Method of Elastic Maps and Its Applications in Data Visualization and Data Modeling

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Abstract
Technology of data visualization and data modeling is suggested. The basic of the technology is original idea of *elastic net* and methods of its construction and application. A short review of relevant methods has been made. The methods proposed are illustrated by applying them to the real economical and sociological datasets.
Keywords: multidimensional data, dimension reduction, visualization, approximation, non-linear factor analysis

1 Introduction

We live in the epoch of exponential growth of information. Internet, Intranet nets, local computers contain a huge amount of information databases and their number grows continually. Often such databases become a kind of "information tombs" because of the difficulties with their analysis.

The basic property of the information is its multidimensionality. Rather than 2-3 a typical object in database has hundreds and thousands features. Because of this information loses its clearness and one can't represent the data in visual form by standard visualization means – graphs and diagrams.

In this paper a technology of visual representation of data structure is described in details. It turns out that many problems concerning data analysis could be solved, at least qualitatively, using visual two-dimensional (or three-dimensional) picture of data and laying on it additional relevant information. This data image should display cluster structures and different regularities in data.

The basic of the technology is original idea of *elastic net* – regular point approximation of some manifold that is put into the multidimensional space and has in a certain sense minimal energy. This manifold is an analogue of principal surface and serves as non-linear screen on what multidimensional data are projected.

Remarkable feature of the technology is its ability to work with and to fill gaps in data tables. Gaps are unknown or unreliable values of some features. It gives a possibility to predict plausibly values of unknown features by values of other ones. So it provides technology of constructing different prognosis systems and non-linear regressions.

The technology can be used by specialists in different fields. There are several examples of applying the method presented in the end of this paper.

International Journal of Computing Anticipatory Systems, Volume 12, 2002
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-9600262-6-8

2 Visualization of Multidimensional Data

As a rule, it is possible to present a database in the form of a big numerical table with "object-feature" structure. A row of such a table contains information about one object, and set of columns contains various numerical features of the object. Providing the set of the features is identical for all of the objects in the table, we may think of the table as a finite collection of points in R^M, where M is the number of features in the set.

Introducing Euclidean metric in the space, we get *geometrical metaphor* of the table of data. In this space close points correspond to the objects with similar properties.

Characteristics of such a cloud of multidimensional points correspond to the empirical laws that could be extracted from a dataset. There are a lot of methods to analyze the cloud. Most of them return as a result some numerical values.

Yet it would be very useful to have a possibility to "glance" over a collection of points to form a clear visual picture of data. Visual presentation of a new dataset helps choose an adequate instrument for further quantitative analysis, and promotes deeper insight in the results of investigation.

Recently a great amount of efforts in practical statistics are given to the methods of data visualization with the help of *dimension reduction*. The most understandable image of data can be obtained from its presentation on two-dimensional picture.

Actually, the method of dimension reduction is a classical approach, but its concrete implementation varies in many ways. Linear methods are most popular in statistics because of their clearness. But in many practical cases these methods are unsatisfactory so developing non-linear methods is really actual problem.

If the structure has higher dimension then three, then one must reconcile to distortions appearing from projecting to the space of lower dimension. On this venue there are several methods that allow to point out regions where mapping gives rise to severe distortions.

3 Methods of Data Visualization

At present several methodological approaches exist concerning the problem of data visualization. In this section we just point out to the three of them.

In the methods of *multidimensional scaling* mapping $P: R^M \to R^2$ is to be found that minimizes some functional calculated for the initial M-dimensional coordinates of the points and for the resulting 2-dimensional coordinates:

$$Q(x^{(1)}, x^{(2)} \ldots x^{(N)}, \hat{x}^{(1)}, \hat{x}^{(2)} \ldots \hat{x}^{(N)}) \to \min,$$

where $x^{(i)}$ is the radius-vector of the i-th point in R^M, $\hat{x}^{(i)}$ is the radius-vector of the same point in the resulting 2-dimensional space, N is the number of points. If the mapping P is assumed to be arbitrary, then functional Q is minimized by varying values of $\hat{x}^{(i)}$, $i = 1..N$, using procedures of gradient optimization.

In the methods of *factor analysis* it is necessarily to find such combinations (usually linear) of the initial coordinates that using them as new coordinate system would make possible to "model" coordinates of given points with allowable accuracy, i.e.

$$x^{(i)} = \sum_{j=1}^{k} \alpha_j^{(i)} f^{(j)} + q,$$

where $f^{(j)}$ is the factor, that actually is a vector in R^M, k is the number of factors, q is random variable that is not dependent on i.

Choosing type of the "description error" to be minimized one gets different variants of factor analysis. In some cases it is rational to project points of data onto the plane spanned by first two *principal components* – directions in the data space, where dispersion of the cloud of points is maximal.

A different ideology lies in the method of *Kohonen Self-Organized Maps* (SOM). In this method *a net of ordered nodes* is placed in the data space. It is common practice to use rectangular or hexagonal two-dimensional net. In the learning process positions of the nodes are adjusted. During one act of adjustment one of the points and the nearest node of the net are selected. Then this node make a small step toward the data point. Besides some of its neighboring nodes on the net are moved in the direction of the selected point. As a result we have a net of nodes that is placed in the multidimensional space and approximates the cloud of data points. In the regions where the data points concentrate the density of nodes is higher and vise versa. In addition the net tends to be "regular" and tries to reconstruct the form of the cloud.

After that the net is unfolded on a plane (it is possible because it was initially two-dimensional) and the data are visualized with the help of different methods (Sammon maps, Hinton diagrams etc., see Kohonen, 1996).

4 Constructing Elastic Net

4.1 Basic algorithm

Method of elastic maps, similar to SOM, for approximation of the cloud of data points uses an ordered system of nodes, that is placed in the multidimensional space.

Lets define *elastic net* as connected unordered graph $G(Y,\mathbf{E})$, where $Y = \{y^{(i)}, i=1..p\}$ denotes collection of graph nodes, and $\mathbf{E}=\{E^{(i)}, i=1..s\}$ is the collection of graph edges. Let's combine some of the adjacent edges in pairs $R^{(i)} = \{E^{(i)},E^{(k)}\}$ and denote by $\mathbf{R}=\{R^{(i)}, i=1..r\}$ the collection of *elementary ribs*.

Every edge $E^{(i)}$ has the beginning node $E^{(i)}(0)$ and the end node $E^{(i)}(1)$. Elementary rib is a pair of adjacent edges. It has beginning node $R^{(i)}(1)$, end node $R^{(i)}(2)$ and the central node $R^{(i)}(0)$ (see Fig. 1).

Figure 2 illustrates some examples of the graphs practically used. The first is a simple polyline, the second is planar rectangular grid, third is planar hexagonal grid, forth – non-planar graph whose nodes are arranged on the sphere (spherical grid), then a non-planar cubical grid, torus and hemisphere. Elementary ribs at these graphs are adjacent edges, that subtend a blunt angle.

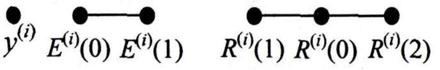


Fig 1: Node, edge and rib

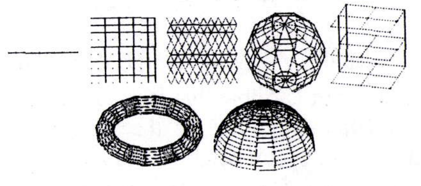


Fig 2: Elastic nets used in practice

Let's place nodes of the net in a multidimensional data space. This can be done in different ways, placing nodes randomly or placing nodes in a selected subspace. For example, it can be placed in the subspace spanned by first two or three principal components. In any case every node of the graph becomes a vector in R^M.

Then we define on the graph G energy function U that summarize energies of every node, edge and rib:

$U = U^{(Y)} + U^{(E)} + U^{(R)}$.

Let's divide the whole collection of data points into subcollections (called *taxons*) $K^{(i)}$, $i = 1 \ldots p$. Each of them contains data points for which node $y^{(i)}$ is the closest one:

$$K_i = \{x^{(j)} : \|x^{(j)} - y^{(i)}\| \underset{Y}{\rightarrow} \min\}.$$

Let's define

$$U^{(Y)} = \frac{1}{N}\sum_{i=1}^{p}\sum_{x^{(j)}\in K^{(i)}}\|x^{(j)} - y^{(i)}\|^2,$$

$$U^{(E)} = \sum_{i=1}^{s}\lambda_i\|E^{(i)}(1) - E^{(i)}(0)\|^2,$$

$$U^{(R)} = \sum_{i=1}^{r}\mu_i\|R^{(i)}(1) + R^{(i)}(2) - 2R^{(i)}(0)\|^2.$$

Actually $U^{(Y)}$ is the average square of distance between $y^{(i)}$ and data points in $K^{(i)}$, $U^{(E)}$ is the analogue of summary energy of elastic stretching and $U^{(R)}$ is the analogue of summary energy of elastic deformation of the net. We can imagine that every node is connected by elastic bonds to the closest data points and simultaneously to the adjacent nodes (see Fig. 3).

Values λ_i and μ_j are coefficient of stretching elasticity of every edge $E^{(i)}$ and coefficient of bending elasticity of every rib $R^{(j)}$. In a simple case we have

$\lambda_1 = \lambda_2 = \ldots = \lambda_s = \lambda(s)$, $\mu_1 = \mu_2 = \ldots = \mu_r = \mu(r)$.

Simplified consideration shows that, if we require that elastic energy of the net remains unchanged in case of finer net, then

$$\lambda = \lambda_0 s^{\frac{2-d}{d}}, \ \mu = \mu_0 r^{\frac{4-d}{d}} \qquad (*)$$

where d is the "dimension" of the net ($d = 1$ in the case of polyline, $d = 2$ in case of hexagonal, rectangular and spherical grids, $d = 3$ in case of cubical grid and so on).

Now we will find such positions of the nodes of graph that it will have minimal energy. It is the elastic net to be constructed that approximates the cloud of data points and has some regular properties. Minimization of term $U^{(Y)}$ gives approximation, using $U^{(E)}$ provides more or less evenness of the net and $U^{(R)}$ makes the net "smooth", preventing it from strong folding and "twisting".

To start let's consider the situation when we separated collection of data points to taxons $K^{(i)}$, $i = 1\ldots p$.

Let's denote

$$\Delta(x,y) = \begin{cases} 1, x = y \\ 0, x \neq y, \end{cases}$$

$$\Delta E^{ij} \equiv \Delta(E^{(i)}(1), y^{(j)}) - \Delta(E^{(i)}(2), y^{(j)}),$$

$$\Delta R^{ij} \equiv \Delta(R^{(i)}(3), y^{(j)}) + \Delta(R^{(i)}(2), y^{(j)}) - 2\Delta(R^{(i)}(1), y^{(j)}).$$

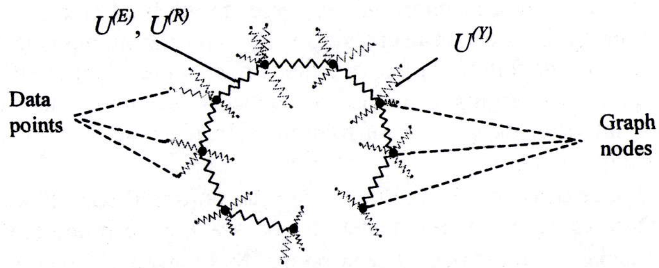


Fig. 3: Energy of elastic net

Then differentiation gives

$$\frac{1}{2}\frac{\partial D}{\partial y^{(j)}} = \sum_{k=1}^{p} y^{(k)}\left(\frac{n_j \delta_{jk}}{N} + e_{jk} + r_{jk}\right) - \frac{1}{N}\sum_{x^{(i)}\in K_j} x^{(i)} = 0,\ j = 1\ldots p,$$

where n_j is the number of points in $K^{(i)}$, $e_{jk} = \sum_{i=1}^{s} \lambda_i \Delta E^{ij} \Delta E^{ik}$, $r_{jk} = \sum_{i=1}^{r} \mu_i \Delta R^{ij} \Delta R^{ik}$ and the system of p *linear* equations to find new positions of nodes in multidimensional space $\{y^i, i=1\ldots p\}$:

$$\sum_{k=1}^{p} a_{jk} y^{(k)} = \frac{1}{N}\sum_{x^{(i)}\in K_j} x^{(i)}, \text{ where}$$

$$a_{jk} = \frac{n_j \delta_{jk}}{N} + e_{jk} + r_{jk},\ j = 1\ldots p, \qquad (**)$$

$$\delta_{jk} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

The values of e_{jk} and r_{jk} depend only on the structure of the net. If the structure does not change then they are constant, n_j depends on the separation of collection of data points to taxons.

Therefore to minimize the energy of graph U, the following algorithm is effective:

1. Place the net of nodes in multidimensional space.
2. Given nodes placement, separate collection of data points to subcollections $K^{(i)}$, $i = 1\ldots p$.
3. Given this separation, minimize graph energy U and calculate new positions of nodes.
4. Go to step 2.

It is evident that this algorithm converges to the final placement of nodes of the elastic net (energy U is a non-decreasing value, and the number of divisions of data points into taxons is finite). Moreover, theoretically the number of iterations of the algorithm before converging is finite. In practice this number may be unacceptable, therefore we interrupt the process of minimization if changes of values of U become less than a small number ε.

To choose constants λ_0 and μ_0 in (*) is a non-trivial task. If we make λ_0 and μ_0 very large then we get a "squeezed" net, its nodes will concentrate in the vicinity of the geometrical center of the cloud of data points. Making λ_0 and μ_0 too small leads to a very irregular net, it will be strongly twisted and its nodes will distribute in the space very unevenly.

Practice showed that it is useful to make λ_0 and μ_0 variable, using the procedure of "annealing". We trained nets in several epochs, starting from large values of λ_0, μ_0 (approximately 10^3) and finishing with small values ($\sim 10^{-1}$) (see Fig. 4). As a result we have a net, approximating the cloud of points, with rather evenly distributed nodes, arranged along a rather smooth d-dimensional surface. The process of "annealing" promises that the resulting net will realize the global minimum of energy U or rather close configuration.

4.2 SOM and elastic net

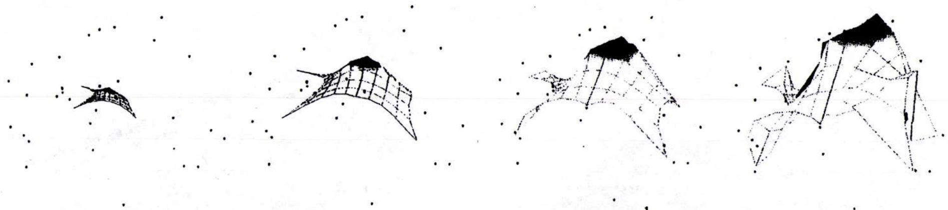
What are the features of elastic nets compared to the methods of SOM?

First, compared to the original SOM algorithm distribution of nodes in the space is more regular and more controllable during the teaching process. This fact can be used to apply the elastic net as a point approximation of some manifold (see next section). Mapping data points onto this manifold is more isometric than in the case of SOM.

Second, the form of resulting manifold does not depend strongly on the number of nodes m, unlikely to SOM. Magnifying m, we just make the point approximation more accurate.

Third, given values λ_i, μ_i, the result of net constructing is optimal because of the minimum of energy of the graph, unlike SOM, where the net is just a result of a stochastic process without optimality criterion.

Fourth, the speed of training of elastic net promises to be higher as compared to SOM. In addition the training process may be easily made highly parallel.



$\lambda_0, \mu_0 \approx 10^3$ $\lambda_0, \mu_0 \approx 10^2$ $\lambda_0, \mu_0 \approx 10^1$ $\lambda_0, \mu_0 \approx 10^{-1}$

Fig.4: Training elastic net in several epochs

5 Building Elastic Map

5.1 Piecewise linear map

Elastic map is a manifold constructed in a multidimensional space using an elastic net as point approximation.

Formally the task is as follows (consider two-dimensional case). We need to reconstruct vector-function $r = r(u,v)$ using its values in a definite set of points $\{y_i = r_i(u_i, v_i),\ i = 1\ldots p\}$. A pair of values u_i, v_i are internal coordinates of the nodes in R^2.

There are many possible ways to construct elastic map. One of the simplest one is to construct piecewise linear manifold. Let's show how to do it.

Let's introduce a *triangulation* of the graph. We define a collection of elementary simplexes $\{T^{(k)}\}$, $k = 1\ldots t$. In case of polyline ($d = 1$) they are simple linear segments. In case of $d = 2$ they are triangles. And in case of $d = 3$ they are tetrahedrons. For clarity we limit ourselves with the case of $d = 2$.

Let's define internal coordinates of the point of the map (u and v in case of $d = 2$). Then using the triangulation we can find the simplex, for which the selected point belongs (it is the closest one). Let's denote numbers of nodes that are corners of this triangle as i_1, i_2, i_3. We can calculate coordinates α, β of the selected point relatively to these nodes:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_{i1} \\ v_{i1} \end{pmatrix} + \alpha \begin{pmatrix} u_{i2} - u_{i1} \\ v_{i2} - v_{i1} \end{pmatrix} + \beta \begin{pmatrix} u_{i3} - u_{i1} \\ v_{i3} - v_{i1} \end{pmatrix}.$$

Then we have equations for finding α and β:

$$\alpha\begin{pmatrix} u_{i2}-u_{i1} \\ v_{i2}-v_{i1} \end{pmatrix}+\beta\begin{pmatrix} u_{i3}-u_{i1} \\ v_{i3}-v_{i1} \end{pmatrix}=\begin{pmatrix} u-u_{i1} \\ v-v_{i1} \end{pmatrix}.$$

Solving it we find the value of the vector-function:

$$r(u,v)=y^{(i1)}+\alpha\left(y^{(i2)}-y^{(i1)}\right)+\beta\left(y^{(i3)}-y^{(i1)}\right).$$

As a result we get faceted surface, an example is shown in Fig. 5.

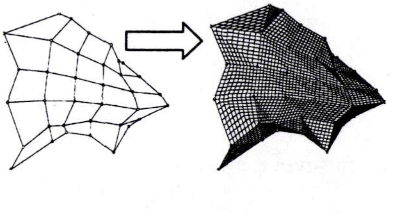


Fig. 5: Piecewise linear manifold

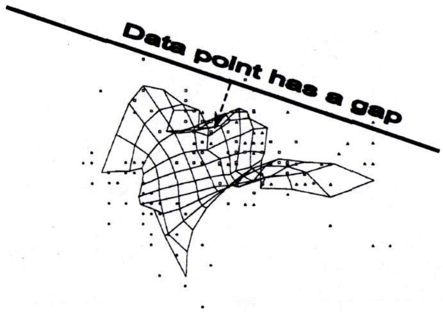


Fig. 6: Projecting data points having gaps

Constructing this piecewise linear manifold is the least labor-intensive method. Although we may use more intricate approaches (like multidimensional Karleman formula). Then as a result we get a continuous smooth surface and all nodes lie on it.

5.2 Projecting Data Points onto Map

In order to have a possibility to analyze data points on a plane, we need to project them onto the constructed two-dimensional manifold. Method of SOM makes use of piecewise constant projecting. It means that every point is transferred to the nearest node.

Because of the regularity of elastic net it is reasonable to apply piecewise linear projecting. We will project a point in the closest point of the *map* (unlikely to SOM where projecting is performed in the closest node of the net).

Lets introduce a *distance from the point to the segment of line*. We will calculate it in this way:

1. Project orthogonally on the line to which the segment belongs. If the projection is on the segment, then the result is the distance to the projection.

2. Otherwise the result is the distance to the closest corner of the segment.

Distance from the point to the triangle is calculated analogously:

1. Project orthogonally on the plane to which the triangle belongs. If the projection is on the triangle, then the result is the distance to the projection.

2. Otherwise the result is the distance to the closest side of the triangle (every of them is a segment).

Distance from the point to the tetrahedron:

1. Project orthogonally in the three-dimensional subspace to which the *tetrahedron* belongs. If the projection is in the tetrahedron, then the result is the distance to the projection.

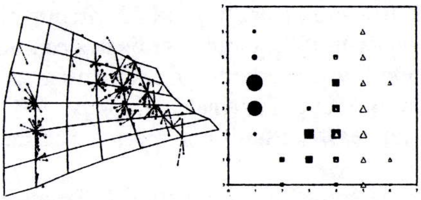
2. Otherwise the result is the distance to the closest side of the tetrahedron (every of them is a triangle).

Analogously we can find the distance from the point to any *k*-dimensional simplex.

One-dimensional map consists of line segments, so the closest point of the map is the closest point of the closest segment. Analogously the closest point of the two-dimensional map is the closest point of the closest triangle and so on.

Note here that in the proposed way of projecting, there exist whole regions of space, from that every point is projected in a node. It may be considered as a principal drawback of such projecting. In case of one-dimensional map other ways of projecting are known, they are free from this defect (see Gorban, Rossiev, 1999).

Piecewise linear projecting gives a more detailed picture of data than in the case of using Hinton diagrams (see Fig. 7).


a)

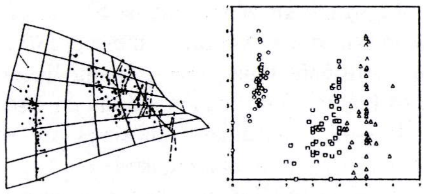

b)

Fig. 7: Comparing piecewise constant (a) and piecewise linear (b) projecting.

5.3 Extrapolation and Interpolation

The constructed manifold is principally bound. It lies in a cloud of points and there are relatively many points such that the closest point of the map is on the border of the manifold. Such "boundary effects" distort the picture of data points considerably. Thus we need to know how to extrapolate the map on its neighborhood. It is desirable for the manifold to slightly extend beyond the cloud of points.

The simplest case is the extrapolation of a piecewise linear map. The manifold is extended in linear fashion by gluing additional simplexes to its borders.

Analogously if we wish to make the grid finer then we can apply the procedure of interpolation of the map.

5.4 Visualizing data points with gaps

In the given methods it is easy to manage with data points that have gaps – missing or unreliable values of some features. Actually real data tables almost always contain gaps. Distribution of gaps is sometimes crucial for using standard methods of statistics.

We can consider a data point that has value of the *i*-th feature unknown as a line parallel to the *i*-th coordinate axis. Then we can calculate for the line the closest point of the map just as we did it in a common situation (see Fig. 6).

6 Using Elastic Map

6.1 Colorings

Let's emphasize here that unlike the SOM technology and methods of multidimensional scaling we get as a result a low-dimensional bounded manifold put in multidimensional feature space. Every point of the manifold has from one hand M coordinates as a point in feature space, from other hand d "internal" coordinates (d – dimension of the manifold).

Thus we can show on the manifold values of any functional in R^M in the points of the manifold. What are the functionals that may be useful for coloring the map?

First, the simplest case is the use of values of coordinates. It gives M variants of the map colorings. Not all of these colorings are really useful. Some coordinates can be more significant than others. Using linear and non-linear methods of evaluating the level of feature significance we can choose most informative coordinate colorings.

If some coordinate colorings are similar (at least partially) it means that the corresponding features are correlated.

Second, on the map we can show values of some linear functional. Practice showed that it is very useful to illustrate on the map results of applying such traditional methods as linear discriminant and regression analysis. This gives an instrument for visual evaluation of the quality of applying these methods.

Third, on the map we can show values of more intricate functionals as an non-parametric estimate of density of data points distribution or density of some subcollection of points. It really helps to visually solve the task of cluster analysis or compare the results of applying numerical procedures of cluster analysis with real data structure.

6.2 Non-linear regression (plausible gaps filling)

The map itself together with the procedure of projecting onto the map gives a possibility to construct regression of some coordinates of data space from the others.

Consider the two-dimensional case. In R^M we have map M and its vector-function $r = r(u,v)$: $R^2 \to R^M$, where u, v are internal coordinates, given in the finite connected domain in R^2. Besides we have mapping $P(x)$: $R^M \to R^2$ that assigns two coordinates u_x,

v_x in the domain to point x from R^M. We could see that the mapping also applicable to the point x with some unknown coordinates.

If we consider the dependent coordinate of point x to be unknown then we can "recover" it by projecting x onto the map, getting internal coordinates u_x and v_x , and read the value in vector $r(u_x,v_x)$.

6.3 Iterative error mapping

It is evident that using two-dimensional manifold as a model of highly multidimensional data distribution leads to considerable error of approximation.

There are several ways to diminish the error.

First is to increase the number of nodes. *Second* is to make the map "softer" making values of elasticity coefficients lower. In both ways the map with better approximation becomes more "twisted" in the data space. It is a rather common situation in the methods of data approximation and it could be called "accuracy-regularity dilemma".

Other way is to construct rather regular ("rigid") map and to build a new space named "space of errors" with a collection of points, each of them representing the vector of error of mapping by the first constructed map. So every point in the space has a vector equal to

$$x_{err}^{(i)} = x^{(i)} - r(u_{x^{(i)}}, v_{x^{(i)}}),$$

where vector $r(u_x,v_x)$ is the projection $P(x)$ of data point x onto the map.

In the new space we can construct another regular map, getting regular model of errors of first map. If the accuracy of this model is not satisfactory then we can make new iteration and to continue to model errors until the quality becomes allowable. In such a manner we can model data points distribution with the error as small as desired.

So we have *the first* map that models the data distribution itself, *the second* map that models errors of the first model, *the third* map models errors of the second model and so on.

Let's denote mapping onto the first map in the data space by $P(x)$, mapping in the space of errors onto the second map by $P_2(x)$ and so on. So every point x in the initial data space is modeled by the vector

$$\tilde{x} = P(x) + P_2(x - P(x)) + P_3(x - P(x) - P_2(x - P(x))) + ...$$

If x is a vector with some coordinates "unknown", then $\tilde{x}$ is the vector with no gaps in coordinates.

This technology allows to make program systems with the ability to prediction just like they do with neural networks, but unlike the latter there is the possibility to deal with the data tables that have unknown cells (gaps), and the intrinsic possibility to make data visualization.

7 Illustrations of the method applications

7.1 Visualization of economical indicators

Let's illustrate the proposed methods by applying them to the visualization of the table of economical indicators of the biggest Russian companies. The table was taken from the russian magazine "Expert". The files of data were retrieved from the official site of the magazine: http://www.expert.ru. The table contains information of some economical characteristics of two hundred biggest companies in Russia, sorted in the order of decreasing of their gross production output. The following fields (only part of them are independent, others are calculated by explicit formula) were in the table:

1) Name of the company
2) Geographic region
3) Branch of industry the company belongs to
4) Gross production output in 1998
5) Gross production output in 1997
6) Rate of the growth
7) Gross production output in 1998, recalculated in dollar equivalent
8) Balance profit
9) Profit after taxation
10) Profitability
11) Number of workers
12) Efficiency of production

In the work of Shumsky, 1998 traditional Kohonen's self-organizing maps and Hintons diagrams were used to visualize the same table but containig information for 1997 year. For data space coordinates it was suggested to use the relations of some independent features.

We enlarged the dimension of data space up to five and as a result obtained the following set of independent indicators:

N	Indicator	Description
1	LG_VO1998	Logarithm of gross production output in 1998
2	RATE	Gross production output in 1998 / Gross production output in 1997
3	PROFIT_BAL	Balance profit / Gross production output in 1998
4	PROFIT_TAX	Company profit after taxation / Gross production output in 1998
5	PRODUCTIV	Profit after taxation / Number of workers

The resulting dataset contains two hundred records and five fields. Parts of the records contain incomplete information (there are gaps in some cells).

The data is normalized with the formula

$$\tilde{x}_i = th\left(\frac{x_i - M}{\sqrt{D}}\right),$$

where $\tilde{x}_i, x_i\, M,\, D$ – are the new and old value of the coordinate, mean value and dispersion respectively.

The map visualizing the data was constructed according to the algorithm of elastic map described above. The method of "annealing" was used to find the local minimum of the map functional. Parameters μ_0 and λ_0 were changing slowly (so after each change the map could reach the closest local minimum) from μ_0 = 5000, λ_0 = 5000 to μ_0 = 0.01, λ_0 = 0.01.

After constructing the elastic map the data points were projected from the multidimensional data space onto the map with the use of algorithm of piecewise linear projecting in the closest point of the map.

As an illustration of analysis of economical data coordinate below we give colorings of the map (only one example of five possible coordinate colorings are given). In addition density of data points distribution is showed. The data points have different forms corresponding to the industry the company belongs. It helps to get insight in economical situation in Russia.

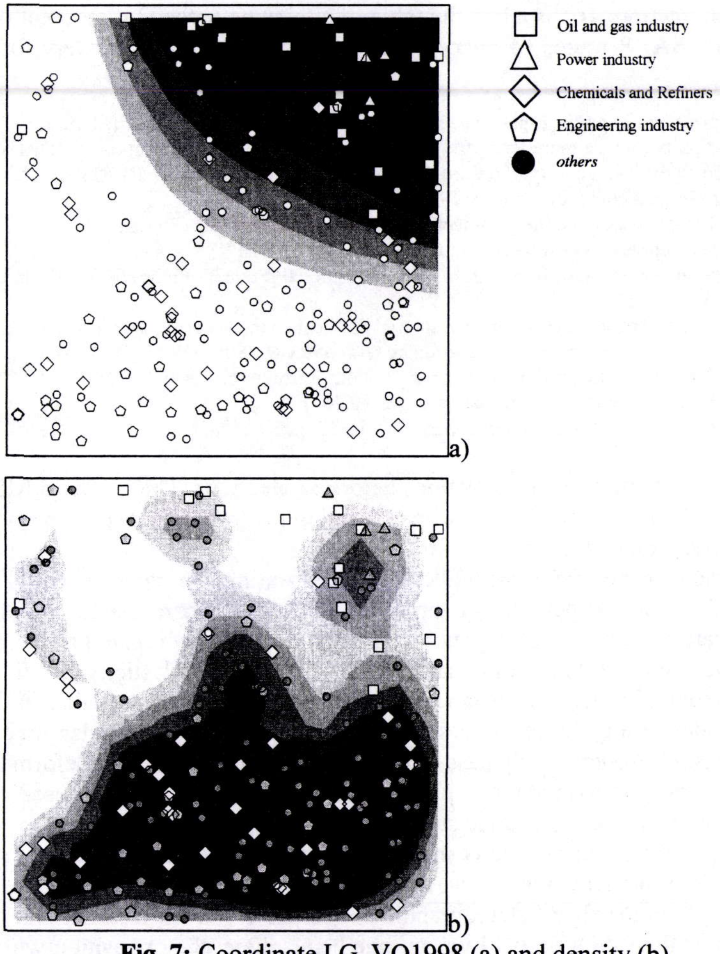


Fig. 7: Coordinate LG_VO1998 (a) and density (b) colorings of the map of economical indicators

The simplest conclusions that the colorings can provide immediately are, for example, that companies with high gross output are not the same as companies with high growth rate. The largest companies belong to oil-gas and power industry. The companies of oil-gas industry are separated for two subclasses with considerably different profitability. Coordinate colorings of PROFIT_BAL, PROFIT_TAX and PRODUCTIV are similar, this points out to the correlation of the last three indicators. At the same time distinctions in colorings allow to distinguish the companies which drop out of the correlation dependence.

7.2 Political forecast

In this section we visualize the table of situations preceding moment of president election in USA. Features of every situation are binary coded answers for 12 simple questions:

1. Has the presidential party (P-party) been in power for more than one term? (MORE1)
2. Did the P-party receive more than 50% of the popular vote in the last election? (MORE50)
3. Was there significant activity of a third party during the election year? (THIRD)
4. Was there serious competition in the P-party primaries? (CONC)
5. Was the P-party candidate the president at the time of the election? (PREZ)
6. Was there a depression or recession in the election year? (DEPR)
7. Was there an average annual growth in the gross national product of more than 2.1% in the last term? (VAL2.1)
8. Did the P-party president make any substantial political changes during his term? (CHANG)
9. Did significant social tension exist during the term of the P-party? (WAVE)
10. Was the P-party administration guilty of any serious mistakes or scandals? (MIST)
11. Was the P-party candidate a national hero? (R_HERO)
12. Was the O-party candidate a national hero? (O_HERO)

Resulting table contains situations before 33 elections. The last field RES contains information about which party has won (1 – ruling party, 2 – opposition party) and gives classification of the elections for 2 classes.

All data points from the collection representing the table actually lie on the vertexes of the unit hypercube. It turns out that using sphere elastic net is a method more accurate for approximating such point distribution as compared to the rectangular grid. Practice showed that with equal number of nodes and elasticity coefficients value of residual sum of squares of distance to the map is considerably smaller in this case. It is quite evident thing (it is impossible to cover cube by rectangular piece of plane without folds, but sphere easily pass through every its vertex without deformation).

So for the approximation a spherical two-dimensional grid was used. To present the sphere on the plane we applied stereographic projection.

In fig.8a the resulting grid is shown in the three-dimensional subspace spanned by the first three principal components. It is apparent that the grid approximates the points rather tightly. On the fig.8b stereographic projection is shown. It turned out that almost all winners of the ruling party "live" on one hemisphere, the opposition winners "live" on other.

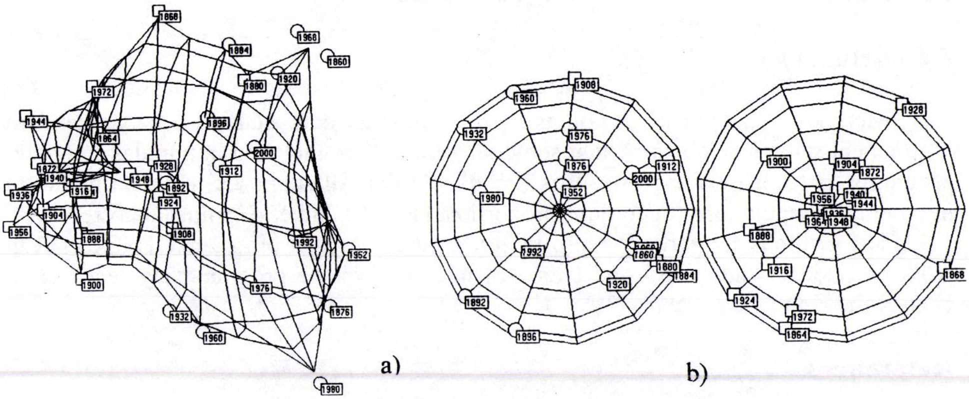


Fig. 8: An elections' map for political forecast

In fig.9 one relevant for classification coordinate coloring is shown. It is black and white coloring because the features are binary. It is apparent the feature CONC is useful for making forecast.

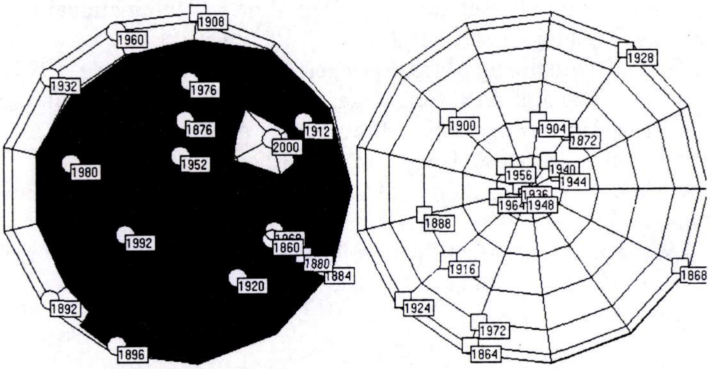


Fig. 9: An example of coloring (by coordinate CONC) of spherical map

We apply the method of linear regression to find the explicit form of dependence of field RES on all other fields. Values of resulting linear function also could be visualized using technique of map colorings. Black and white color corresponds to the areas of reliable prognosis, gray tint correspond to the areas of uncertainty.

Finally we applied methods of visualization to the "transposed" table. In this table features become objects and vice versa. So close in a new space objects correspond to directly correlated features, distant objects correspond to inversely correlated ones. RES feature is also included in this space. Here one can see that CONC feature is mostly correlated with RES, PREZ feature is inversely correlated, THIRD, MIST, O_HERO, DEPR, CHANGES, WAVE features group together.

8 Conclusion

Methods of data visualization as a part of primary data analysis become standard in practical statistics. We think that some of them will be included in popular computer programs for statistical analysis. It is remarkable that all supposed methods require a number of computational operations proportional to the number of points analyzed.

The methods are partially realized in freeware computer program ViDaExpert 1.0, working under Windows'95 and later. All interested in the program may feel free to contact the authors by e-mail.

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