserl stated that logic is the "science of logos in the form of science," i.e., that it determines the general conditions for the possibility of science. Hence, in the

first place it is analytic apophantic, i.e., it is a morphology of judgments and logic of consequences, and, as such, it focuses the categorical objectualities in general (Hua 17, pp. 139-140):

> Das in einem Urteilen Geurteilte ist die geurteilte, die urteilend vermeinte kategoriale Gegenständlichkeit. Erst, wie wir feststellten, in einem Urteilen zweiter Stufe wird der Satz im Sinne der Logik — der Satz als Sinn, die vermeinte kategoriale Gegenständlichkeit als solche — zum Gegenstand, und sie ist in diesem neuen Urteilen urteilend vermeinte schlechthin.

While analytic (or "formal") logic is the objective aspect of logic, transcendental logic is the "subjective" aspect of logic, i.e., the aspect where the focus is on a "theory of knowledge." The main aim of transcendental logic is to point out the *idealizing presuppositions* of analytic logic, i.e., its surreptitious assumptions, and to evaluate their ground, i.e., to establish whether there is any evidence to support them. This inquiry is to be understood in close connection with the division of logic into pre-analytic logic (dealing with the pure possibility of judgments), the logic of consequence (dealing with the non-contradictoriness of true judgments) and the logic of truth (dealing with the truth of judgments).

As for mathematics, Husserl emphasized the fact that either it is directly apophantic (it is this sort of mathematics that treats of propositional forms by computing with them like with numbers) or (this is the case of set theory and cardinal numbers theory) it deals with the "something in general," with the object in general, and, for this reason, is defined as a "formal ontology" ("formal" because it leaves aside any concrete determination of objects).

This view is very different from the intuitionist one. Husserl conceives of mathematics as a formal discipline that does not ask about truth, and limits itself to non-contradictoriness and to the relation "is a consequence of." Mathematics remains outside the domain of evidence, i.e., outside the domain of truth. Apophantic logic coincides with mathematics (without either of them absorbing the other), but transcendental logic lies at a higher level.

A possible use of phenomenology to support intuitionism should not focus on the issue of reaching an agreement between Heyting and Husserl, but it should consider that some aspects of intuitionism are required to be changed so that phenomenology may be applied to it. The status of evidence and the role of logic (with respect to mathematics) are surely the first things that need to be modified.

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