Local Semantics as a Lattice Based on the Partial-all Quantifier

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Abstract
The main problem of system theory is summarized by; how one can express the growing universe in a universe? In a set theory, the universe appeared in the diagonal argument is a candidate of the expression of the growing universe, however it is not in a universe. To resolve this problem, we propose the dynamic quantifier and partial-all quantifier that mimic infant's eyes. A universe is defined as a concept lattice that is obtained from a binary relation between two sets. Because a formal concept is defined through a particular operator with all-quantifier, $\forall$, a concept lattice is complete and the relationship between a relation and a lattice is unique. By contrast, partial-all quantifier, $\forall_p$, no longer surveys all elements in a given set, and then an obtained lattice is different from a normal concept lattice. In this scheme, a lattice is perpetually changed. If a lattice polynomial is used as syntax, a lattice is used as semantics. Due to partial-all quantifier, a lattice is destined to be local semantics. Especially, lattices that are derived through partial-all quantifier can constitute the hierarchy of distributive law, and then the domain in which perturbation is applied is perpetually changed in the model that a lattice polynomial keeps on transform binary sequences.

1 Introduction

Science in the 21th century will be focused on the origin of consciousness. The central problem on consciousness is strongly relevant to the interface of epistemology and ontology. The quolia (Chalmers, 1996), for example, can be the interface of a natural thing and the expression of it. Qualia of red as an apple is not a particular frequency of light, however the latter is a particular expression of it. There is a great gap between the expression and an object expressed before, however the consciousness exists as far as a natural object and its expression is connected. Of course it is not a unique problem up to the science of consciousness.

The new system theory is expected to resolve the problem arising from the origin of consciousness. The problem of the qualia can be summarized as the problem of an evolutionary hierarchical system (Ehresmann & Vanbremeersch, 1987; 1997), because

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the qualia is established in the interface of different logical criteria. Such a system contains logical hierarchy. In logical sense, the mixture of different logical criteria is prohibited because it entails to a contradiction. A system, however, generates new hierarchical criterion in its own right, and then a system has to see and compare various kinds of logical criteria. Such a system, therefore, cannot be failed even if a system is exposed to a logical contradiction. Conrad (1985, 1993) also concentrates on living systems as evolutionary hierarchical systems. He points out the aspect in which parallelism among cells can proceed and vertical interactions among different logical criteria. In a logical sense, such a system fails to a paradox. The evolutionary hierarchical system, however, contains a particular boundary condition in which a paradox is established. In other words, a boundary condition in which a system is logically defined is regarded as materials in the sense of evolutionary hierarchical systems.

Rosen explicitly manifests the paradox in evolutionary hierarchical systems, and calls them complex systems (Rosen, 1985). He and his colleagues find a particular complementary relationship called adjunction (MacLane, 1971, Birkhoff, 1967) between formal and natural systems (Luie, 1985), and expands the idea to a system with anticipation (Rosen, 1985). If the system keeps on touching the outside that is different from the inside with respect to logical status, it entails to some entity corresponding to a contradiction inside of the system. This entity is used as some information of the surrounding, and that is the origin of anticipation. The aspect of anticipation can be expressed as backward-dynamics (Dubois, 1992, Gunji & Nakamura, 1991). Dubois expands the idea and general theory of anticipatory systems is established (Dubois & Resconi, 1992, Dubois, 1997; 2000).

The eternal interface of different logical criteria is the interface of an observer and an object. In general, the boundary condition in which an object is regarded as a particular unity separated from its surroundings is hidden in an observer, and what a system is observed as an object is regarded as the trivial. Against this attitude, such a boundary condition also has to be regarded as materials if one concentrates on evolution of a system. It can entail to innumerable hierarchical boundary conditions. Of course there is no way to see whole universe containing infinite hierarchical boundary conditions, and the natural things by no means see the whole universe. Next question arises how one can construct the boundary condition, in which an object is separated from a universe, as materials. This is discussed in the science of endo-physics (Rossler, 1987; 1996, Svozil, 1993), and internal measurement (Matsuno, 1989; 1997, Gunji et al., 1997). We call such a particular boundary condition materialized boundary condition.

Materialized boundary condition is the growing universe in a universe. Infant's eyes are typical examples of materialized boundary condition (Gunji et al., 2001a). It is very difficult to formalize infant's eyes, because infant's eyes contain a touch between different logical criteria. If one defines a universe by a set and regards the infants' universe as a subset, there is no different logical criterion. On the other hand, if one introduces the mixture of different logical criteria, then there is no consistent distinction between infant's universe and its surroundings. In this paper, we show new attempt to construct the infant's eyes based on a set theory. First we define adult's universe (i.e., exo-physics, external measurement, objective perspective) as a concept lattice (Ganter & Wille, 1999). It also contains two different categories, natural universe and formally expressed universe, however the relationship between them is well-defined in a term of adjunction. Second, we define infant's eyes in an objective universe. Infant's eyes are defined by dynamic-all quantifier and by partial-all quantifier. Readers see that both natural universe and formally expressed universe are perpetually changed, and that is called evolution. Infant's perpetually gathers his own epistemic data and constructs his own semantics through which expressed universe makes a sense. That is why semantics for an evolutionary universe is destined to be local, and the interface of different logical criteria can be expressed as local semantics.

2 Local Semantics by Dynamic Quantifier

2.1 Global Semantics Based on Adjunction

First we define adult's universe, or exo-physics in a set theory. It consists of epistemic universe and conceptual universe. They can be compared to Rosen's natural and formal systems (Rosen, 1985, Luie, 1985). Epistemic universe is defined by formal context denoted by $\boldsymbol{K} = (G, M, I)$, where G, M and sets and I is relation between them. According to Ganter & Wille (1999), any entities can be expressed as a set of attribute or a set of objects. Various concepts are re-examined in terms of formal concepts (Wolff, 2000). Then, G and M are assumed as a whole set of objects and a whole set of attributes, respectively.

From formal context, formal concept (A, B) with $A \subseteq G$ and $B \subseteq M$ is defined by $A' = B$ and $B' = A$ (Ganter & Wille, 1999), where

$$A' = \{ m \in M \mid gIm, \ ^{\forall} g \in G \} \quad (1a)$$
$$B' = \{ g \in G \mid gIm, \ ^{\forall} m \in M \}. \quad (1b)$$

Among formal concepts, partial order is defined by

$$(A_1, B_1) \leqq (A_2, B_2) \ :\Leftrightarrow \ A_1 \subseteq A_2 \ \text{ and } \ B_2 \subseteq B_1. \quad (2)$$

Adult's conceptual universe is expressed as a concept lattice defined by a set of concepts and the partial order defined by (2). In general a lattice is a partial ordered set, S, that is closed with respect to supremum $(x \cup y)$ and infimum $(x \cap y)$ for all two-elements-sets, $\{x, y\}$ with $x, y \in S$. Because of definition (8), all concept lattices are complete [22], and the following statements are proven: for A, A_1, $A_2 \subseteq G$ or M,

$$A_1 \subseteq A_2 \quad \Rightarrow \quad A_2' \subseteq A_1 \quad (3a)$$
$$A \subseteq A'' \quad (3b)$$
$$A' = A'''. \quad (3c)$$

From statements (3), A' can be regarded as a closure of A. It means that two operators ' : $PG \rightarrow PM$ and ' : $PM \rightarrow PG$ gives a Galois connection, where PG and PM represent power sets of G and M. It is also proven that that $A \subseteq B' \Leftrightarrow B \subseteq A'$, and this one-to-one relationship is called adjunction.
Adult's conceptual universe is based on adjunction. Technically, it is stated that the operator ' : $G \rightarrow M$ is left-adjoint to the operator ' : $M \rightarrow G$. These operators constitutes adjunction because it contains all-quantifier, $\forall$. Due to all-quantifier, the adult can survey whole universe expressed as relation, I. In other words, all-quantifier is an expression of adult's eyes. In terms of an adult's conceptual universe some statements can be proven (for proof see Gunji et al., 2000b).

Proposition 1.

There exists $h \in G$ and $n \in M$ such that $hIn \wedge$ (for $^{\forall}g$ $(g \neq h)$ $gJn) \wedge$ (for $^{\forall}m$ $(m \neq n)$ $hJm)$ in $\boldsymbol{K}(G, M, I) \Rightarrow \boldsymbol{L}(G, M, I)$ is not distributive.

Proposition 2.

If, for $^{\forall}m \in M$, $^{\exists}h \in G$ such that hIn for $^{\forall}n \in M$ $(n \neq m)$ and hJm (i.e., h has no relation to m) in $\boldsymbol{K}(G, M, I)$, then $\boldsymbol{L}(G, M, I)$ is a Boolean lattice.

2.2 Infant's Eyes as Dynamic Quantifier

Compared with adult's eyes, infant's eyes are defined by a particular quantifier, because an infant constitutes his own conceptual universe by surveying a part of the universe. The first attempt to construct infant's eyes is expressed as a dynamic quantifier. Given an adult's epistemic universe, $\boldsymbol{K}(G, M, I)$, infant's epistemic universe is expressed as a local context, $\boldsymbol{K_L}(G^*, M^*, I^*)$, where

$$G^* = G \cup \{*\} \qquad (4)$$
$$M^* = M \cup \{*'\} \qquad (5)$$
$$I^* = I \cup I_1 \cup I_2. \qquad (6)$$

And $\{*\}$ and $\{*'\}$ represents a singleton set. The new relations I_1 and I_2 defined by

$$I_1 = \{(*, m) \mid 4Im, \ ^{\forall(t)}m \in A\} \qquad (7)$$
$$I_2 = \{(g, *') \mid gIe, \ ^{\forall(t)}g \in B\}, \qquad (8)$$

where $A \subseteq G$, $B \subseteq M$ and $\forall(t)$ called a dynamic quantifier is defined by

$$^{\forall(t)}x \in X \ :\Leftrightarrow n \text{ elements (randomly chosen) in } X, \quad \text{if } |\forall(t)| = n \neq 0; \qquad (9)$$
$$:\Leftrightarrow \ ^{\forall}x \in X, \quad \text{if } |\forall(t)| = 0.$$

A dynamic quantifier is designated by number of chosen elements in a set, that is represented by $|\forall(t)|$. Depending on both a chosen subset, A, and time step, t, $|\forall(t)|$ is dynamically changed. It is defined by; for ${}^{\forall(t)}x \in A$,

$$|\forall(t+1)| = 0, \quad \text{if } |\forall(t)| = |X|; \tag{10}$$
$$= 1, \quad \text{if } |\forall(t)| = 0;$$
$$= |\forall(t)|+1, \quad \text{otherwise,}$$

where $\forall(t)$ is used as a normal all-quantifier of $\forall$ if $|\forall(t)|= 0$.

Infant's conceptual universe, $\boldsymbol{L_L}(G^*, M^*, I^*)$, is constructed from infant's epistemic universe, $\boldsymbol{K_L}(G^*, M^*, I^*)$. It is defined as a concept lattice. On the infant's conceptual universe, the following statements can be proven (for proof see Gunji et al., 2000b).

Proposition 3.

Given a context, $\boldsymbol{K} = (G, M, I)$ and $\boldsymbol{K_L}(G^*, M^*, I^*)$, where $G^* = G \cup \{4\}$, $M^* = M \cup \{e\}$, and $I^* = I \cup I_1 \cup I_2$, I_1 and I_2 are defined by

$$I_1 = \{(4, m) |\ 4Im,\ {}^{\forall(t)}m \in \emptyset\} \tag{11a}$$
$$I_2 = \{(g, e) |\ gIe,\ {}^{\forall(t)}g \in \emptyset\}, \tag{11b}$$

$\boldsymbol{L_L}(G^*, M^*, I^*)$ is not distributive.

One of the most important properties in the relationship between $\boldsymbol{K}(G, M, I)$ and $\boldsymbol{K_L}(G^*, M^*, I^*)$ is compatibility of a local context. Given $\boldsymbol{K}(G, M, I)$, and $\boldsymbol{K_L}(G^*, M^*, I^*)$, $\boldsymbol{K}$ is called a compatible sub-context if and only if $\Pi_{G,M}: \boldsymbol{L_L}(G^*, M^*, I^*) \rightarrow \boldsymbol{L}(G, M, I)$ is a surjective complete homomorphism, where $\Pi_{G,M}$ is defined by

$$\Pi_{G,M}(A, B) = (A \cap G, B \cap M) \tag{12}$$

for any concepts $(A, B) \in \boldsymbol{L_K}(G^*, M^*, I^*)$.

A compatible subcontext can be verified by using a double arrow relation [22]. Double arrow relation $g \downarrow\downarrow m \in \downarrow\downarrow \subseteq G \times M$ is defined by the existence of two sequences of $g = g_1, g_2, .., g_n$ and $m_1, m_2, .., m_n = m$ such that

$$g_i \downarrow m_i \quad \text{and} \quad g_i \uparrow m_{i-1} \tag{13}$$

with $i = 1, 2, .., n$. Note that for $g \in G$ and $m \in M$, $g \downarrow m$ and $g \uparrow m$ are defined as

$$g \downarrow m \Leftrightarrow gJm \wedge ({}^{\forall}h(g \subset h), hIm) \tag{14}$$
$$g \uparrow m \Leftrightarrow gJm \wedge ({}^{\forall}n(m \subset n), gIn). \tag{15}$$

Double arrow relation induces the following theorem; A context (G, M, I) that is a subcontext of a context (G^*, M^*, I^*) is compatible, if and only if $(G^* \setminus G, M, \text{not}(\downarrow\downarrow))$ is a concept, where not($\downarrow\downarrow$) represents that $\downarrow\downarrow$ does not hold. It, therefore, means that

$$(G^* \setminus G)' = \{ m \in M | \ g \ \text{not}(\downarrow\downarrow) m \ \text{for} \ ^{\forall} g \in (G^* \setminus G)\} = M \quad (16)$$
$$M' = \{ g \in G | \ g \ \text{not}(\downarrow\downarrow) m \ \text{for} \ ^{\forall} m \in M\} = G^* \setminus G. \quad (17)$$

We obtain the propositions with respect to the interpolation from (G, M, I) to (G^*, M^*, I^*).

Proposition 4.

Given a local context K_L (G^*, M^*, I^*) is defined by $G^* = G \cup \{h\}$, $M^* = M \cup \{n\}$, and $I^* = I \cup I_1 \cup I_2$. If the context (G, M, I) satisfies the condition; for a particular $s \in G$ and all $g \in G$ $(g \neq s)$,

$$sIn, \quad gJn, \quad hJn \quad (18)$$

and there exists only m_1 and m_2 such that

$$n' \subset m_1', \ n' \subset m_2', \ hIm_1, \ hIm_2, \ (gJm_1 \vee gJm_2) \quad (19)$$

for all $g \in G$ $(g \neq s)$, then for all $g \in G$,

$$g \uparrow n \ \text{does not hold, and} \ h \uparrow n. \quad (20)$$

Proposition 5.

Given $\boldsymbol{K}(G, M, I)$, a global context (G^*, M^*, I^*) is defined by $G^* = G \cup \{h\}$, $M^* = M \cup \{n\}$, and $I^* = I \cup I_1 \cup I_2$. If the context satisfies the condition; there exists $s \in G$ such that,

$$sIn, \quad hJn \quad (21)$$

and for all $m \in M$

$$sIm \ \Leftrightarrow \ hIm, \quad (22a)$$
$$sJm \ \Leftrightarrow \ hJm, \quad (22b)$$

and there is no r but s such that $h' \subset r'$, then for all $m \in M$,

$$h \downarrow m \ \text{does not hold, and} \ h \downarrow n. \quad (23)$$

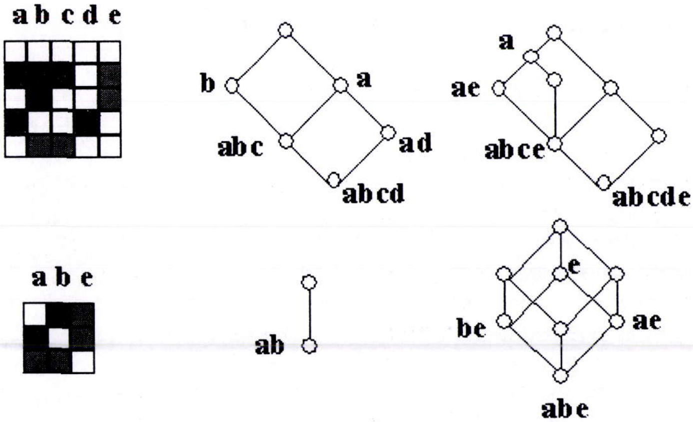


FIGURE 1. Adult's lattice (center) resulting from the context (left, 4x4 matrix above and 2x2 matrix below) is compared with infant's lattice (right) resulting from the expanded context (left, 5x5 matrix above and 3x3 matrix below). Symbols near elements represent intent of a concept. For example, ab represents {a, b}.

From these propositions we can finally obtain the following.

Proposition 6.

Given a context, $\boldsymbol{K_0}(G_0, M_0, I_0)$ with $s \in G$ satisfying following condition;

$$\text{no } g \in G\ (s' \subset g') \tag{24}$$
$$\exists m_1, m_2 \in M\ (sIm_1,\ sIm_2) \tag{25}$$
$$\text{no } n \in M\ (m_1' \subset n',\ m_2' \subset n'), \tag{26}$$

if $\boldsymbol{K_{n+1}}(G_{n+1}, M_{n+1}, I_{n+1})$ is recursively defined by $G_{n+1} = G_n \cup \{h_n\}$, $M_{n+1} = M_n \cup \{n_n\}$ and $I_{n+1} = I_n \cup I(h_n) \cup I(n_n)$ where

$$I(h_n) = \{(h_n, m) \mid \forall m \in M(sIm),\ h_nIm;\ \forall r \in M(sJr),\ h_nJr\ \} \tag{27}$$
$$I(n_n) = \{(g, n_n) \mid \forall g \in G(g \neq s),\ gJn_n;\ sIn_n\ \}, \tag{28}$$

$\boldsymbol{K_n}$ is compatible to $\boldsymbol{K_{n+1}}$(see Gunji et al. (2000b) for proof of all propositions).

Fig. 1 shows comparison between adult's concept lattice and infant's concept lattice. Because an infant surveys whole universe and constructs its epistemic universe depending on his own survey and the property of dynamic all-quantifier, there can be possible other various local contexts.

3 Local Semantics Based on Partial-all Quantifier

3.1 Infant's Eyes as Partial-all Quantifier

The second and more essential attempt to construct infant's eyes is expressed as partial all-quantifier. In this scheme, context, ***K***(*G, M, I*) is regarded as a universe. This objective universe can be interpolated as far as the context is compatible. For example, if ***K***(*G, M, I*) is defined by; $G = \{0, 1\}$, $M = \{a, b\}$ and $I = \{0Ia, 1Ib\}$, resulting concept lattice (adult's conceptual universe) is a Boolean lattice. Then lattices resulting from all contexts compatible to ***K***(*G, M, I*) is also Boolean (Fig. 2).

Even if compatible transformation of contexts is allowed, adult's conceptual universe is invariant. Because concepts are based on adjunctive operators and normal all-quantifier, compatible transformation of context just increases the number of combinations. For example, 2-Boolean concept lattice consists of 0 and 1, and it is obtained from relation $I \subseteq \emptyset \times \emptyset$ (it is expressed as 1x1 matrix). 2^2-Boolean concept lattice (Fig.2) consists of 2^2-elements, (0,0), (0,1), (1,0) and (1,1), and that is just refinement. That is why lattices obtained from compatible contexts are isomorphic to one another. In a universe in which compatible transformation of context is allowed, an infant surveys a universe and constructs his own conceptual universe. In this sense, difference between adult's eyes and infant's eyes can be expressed as difference of all-quantifier.

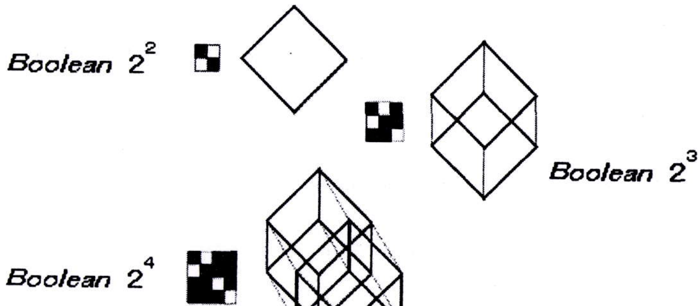


FIGURE 2. The hierarchy of a Boolean lattice derived from compatible lattices.

As well as Boolean lattices, if ***K***(*G, M, I*) is a context by which a modular lattice is constructed as a concept lattice, all contexts compatible to ***K***(*G, M, I*) lead to modular lattices (Fig. 3).

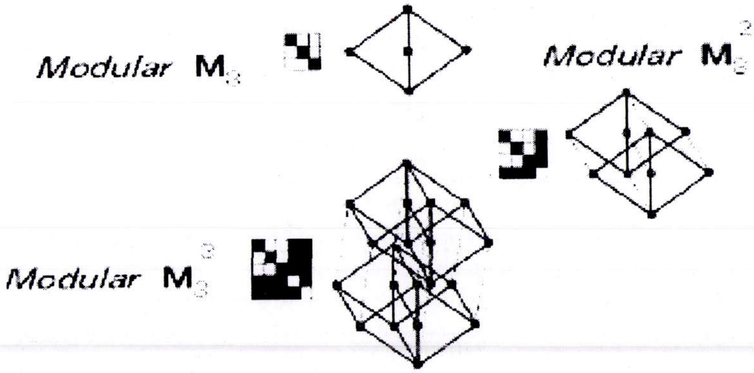


FIGURE 3. The hierarchy of a Modular lattice derived from compatible lattices.

Compared with adult's eyes, infant's eyes are defined by partial all-quantifier denoted by $\forall$p. It is defined by

$$\forall_{p} g \in A \;\Leftrightarrow\; \forall g \in A \cap ((A')^{C})^{+} \tag{29}$$

where B^C represents complement of B such that $B \cap B^C = 0$ and $B \cup B^C = 1$. The value 0 and 1 are the least and the greatest element in a lattice, respectively. The operation B^+ is defined by

$$B^{+} = \{\, m \in G \mid gIm,\ \exists m \in B \,\}. \tag{30}$$

Definition (29) is derived from consideration on the diagonal argument [24]. The growing universe in the diagonal argument is defined by Intent $\cap$ Extent = All-parts $\cap$ (All-parts)^C. Because the infant's universe is in a universe, the outside of infant's universe can be defined by a set of objects separated from a set of attribute. It, therefore, leads to the definition (7). Also for an infant, his own empirical universe is defined by $I \subseteq G \times M$. Using infant's eyes, an infant can construct infant's own conceptual universe from his epistemic universe. The infant's conceptual universe by a lattice $(L, \subseteq)$, where an ordered set L is defined by

$$L = \{\, FA \subseteq M \mid A \subseteq G \,\}, \tag{31}$$
$$FA = \{\, m \in M \mid gIm,\ \forall_{p} g \in A \,\}. \tag{32}$$

Given a lattice, $(L, \subseteq)$, infant's new empirical universe, I^*, can be constructed from a lattice, by the following: (1) At first, for all $B \in L$, a pair $(\emptyset, B)$ is defined and $\emptyset \in G^*$. (2) If $B \in L$ is a co-atom (i.e., if $B \subseteq C$ with $C \in L$, then $C = B$ or $C = M$), a pair (A,

B) is defined and A is defined by a singleton set $\{g\}$ (i.e., a set consisting of one element), and $g \in G^*$, where if two atoms satisfy $B_1 \neq B_2 \in L$, (A_1, B_1) and (A_2, B_2), then $A_2 = \{h\}$ and $h \neq g = A_1$. (3) If $B_1 \subseteq B_2$ with $B_1, B_2 \in L$, (A_1, B_1) and (A_2, B_2), define A_1 such that for all $g \in A_2$, $g \in A_1$ (It means that $A_2 \subseteq A_1$). (4) If $B_1 \subset B_2$ with $B_1, B_2 \in L$, (A_1, B_1), (A_2, B_2) and $A_1 = A_2$, add a new element g to G^* and define $A_1 = A_2 \cup \{g\}$. (5) If there is a pair, (A_1, B_1), (A_2, B_2) with $B_1 \subset B_2$ and $A_1 = A_2$, apply the statements (3)and (4). Finally we obtain new context $\boldsymbol{K}(G^*, M, I^*)$.

However, the original empirical universe is not generally re-constructed, and is perpetually changed. To distinguish infant's universes in time development, the suffix, t, is introduced. Infant's empirical universe at the tth step is denoted by $I(t) \subseteq G(t) \times M(t)$, and infant's conceptual universe is denoted by $(L(t), \subseteq)$. Even if a universe is assumed to be compatible contexts, infant's conceptual universe (i.e. $(L(t), \subseteq)$) and his empirical universe (i.e. $\boldsymbol{K}(G(t), M(t), I(t))$) is perpetually changed. Fig. 4 shows an example of time development of $(L(t), \subseteq)$) and $\boldsymbol{K}(G(t), M(t), I(t))$.

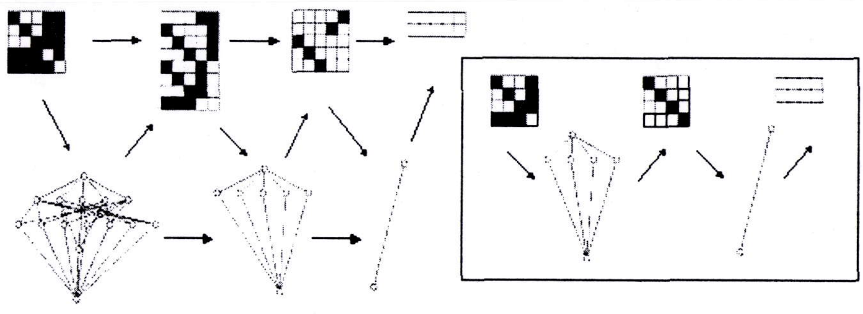


FIGURE 4. Time development of an infant's conceptual universe, $(L(t), \subseteq)$). Even if initial contexts are compatible, generated lattices are different with respect to the distributive law.

Two time developments are shown and the initial contexts, $\boldsymbol{K}_0(G(0), M(0), I(0))$ (left) and $\boldsymbol{K}_1(G(0), M(0), I(0))$ (right, surrounded by a rectangle) are compatible in the sense of an adult. These contexts, however, lead to concepts that are not isomorphic to one another. The former one is non-modular ortho-modular lattice and the latter is non-distributive modular lattice. Final lattices are the same as a Boolean lattice. Fig. 5 also shows another time development of $(L(t), \subseteq)$) and $\boldsymbol{K}(G(t), M(t), I(t))$, where the initial context leads to a Boolean lattice as far as a concept is defined by adult's eyes. Due to infant's eyes, a non-distributive modular lattice is obtained.

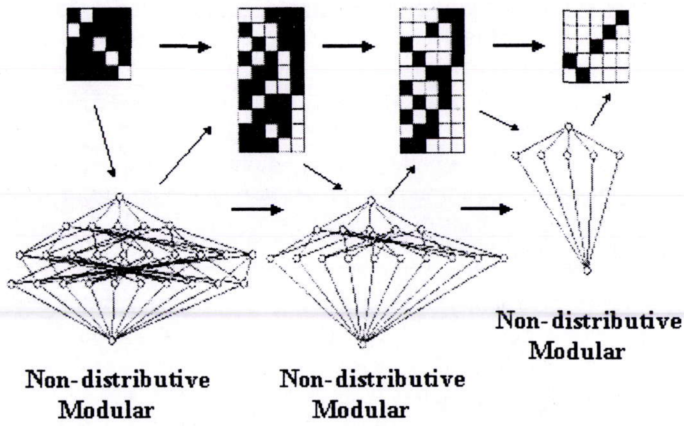
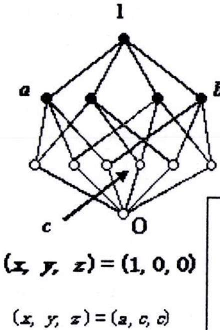


FIGURE 5. Time development of an infant's conceptual universe, $(L(t), \subseteq))$, where initial context leads to a Boolean lattice if all-quantifier is applied instead of partial-all quantifier.

Even if initial lattices are compatible (also see Fig. 3), generated lattices are different with respect to the distributive law. A lattice, L, is a distributive lattice if and only if all elements x, y, $z \in L$ satisfy the distributive law, $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$. Lattices constitute a hierarchy with respect to the distributive law. The ratio of elements satisfying the distributive law in a lattice is decreased in the order; distributive lattice → non-distributive modular lattice → non-modular ortho-modular lattice → non-ortho-modular lattice. We call it the hierarchy of distributive law.

3.2 Cellular Automata Driven by Local Semantics

Thanks to the hierarchy of distributive law, infant's eyes are significant if a lattice is used as semantics. How does one use a lattice as semantics? We define a particular lattice polynomial as syntax. It is an expression consisting of variables, operators $\cap$, $\cup$ and c (complement). If elements of a lattice are substituted into a lattice polynomial, it can be computed. It, however, depends on a lattice structure. Such an example is shown in Fig. 6. We here give a transition rule of elementary cellular automaton expressed as a lattice polynomial (right, above). This transition rule transforms a binary sequence if it is applied to all cells with synchronous updating. For this transition rule a non-distributive modular lattice (left) is used as semantics. Because the input and output of computation is expressed in the form of a binary sequence, elements of a lattice are divided into either 0 (white circle) or 1 (black circle). Assume that the input is (1, 0, 0). First these values are mapped to elements of a lattice. If the input is 1, then it is mapped to elements of a lattice painted black. If it is 0, it is mapped to elements painted white. After that computation following a lattice polynomial is actualized in this lattice. Because this lattice is not distributive, complement of a particular element is not uniquely determined. Imagine that the input (1, 0, 0) is mapped to (*a, c, c*) in a lattice. For *c,* both *a* and *b* are complements, because they satisfy the definition of complement of *a* such that $c \cap x = 0$ and $c \cup x = 1$. Symbols 0 and 1 in the definition of complement are the least and the greatest value, respectively. If $c^c = b$, the output of a polynomial is mapped to 1 because an element, *a,* corresponds to 1 in a binary expression. If $c^c = a$, the output of a polynomial is mapped to 1 because an element, *a,* corresponds to 1 in a binary expression (see calculation surrounded by a rectangle). That is why it is inevitable to introduce, what is called, a perturbation in calculation, if a lattice is not distributive.



$$a_k(t+1) = f(a_{k-1}(t), a_k(t), a_{k+1}(t))$$
$$= f(x, y, z)$$
$$= (x^c \cap y^c \cap z) \cup (x \cap y^c \cap z^c)$$

$$a_k(t+1) = (a^c \cap c^c \cap c) \cup (a \cap c^c \cap c^c)$$
$$= (c \cap a \cap c) \cup (a \cap a \cap a)$$
$$= 0 \cup a = a = 1$$
$$a_k(t+1) = (c \cap b \cap c) \cup (a \cap b \cap b)$$
$$= 0 \cup 0 = 0$$

FIGURE 6. It shows an example of calculation of a given lattice polynomial based on a non-distributive lattice. Because complement of an element is not uniquely determined, the output is not uniquely determined. See text.

We propose an abstract model of the learning process of an infant. The task of learning is uniquely determined, if semantics is assumed to be global. In real learning process just an image of the task is acquired by a subject. It is very difficult to coincide the subject's image of the target and the image determined in advance. In other words, a subject always constructs his own image of a task in surveying his own epistemic universe.

In this perspective, we define the task of leaning process by the calculation of an elementary cellular automaton, expressed as a lattice polynomial,

$$a_k(t+1) = f(a_{k-1}(t), a_k(t), a_{k+1}(t)) \\ = (x^c \cap y^c \cap z^c) \cup (x \cap y \cap z), \quad (33)$$

where x, y and z represents $a_{k-1}(t)$, $a_k(t)$ and $a_{k+1}(t)$, respectively. Given a binary sequence of $\boldsymbol{b}(0) = (a_1(0), a_2(t), a_2(t), \ldots)$, the transition rule (33) is applied to it with synchronous updating and periodic boundary condition, and the next binary sequence is generated. This procedure is iterated. Compared to the learning process, generated binary sequences are the images of the task. Actual-ization of the task is possible if a subject surveys his own empirical universe and constructs his own conceptual universe that is local semantics.

The interaction between the image of the task and local semantics is defined as follows. A binary sequence of $\boldsymbol{b}(t)$ indicates the degree of compatible transformation of a context, $\boldsymbol{K}_t(G(t), M(t), I(t))$, is determined by some information of $\boldsymbol{b}(t)$. For the sake of convenience, we define that $(2\text{x}a_1(t)+a_2(t))$- rows and columns are added to a context, $\boldsymbol{K}_t(G(t), M(t), I(t))$. Such a transformed context is denoted by $\boldsymbol{K}_t^*(G(t), M(t), I(t))$. From $\boldsymbol{K}_t^*(G(t), M(t), I(t))$ and partial-all quantifier, a lattice, $(L(t), \subseteq))$ is obtained. Then, $\boldsymbol{b}(t)$ is mapped to $(L(t), \subseteq))$ by $f:\{0,1\}\rightarrow(L(t), \subseteq)$ expressed as $f(0)=w$ and $f(1)=q$, where w is an element of a lattice corresponding to 0 in a binary expression, and q is an element of a lattice corresponding to 1. After this mapping, a lattice polynomial is calculated, and the output is mapped to a binary sequence by $g : (L(t), \subseteq)\rightarrow\{0,1\}$ expressed as for all x in $(L(t), \subseteq)$, $g(x) = y$ with $f(y) = x$. Simultaneously next context $\boldsymbol{K}_{t+1}(G(t+1), M(t+1), I(t+1))$ is also calculated by the procedure mentioned before. Initial context $\boldsymbol{K}_0(G(0), M(0), I(0))$ is given as $G(0)=M(0)=$empty.

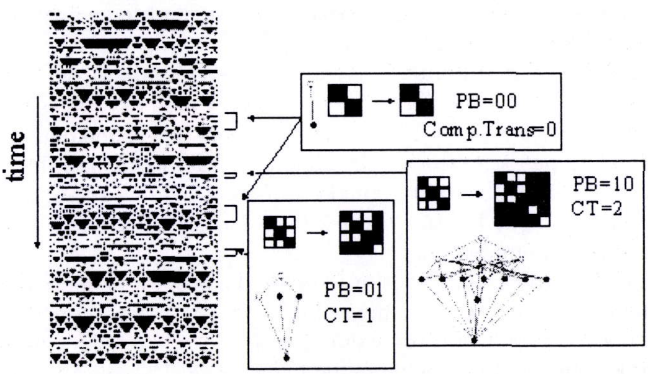


FIGURE 7. It shows time development of an elementary cellular automaton (33), where a lattice used as semantics is perpetually changed due to the partial-all quantifier. PB represents a sequence of $(a_1(t), a_2(t))$, and the corresponding lattice is shown as a Hasse diagram.

Fig. 7 shows a typical time development of this model. In spite of a unique transition rule, thanks to perpetual transformation of a lattice, this system can show complex behavior, as if the transition rule was changed. It results from infant's eyes or partial-all quantifier. If one uses adult's eyes or all-quantifier instead of partial-all quantifier, a lattice cannot be changed along time. Because just a compatible transformation of the context, a lattice is isomorphic along time if it is constructed by adult's eyes. For this initial context, it keeps Boolean lattice.

Generated patterns are often perturbed and one can see the drastic change like a horizontal line. It results from autonomous perturbation due to the infant's partial survey in his own epistemic universe. Compared with a simple model of the thermal bath, the domain in which perturbation can be applied is always changed.

4. Conclusion

Consciousness exists in the interface of the growing universe in a universe. If one compares the inside and the outside of the growing universe to the expression and quolia, one can find the essence of the problem of consciousness. For example, if one feels pain, he can say "Ouch!" that is a particular expression of the qualia of pain. In this scheme, one generally imagines that qualia of the pain causes the expression of the pain, and that scientists at an objective stance can observe just expressions of the pain. That is why one concludes that science cannot approach what qualia is and/or the origin of consciousness. Is it easy to prove that the expression of the pain always follows qualia of a pain? One sometimes feels pain when another one tells him "Aren't you fine?" There is no definite causality, and there is only distinction between qualia and pain. One, therefore, concludes that feeling quolia just appears when a definite distinction of quolia and its expression appears. The interface exists and it brings forth the distinction quolia and its expression or feeling the quolia.

The problem on the origin of consciousness is as same as the problem on the interface of the growing universe in a universe. It is very hard to express this aspect. On one hand, one finds difference of logical criteria between inside and outside, and on the other hand one has to pay attention to the contact of the inside and the outside. It is known that the mixture of different logical criteria entails to a contradiction in a set theory. Against this fact, one has to construct the interface of different logical criteria, which cannot falls to a contradiction.

In front of this problem, one has to focus on the proof of a contradiction in a set theory, for example, the diagonal argument. It is clear to see that a contradiction results from surveying the wholeness of infinite universe. It looks like fallacy because infinity is defined by impossibility of surveying in the finite sense. The question arises whether a contradiction by the diagonal argument is true contradiction or not. If one takes the sense of finite things, one cannot survey a whole universe. It makes us the sense of finite things called infant's eyes.

We here propose two kinds of formal infant's eyes, dynamic quantifier and partial-all quantifier. Especially, partial-all quantifier mimics infant's eyes. We prescribe infant's eyes as how to survey a universe from the growing universe. Then first we define a universe in a set theory. It is expressed as a formal concept lattice. A universe is recognized as the universe, and then the universe is objective but epistemic. A universe is expressed as the infinite hierarchical epistemic universe, and that is expressed as a hierarchy of compatible formal contexts, and a hierarchy of isomorphic lattices. Given a universe, an infant surveys a universe by his own eyes. As a result, infant's conceptual universe in the form of a lattice is constructed. In our model, adult's objective eyes are expressed by all-quantifier, and infant's eyes are expressed by partial-all-quantifier. Only thanks to infant's eyes, generated lattices are changed in spite of compatible contexts.

An infant no longer surveys a whole universe, but just a part of a universe. It results in non-contradictory leaning, development and evolution. If one remembers that a contradiction is a particular expression of infinite consuming time in a term of a computation theory, partial survey leads to an efficient consuming time and makes saving of time-resources possible. The partial-all-quantifier may be an adequate device to save time-resources in terms of consuming time.

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