Linguistic Variables: a Powerful Concept for Knowledge Representation

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Abstract The power of daily communication and commonsense reasoning lies in the use of natural language. The importance of computing with words has increased tremendously over the last decades and will continue to do so during the ones to come. A key-role in this process is played by linguistic variables, i.e. variables whose values are linguistic terms. Since these terms are often vague, they cannot be modelled by classical set theory in se. In this paper we point out the importance of linguistic terms for the representation and the manipulation of knowledge. We describe how atomic terms, logically composed terms and modified terms can be represented using the framework of fuzzy set theory.
Keywords: linguistic variable, linguistic term, modifier, vague, fuzzy set theory

1 Introduction

During millenia yet, people tend to express real-life information by means of natural language; it allows them to reason about everyday issues within a certain (tolerated) degree of imprecision. A successful approach to close the gap between artificial and human intelligence could therefore be the use of natural language as a tool for knowledge representation in computer systems. Indeed, the importance of computing with words has increased tremendously over the last years (Zadeh, 1999 [27, 28]) and it will grow even more in the years to come (Zadeh, 2000 [29]). A key-role in this process is played by linguistic variables, i.e. variables whose values are linguistic terms contrastively to the numerical variables with numbers as values.

Linguistic terms however are often intrinsically vague, which means that it is impossible to give exact bounds for them. A query like "*List the young salesmen who have a good selling record*" (Gaines, 1977 [7]) is perfectly comprehensible to humans despite the appearance of vague terms such as young and good. However in order to make the sentence suitable for a classical database system, it needs to be transformed in a query like "*List the salesmen under 25 years old who have sold more than 20000 pounds of goods.*" As a consequence the system will not select the girl of 26 with a very good selling record, nor the guy of 19 years old with a selling record of 19000 pounds. This counter-intuitive result is due to the fact that there is actually *no* particular age at which one abruptly stops being young (and likewise for the amount of pounds and the term good). One can enumerate some ages which are definitely young and some ages which are definitely not young, but the transition between being young and not being young is gradual. Classical set theory (also called crisp set theory) in se lacks the ability to deal with vague information in an appropriate manner. In this paper we will outline how such vague linguistic terms can be modelled by means of fuzzy set theory (Zadeh, 1965 [24]), a framework built on top of crisp set theory.

2 What is a linguistic variable?

Informally a variable is called linguistic as soon as its values are linguistic terms rather than numerical ones (Kerre, 1993 [10]). In his founding papers, Zadeh (1975 [26]) gives a formal definition which we slightly modify here:

Definition 1 Linguistic Variable *A linguistic variable X is characterized by a quadruple $(T(X), U, G, M)$ with*

- *$T(X)$ being the term set, i.e. the set of linguistic values of the variable*
- *U being a universe of discourse*
- *G being a syntactic rule for generating the linguistic values*
- *M being a semantic rule for attaching meaning to the linguistic values*

Note that this definition covers a wide range of possibilities. E.g. "rather poorly" can be a value of the variable driving skills, "not very young and not very old" can be a value of the linguistic variable age, "sweet lemonade" can be a value of the variable soft drink, "singing" can be a value of the variable hobby, "pencil" can be a value of the variable tool, etc. In all of these examples the terms in $T(X)$ are composed of adverbs (e.g. rather, very), adjectives (poorly, young, old, sweet), logical connectives (not, and), nouns (lemonade, pencil) and verbs (singing). In theoretical studies and in applications however this class is restricted to a much smaller set of linguistic terms, which is still very expressive. The narrowing process can be done by imposing a specific rule G.

Syntactic rule G

A term is either an atomic term, a logically composed term or a modified term.

1. An **atomic term** is an adjective.

Linguistic Variable	Linguistic Terms
age	young, not very old, not young and not old, middle-aged
height	tall, short, rather short
weight	heavy, medium, extremely light
blood pressure	rather high and not very high
temperature	hot, very hot, extremely hot, absolutely hot

Table 1: Linguistic variables and some of their terms

2. A **logically composed term** is a logical composition of terms using not, and and or.

3. A **modified term** is a term generated by applying a linguistic modifier (very, rather, more or less, slightly,...) to a term. □

Table 1 shows some examples of linguistic variables and their terms, generated by means of G.

In (Novák, 1999 [16]) the terms generated by G are called evaluating syntagms. Among the atomic terms, one can distinguish a primary term (e.g. old), its antonym (young) and a medium term (middle-aged). In (Novák, 1999 [16]) these are called an evaluation linguistic trichotomy.

Most often the universe U is a subset of $\mathbb{R}$: e.g. $[0, 120]$ as a universe corresponding to the linguistic variable age, $[0, 30]$ corresponding to blood pressure, $\mathbb{N}$ corresponding to the amount of people present etc. However the universe U can just as well be non-numerical: e.g. the universe of female students on which terms as "beautiful", "very beautiful" and "average" of the linguistic variable appearance can be modelled.

The most intriguing question is of course how to attach meaning to these linguistic terms. In this paper we will address the question of constructing a semantical rule M to this purpose. In Section 4 we will explain that the meaning associated with a linguistic term can be mathematically represented by a fuzzy set on U. However first we will show that the class of linguistic terms that can be generated by G is an expressive tool to tackle many real-life problems.

3 Where do linguistic variables appear?

As we stated in the introduction, the power of daily communication and commonsense reasoning is based on the use of natural language. The linguistic terms that are used in this process are most often vague. Sometimes this is due to a lack of precise information, for instance "*I do not exactly know his age, but he looks old.*" However even when exact numerical values are available, experts tend to transform these values into linguistic ones. For instance a physician will usually translate a numerical measurement of a blood pressure into linguistic specifications such as normal, very high, too low,... This is due to the fact that he has to combine this information with other data, which makes diagnosing a complex task. Computing with numbers demands a high level of precision, which is incompatible with high complexity (the so-called principle of incompatibility (Zadeh, 1975 [26])). Computing with words however requires a lower level of precision; hence it is far more suitable to perform a highly complex task within a certain tolerated degree of imprecision. Humans seem to apply this technique spontaneously (cfr. the diagnosing doctor). Computing with linguistic terms however can also aid computers to deal with problems of a high level of complexity.

There is even more benefit: since natural language is mankind's favourite way of communication, expressing information in computer systems in a linguistical manner makes it far more easy for non-computer experts to implement their knowledge in the system, to give the correct input, to interpret the output,... to understand machines. The following examples indicate some of the potentials of computing with linguistic terms to tackle real-life problems.

Databases In the introduction we mentioned a part of the query "*List the young salesmen who have a good selling record for household goods in the north of England*" (Gaines, 1977 [7]). When being able to compute with linguistic terms, this query can be launched on a database containing crisp values, i.e. ages, amounts of pounds and locations expressed in numbers, and goods expressed in numbered categories. However the attribute values in a database can also be linguistic terms. When constructing a table with information on language skills of candidates for a job opening, it might be far more easy to express these using some linguistic term instead of a number. The table will contain e.g. the following attribute-value pairs for a candidate: (dutch, excellent), (english, very good), (french, good), (german, poor). See also [23].

Expert systems Using linguistic terms allows for **approximate reasoning**, i.e. a reasoning which is neither very precise, nor very imprecise (Zadeh, 1975 [26]). The linguistic terms can be used to express knowledge with relation to the variables used in the facts and the rules. For instance a diagnose expert system may contain the following rule:

> IF cholesterol level is very high and patient is corpulent and age is middle
> THEN heart disease risk is high

which is a typical example of a rule handled by a physician. Computing with linguistic terms allows for the direct implementation of this rule in an expert system. A possible fact that can be matched with the above mentioned rule is

cholesterol level is rather high and patient is corpulent and age is rather young which allows for the deduction of a new fact, e.g. "*heart disease risk is more or less high.*" (For details regarding the deduction we refer to (Ruan, 2000 [18]).) In this case the facts are also formulated by means of linguistic terms. It is however very common to implement the reasoning process by means of linguistic rules, but to give the system a crisp input, e.g. a number generated by some measurement tool. For example the intelligent washing machine, with rules like

IF dirtiness of clothes is large and type of dirt is greasy
THEN washing time is very long

receives its two numerical inputs from a sensor. The calculated washing time is again a number, used to control the washing machine.
The benefits of such systems are obvious:

- One single linguistic rule can replace many numerical rules. E.g. in the diagnose expert system it is practically impossible to formulate all the knowledge in numerical rules like

  IF cholesterol level is 225 and body mass index is 26 and age is 35
  THEN heart disease risk is 10

- Humans can implement their knowledge in the system in a natural, linguistic manner. The same applies for the inputs and the outputs of the system.

Of course the meaning of the linguistic terms involved still needs to be defined by the expert and represented mathematically. In the next section we will discuss how this can be done by means of fuzzy set theory. If the reasoning mechanism of an expert system is based on fuzzy set theory, it is called a fuzzy expert system. If the input and output are crisp numbers, it is called a **fuzzy control** system. These kinds of systems are probably the most popular application of fuzzy set theory nowadays, in all kinds of domains ranging from washing machines over photo cameras to power plants and many more. See e.g. (Babuška, 1998 [1], Ruan, 2000 [19]).

4 How to represent linguistic terms?

To be able to compute with linguistic terms, we need a way to represent them mathematically. Thanks to the syntactic rule G, the linguistic terms of a variable are structured. This facilitates the construction of the semantic rule M. After discussing the representation of terms by means of fuzzy sets in general, we will go into the representation of atomic terms, the representation of logically composed terms and finally the representation of modified terms. If X is a variable as defined in Definition 1, a term of $T(X)$ can be represented by a fuzzy set on U.

Definition 2 Fuzzy set *(Zadeh, 1965 [24]). A fuzzy set A on U is a* $U - [0,1]$*-mapping, also called the membership function of A. For all x in U, A(x) is called the membership degree of x in A. The class of all fuzzy sets on U is denoted* $\mathcal{F}(U)$.

The semantic rule M is therefore a mapping from $T(X)$ to $\mathcal{F}(U)$ that associates with every term t in $T(X)$ its meaning $M(t)$, which is a fuzzy set on U. For all x in U, $M(t)(x)$ denotes the degree to which x belongs to the fuzzy set $M(t)$. $M(t)(x)$ can also be considered as the degree to which x "satisfies t" or "is compatible with t."

As we said above, in this section we will go into $M(t)$, $M(\text{not } t_1)$, $M(t_1 \text{ and } t_2)$, $M(t_1 \text{ or } t_2)$ and $M(h\ t_1)$ with t being an atomic term, t_1 and t_2 arbitrary terms and h a linguistic modifier. First however we recall some important concepts of fuzzy set theory.

Definition 3 Kernel, Support *Let A be a fuzzy set on U. The kernel of A is defined as*

$$ker\ A = \{x | x \in U \wedge A(x) = 1\}$$

It is the set of all objects that definitely belong to A. The support of A is defined as

$$supp\ A = \{x | x \in U \wedge A(x) > 0\}$$

It is the set of all objects that belong to A to some degree greater than 0.

Definition 4 Inclusion *For A and B fuzzy sets on U the inclusion is defined as:*

$$A \subseteq B \text{ iff } A(x) \leq B(x), \text{ for all } x \text{ in } U$$

4.1 Representation of atomic terms

One of the most difficult tasks in designing a fuzzy system is the construction of appropriate membership functions for the linguistic terms involved. Membership functions are context dependent like the terms they represent (e.g. large in "a large mouse" versus in "a large elephant"), and dependent on the observer (e.g. the meaning of young according to a 5 year old child versus a 70 year old man). Despite the importance of the matter, not much attention has been paid yet in the literature to the design of membership functions. The methods proposed in (Klir, 1995 [12], Mason, 1998 [14], Verkeyn, 2000 [21]) turn results of an inquiry into membership functions or help experts to transform their knowledge into membership functions.

In most applications (especially in fuzzy control), the universe U is numerical (often a subset of $\mathbb{R}$). In these cases typical shape functions are at hand, which simplifies the task. Such general shape functions are dependent on some parameters that can be adapted to the context as well as to the observer. Their shape is considered to be acceptable from a psycholinguistic point of view for the representation of linguistic terms (Hersh, 1976 [9]). We will give an overview of the most popular shape functions; for more details we refer to Kerre (1993 [10]).

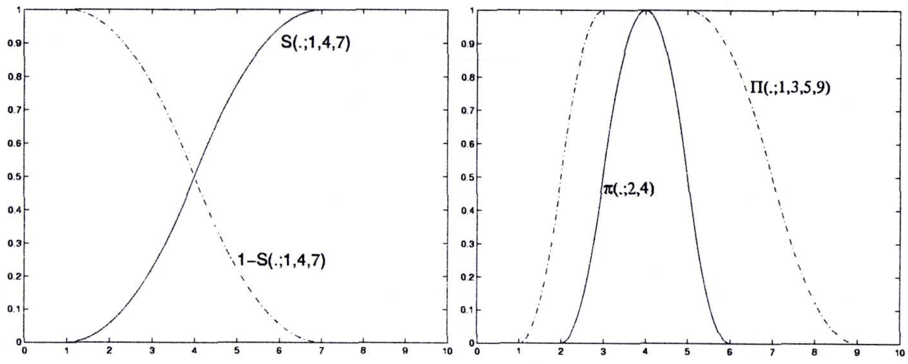

Fig. 1: a) S-membership function and complement of S-membership function b) π-membership function and Π-membership function

S-membership function The S-membership function $S(.;\alpha,\beta,\gamma)$ is an increasing function defined by three real parameters α, β and γ. As shown in Figure 1a α determines the support (equal to $]\alpha,+\infty[$) and γ determines the kernel (equal to $[\gamma,+\infty[$), while $\beta = \frac{\alpha+\gamma}{2}$ corresponds to the so-called "crossover point." The complement of the S-function is defined by $(1-S(.;\alpha,\beta,\gamma))(x) = 1-S(x;\alpha,\beta,\gamma)$, for all x in $\mathbb{R}$. The S-memberhip function is suitable for the representation of increasing notions (old, tall, high,...) while the complement can be used for decreasing notions (young, small, low,...).

π-membership function The π-membership function $\pi(.;\beta,\gamma)$ is defined by two parameters β and γ. As Figure 1b shows, the kernel of this membership function is $\{\gamma\}$ while the support is $]\gamma-\beta,\gamma+\beta[$.

If the increasing and the decreasing part of the membership function are both linear, the fuzzy set is called triangular. The π-membership function is used for the representation of approximating notions, notions that are supposed to have a more or less fixed value (about 140 (pounds), approximately 2 (years)). Furthermore it is also used to represent increasing and decreasing notions in the non-inclusive interpretation. In this paper however we only consider the inclusive interpretation. For details regarding the non-inclusive interpretation, we refer to Kerre (1999 [11]).

Π-membership function The Π-membership function $\Pi(.;\alpha,\beta,\gamma,\delta)$ is characterized by four parameters. As shown in Figure 1b the support is $]\alpha,\delta[$ and the kernel is $[\beta,\gamma]$. If the increasing and the decreasing part of the membership function are both linear, the fuzzy set is called trapezoidal. The Π-membership function is suitable for the representation of notions that can vary between some bounds, like approximately between 4 and 5 p.m., and medium terms like middle-aged, medium weight.

Thanks to the structure imposed by the syntactic rule G, only the membership functions of the atomic terms have to be constructed from scratch, for the fuzzy sets representing logically combined terms and modified terms can be derived from them.

4.2 Representation of logically composed terms

The representation of logically composed terms can be realized by means of tools from fuzzy logic (in the narrow sense (Novák, 1999 [16])), i.e. the many-valued ($[0,1]$-valued) logic dealing with vagueness. In fuzzy logic, negation is a $[0,1]-[0,1]$ mapping:

Definition 5 Negation *A negation $\mathcal{N}$ is a decreasing $[0,1]-[0,1]$ mapping satisfying $\mathcal{N}(0)=1$ and $\mathcal{N}(1)=0$.*

Definition 6 $\mathcal{N}$-complement *For a negation $\mathcal{N}$ and a fuzzy set A on U, the $\mathcal{N}$-complement of A is a fuzzy set on U denoted by $co_{\mathcal{N}}(A)$ and defined by*

$$co_{\mathcal{N}}(A)(x) = \mathcal{N}(A(x)), \text{ for all } x \text{ in } U$$

Negations belonging to the Sugeno class are determined by a parameter λ in $]-1, +\infty[$ and defined by $\mathcal{N}^s_\lambda(x) = \frac{1-x}{1+\lambda x}$, for all x in $[0,1]$. Those of the Yager class are determined by a parameter w in $]0,+\infty[$ and defined by $\mathcal{N}^y_w(x) = (1-x^w)^{\frac{1}{w}}$, for all x in $[0,1]$. $w=1$ characterizes the most popular one, also called the standard negation $\mathcal{N}^y_1(x) = 1-x$. This negation is widely used for the representation of not, i.e.

$$M(\text{not } t_1) = co_{\mathcal{N}^y_1}(M(t_1))$$

although others can be used as well.

Let $[0,100]$ be the universe of ages expressed in years. To represent old in Figure 2a the S-membership function $M(\text{old}) = S(.;50,60,70)$ is used. Hence not old can be modelled by $M(\text{not old}) = 1 - S(.;50,60,70)$. Note that this is different from $M(\text{young}) = 1 - S(.;20,30,40)$.

In fuzzy logic, conjunction and disjunction are represented by $[0,1]^2-[0,1]$ mappings called triangular norms and triangular conorms respectively.

Definition 7 Triangular norm *A triangular norm (or shortly t-norm) $\mathcal{T}$ is an increasing, associative and commutative $[0,1]^2-[0,1]$-mapping that satisfies $\mathcal{T}(1,x) = x$, for all x in $[0,1]$.*

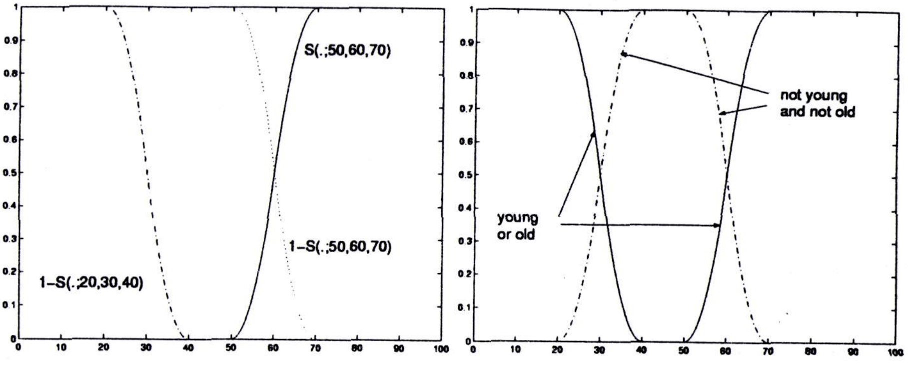

Fig. 2: a) Complement b) Intersection and Union

Definition 8 Triangular conorm *A triangular conorm (or shortly t-conorm) $\mathcal{S}$ is an increasing, associative and commutative $[0,1]^2 - [0,1]$-mapping that satisfies $\mathcal{S}(x,0) = x$, for all x in $[0,1]$.*

Some examples of t-norms and t-conorms are given below:

t-norms	t-conorms
$\mathcal{T}_M(x,y) = \min(x,y)$	$\mathcal{S}_M(x,y) = \max(x,y)$
$\mathcal{T}_P(x,y) = x.y$	$\mathcal{S}_P(x,y) = x + y - x.y$
$\mathcal{T}_L(x,y) = \max(0, x+y-1)$	$\mathcal{S}_L(x,y) = \min(1, x+y)$

Definition 9 $\mathcal{T}$-intersection, $\mathcal{S}$-union *For a t-norm $\mathcal{T}$, a t-conorm $\mathcal{S}$ and two fuzzy sets A and B on U, the $\mathcal{T}$-intersection and the $\mathcal{S}$-union of A and B are the fuzzy sets on U defined by*

$$(A \cap_{\mathcal{T}} B)(x) = \mathcal{T}(A(x), B(x)), \text{ for all } x \text{ in } U$$

$$(A \cup_{\mathcal{S}} B)(x) = \mathcal{S}(A(x), B(x)), \text{ for all } x \text{ in } U$$

and and or can now be modelled by means of intersection and union respectively:

$$M(t_1 \text{ and } t_2) = M(t_1) \cap_{\mathcal{T}} M(t_2)$$

$$M(t_1 \text{ or } t_2) = M(t_1) \cup_{\mathcal{S}} M(t_2)$$

Figure 2b depicts the membership function of young or old computed for $\mathcal{S} = \mathcal{S}_M$, as used in the rule "If you are young or old, you get a 50 percent reduction on the train fare." Furthermore the fuzzy set for not young and not old was constructed using $\mathcal{N}_1^y$ and $\mathcal{T}_M$. The resulting Π-membership function can be used for middle-aged.

4.3 Representation of modified terms

A linguistic modifier h such as very, more or less, rather etc. can be represented by a $\mathcal{F}(U) - \mathcal{F}(U)$ mapping m, which is called a fuzzy modifier (Thiele, 1998 [20]), i.e.

$$M(h\ t_1) = m(M(t_1))$$

Definition 10 Fuzzy Modifier *(Kerre, 1999 [11]) A fuzzy modifier m on U is a $\mathcal{F}(U)-\mathcal{F}(U)$ mapping. m is called restrictive (expansive) iff $m(A) \subseteq A$ ($A \subseteq m(A)$) for all A in $\mathcal{F}(U)$.*

If a $U - U$ mapping s and a $[0,1] - [0,1]$ mapping r exist such that $m(A)(x) = r(A(s(x)))$ for all x in U, then m is called decomposable, and s and r are called the pre- and the postmodifiers respectively. These kind of fuzzy modifiers are often used to model linguistic modifiers, especially for r the identical $[0,1]-[0,1]$ mapping (pure postmodification) or for s the identical $U-U$ mapping (pure premodification). In this section we will give representative examples for both categories and we will point out their main (dis)advantages. Then we will go into a recently developed new approach which helps to overcome the previously mentioned disadvantages by providing the fuzzy modifiers with a clear semantics.

4.3.1 Postmodification

Let A and B denote fuzzy sets on U and x in U. For $\alpha \in [0, +\infty[$, the **powering modifier** P_α is defined by $P_\alpha(A)(x) = A(x)^\alpha$ (Zadeh, 1972 [25]). For $\alpha \geq 1$, P_α is a restrictive fuzzy modifier, while for $\alpha \leq 1$, P_α is expansive. The α-values 0.5 and 2 are often used (see e.g. (Babuška, 1998 [1], Yasmuk, 1998 [22]):

$$M(\text{more or less } t_1) = P_{0.5}(M(t_1)) \qquad M(\text{very } t_1) = P_2(M(t_1)).$$

Figure 3a shows an example of the application of P_2 on an S-membership function for old, generating very old. The powering modifiers are easy to use on all kinds of universes. They respect nice properties w.r.t. complement, union and intersection. For example:

$$\begin{aligned} M(\text{very (young or old)}) &= P_2(M(\text{young}) \cup_{S_M} M(\text{old})) \\ &= P_2(M(\text{young}) \cup_{S_M} P_2(M(\text{old})) \\ &= M(\text{very young}) \cup_{S_M} M(\text{very old}) \\ &= M(\text{very young or very old}) \end{aligned}$$

In other words in this representation the meaning of very (young or old) and of very young or very old is the same. For a detailed list of properties we refer to Kerre (1993 [10], 1999 [11]).

The main disadvantage of powering modifiers is that they keep the kernel and the support, i.e. $ker(P_\alpha(A)) = ker(A)$ and $supp(P_\alpha(A)) = supp(A)$. Hence the

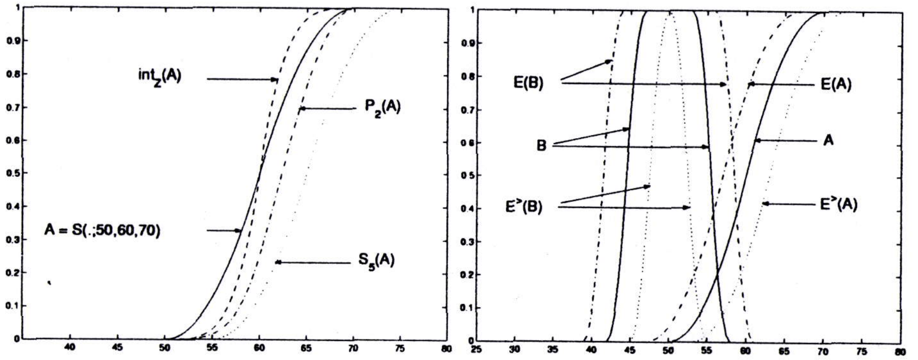

Fig. 3: a) Powering modifier, shifting modifier, contrast intensifier b) Modifiers based on fuzzy relations

representation presented above does not make any distinction between being A to degree 1 and being very A to degree 1. One might feel however that a person of 80 years is old to degree 1 but very old only to a lower degree (e.g. 0.7), but this can not be modelled by means of powering modifiers (or pure modifaction in general). The same process also causes an increase in the slope of $P_2(A)$ while psycholinguistic experiments showed that the slopes of t_1 and very t_1 should be approximately equal (Hersh, 1976 [9]). We believe these intuitive shortcomings are due to the fact that powering modifiers are only technical tools, lacking inherent meaning. At the end of this section we will therefore propose another approach.

Contrast intensification effects an increase of the degrees of membership greater than or equal to 0.5 and a decrease of the degrees of membership smaller than 0.5. To this purpose the operators int_G (Ragade, 1977 [17]) and int_Z (Zadeh, 1972 [25]) were introduced.

$$int_G(A)(x) = \begin{cases} A(x)^2 & \text{if } A(x) < 0.5 \\ \sqrt{A(x)} & \text{if } A(x) \geq 0.5 \end{cases}$$

$$int_Z(A)(x) = S(A(x); 0, 0.5; 1)$$

Combined with powering modifiers they are used to model slightly, namely (Zadeh, 1972 [25]):

$$M(\text{slightly } t_1) = int_Z(M(\text{plus } t_1 \text{ and not very } t_1))$$

in which $M(\text{plus } t_1) = P_{1.25}(M(t_1))$ and the rest is modelled as explained above.

4.3.2 Premodification

Let A and B be fuzzy sets on $\mathbb{R}$ and x in $\mathbb{R}$. For $\alpha \in \mathbb{R}$, the **shifting modifier** S_α is defined by $S_\alpha(A)(x) = A(x - \alpha)$ (Lakoff, 1973 [13], Hellendoorn, 1990 [8], Bouchon-Meunier, 1993 [2]). Shifting modifiers are not in general restrictive of expansive. However the following properties do hold (Kerre, 1993 [10]):

1. If A is increasing and $\alpha \geq 0$ then $S_\alpha(A) \subseteq A$
2. If A is increasing and $\alpha \leq 0$ then $A \subseteq S_\alpha(A)$
3. If A is decreasing and $\alpha \leq 0$ then $S_\alpha(A) \subseteq A$
4. If A is decreasing and $\alpha \geq 0$ then $A \subseteq S_\alpha(A)$

Hence for p a suitable positive number, for increasing membership functions we can use S_p to model very and S_{-p} to model more or less, while for decreasing membership functions just the opposite. Figure 3a shows the application of S_5 to old, generating very old. These representations respect similar properties w.r.t. complement, intersection and union as the powering modifiers (see Kerre (1993 [10])) and they comply better with experimental results (Hersh, 1976 [9]). However when the membership function of a term t_1 is partly increasing and partly decreasing (e.g. about 5 o'clock) there is no straightforward way to model more or less t_1 by means of shifting modifiers. Furthermore these modifiers can only be used on numerical universes because of the need for a shifting operation.

In (Novák, 1992 [15]) a combination of pre- and postmodification is proposed. While this approach more or less helps to overcome some of the above mentioned shortcomings, its application tends to be complicated and very dependent on the kind of membership function of the original term.

4.3.3 Fuzzy modifiers based on fuzzy relations

Although already useful, the above mentioned fuzzy modifiers have important shortcomings, which are in our opinion due to the fact that they are designed simply to perform a technical transformation, but have no further meaning of their own. Recently two new approaches were developed in which the representation of linguistic modifiers is endowed with an inherent semantics: the so-called horizon approach (Novák, 1999 [16]), and the framework of fuzzy modifiers based on fuzzy relations (De Cock, 2000 [4]). In this section we will discuss the latter. The strength of this approach is that in determining the degree to which y is very t_1 the context is taken into account, namely the fuzzy set of all objects resembling to y. Resemblance is modelled by means of a fuzzy relation.

Definition 11 Fuzzy relation *A fuzzy relation R on U is a fuzzy set on $U \times U$. For y in U the R-foreset of y is the fuzzy set Ry on U defined by $(Ry)(x) = R(x, y)$.*

If E is a resemblance relation on U, i.e. E is a fuzzy relation on U such that for all x and y in U, $E(x, y)$ is the degree to which x and y resemble to each other, then Ey is the fuzzy set of objects resembling to y. The general idea is that an object y can be called more or less t_1 if it resembles to an object that can be called t_1. Likewise an object y can be called very t_1 if every object it resembles to can be called t_1. In the first case we need to represent the intersection of Ey and $M(t_1)$ for which we will use a t-norm. In the second case we have to study the inclusion of Ey in $M(t_1)$; to this end we will need another tool from fuzzy logic, namely implication.

Definition 12 Implication *An implication $\mathcal{I}$ is a $[0,1]^2 - [0,1]$-mapping with decreasing first partial mappings $\mathcal{I}(.,x)$ and increasing second partial mappings $\mathcal{I}(x,.)$ that satisfies $\mathcal{I}(1,x) = x$, for all x in $[0,1]$.*

Definition 13 *For A and B two fuzzy sets on U the* **degree of inclusion** *and* **the degree of overlap** *are defined by:*

$$\mathsf{INCL}(A,B) = \inf_{x\in U} \mathcal{I}(A(x), B(x)) \quad \mathsf{OVERL}(A,B) = \sup_{x\in U} \mathcal{T}(A(x), B(x))$$

Using these notions the following representation can be constructed:

$$M(\textsf{more or less } t_1)(y) = \mathsf{OVERL}(Ey, M(t_1))$$

$$M(\textsf{very } t_1)(y) = \mathsf{INCL}(Ey, M(t_1))$$

These representations coincide with the direct and the superdirect image (Kerre, 1993 [10]) of $M(t_1)$ under E, i.e.

$$M(\textsf{more or less } t_1) = E(M(t_1)) \text{ and } M(\textsf{very } t_1) = E^{>}(M(t_1))$$

These images respect all kinds of mathematical properties (De Cock, 2000 [5]) which can be interpreted for linguistic terms (De Cock, 2000 [6]). Figure 3b depicts the application to old of modifiers based on the resemblance relation E defined by $E(x,y) = min(1, max(0, 2.5 - 0.5.|x-y|))$, using the implication $\mathcal{I}_L(x,y) = min(1, 1-x+y)$ and the t-norm $\mathcal{T}_L$. Note that the kernel and the support are changed, which makes these modifiers intuitively more correct than the powering modifiers. In the same figure it is shown that these modifiers can also be applied in the same way to a Π-function like $B = \Pi(.; 42, 47, 53, 58)$ modelling about 50, which was not possible for shifting modifiers. In (De Cock, 2000 [3]) a similar framework is presented for the representation of at least t_1 and at most t_1 by means of ordering relations.

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