Regulation of Great Lakes Reservoirs System by a Neuro-Fuzzy Optimization Model

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Abstract

Great Lakes reservoirs system is a complex natural system containing a large percentage of the fresh water resources of the world. It is located in Canada and U.S.A. serving about 40 Million people and is managed by an International Joint Commission made up of engineers from these two countries. Management of this system is still based on rule curves and much more could be done to improve this situation. The system is complex also due to highly differing scales of variables, nonlinearity, uncertainty, and dimensionality. An implicit stochastic method is applied using successive approximation optimization to obtain the optimal state and control variables of the reservoirs using the 90 years monthly data. However, when simulating the derived policies a re-optimization in each time period is needed due to inequalities and nonlinear relations existing among variables. The optimal values obtained from simulation are used as input-output data in training an Adaptive-Network-based Fuzzy Inference System (ANFIS) model for one of the months that required a non-constant release policy. ANFIS derives the general operating rules of the reservoirs in the form of fuzzy "if-then" rules. The parameters of a Sugeno-type Fuzzy Inference system (FIS) are optimized through an Artificial Neural Network (ANN) using back-propagation learning algorithm and least square method. The model of our system is anticipatory in nature given the fact that we base our current decision from the expectation of a future state. In this paper, we discuss the various aspects related to our implementation and the computational issues. Simulation of operating policies obtained from the ANFIS model, and comparison of its performance with other policies shows the potential capability of the proposed approach to tackle optimal operations of the system.

Keywords: Optimization, Successive approximation, Simulation, Anticipatory, ANFIS

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1 Introduction

The Great Lakes consist of Lakes Superior, Michigan-Huron, St.Clair and Ontario. They form a chain of natural reservoirs with each one draining to the next. Lake Superior is the uppermost and the largest lake while Lake Ontario is the lowest lake in the chain. The schematic of the system is shown at Figure 1. The fluctuation of the water levels and the releases in this system continues to be of major concern to several interest groups. Recently, an international study was conducted to consider additional control structures for the three middle lakes. The study was done for the International Joint Commission for which the first author provided a network flow algorithm requested by Environment Canada (International Joint Commission, 1993). The study concluded that the additional structures were not highly beneficial. However much can be improved in terms of the methodology used thus providing motivation for the work presented here.

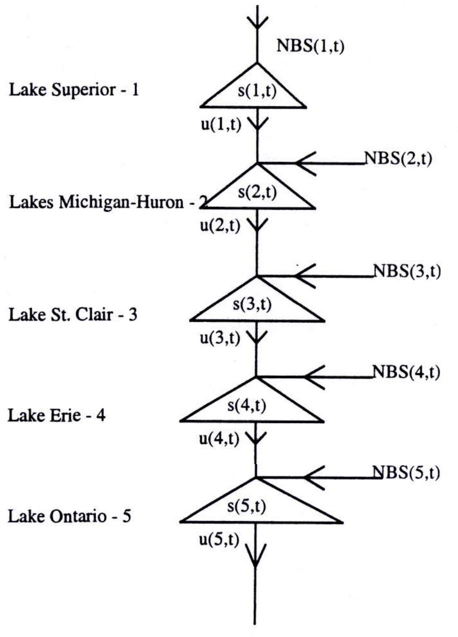

Fig. 1: Schematic of Great Lakes System showing Net Basin Supply NBS, Lake Storage (s) and Outflow (u), for each lake at time t

In this paper, the usefulness of the formulation of the Neuro-Fuzzy model is examined in its application to the Great Lakes levels regulation problem. The various constraints and operating conditions critically test the proposed model's ability to produce results that would compare favorably with results obtained from a system simulation under real-world conditions.

The formulation developed thus far considers deterministic control. However, stochastic inflows are considered in simulation with the actual discharge requirements for the three middle lakes based on highly nonlinear functions of the upstream and downstream lake elevations. Depending on how the hydrologic information is treated, the approaches to the solution of the reservoir operation problem falls into either of three general classes, as described by Mejia et al. (1974). These are the simulation approach, the implicit stochastic approach, and the explicit stochastic approach. In the simulation approach, the consequences of alternative operating strategies for multi-reservoir systems can be examined. Synthetic stream flows generated using complex inflow model structures together with sophisticated release policies can also be accommodated in a simulation analysis. However, simulation on its own is not recommended for the analysis of multi-reservoir systems where the objective is to develop the best operating strategies, given specified objectives.

In the implicit stochastic approach, as presented by Young (1967), for example, Monte Carlo generated flow sequences and deterministic dynamic programming are used to develop optimal release strategies for a reservoir system. However, because this approach depends on the simulation of large numbers of sequences, the approach is not very practical for large problems. Moreover, deriving optimal policies on the basis of the simulated results becomes a tedious exercise in statistical regression (Unny et al., 1981), and may not yield useful results in any case.

In the explicit stochastic optimization of multi-reservoir systems the probability distributions of the inflows are considered instead of inflow sequences. However, very few successful techniques have been advanced based on this approach. Those that have been proposed usually incorporate some approximation scheme to deal with the dimensionality problem, as for instance, the aggregation/decomposition technique using dynamic programming (A/D - DP) of Turgeon (1980, 1981), Romanowicz (1983), and Ponnambalam and Adams (1996). See Trezos and Yeh (1987) and Braga Jr. et al. (1990) for alternative techniques that are based on stochastic dynamic programming. Apart from these approximate techniques, the numerical stochastic dynamic programming solution to multi-reservoir control problems is generally limited to a consideration of two or three state variables. Even in these cases, factors like the cross-correlation of inflows to the reservoir system have to be neglected to keep the dimensions of the DP problem at a manageable level.

Because of stochastic inflows, nonlinear constraints and multiple reservoirs, using a large-scale nonlinear optimization method that implement all the needed details of the system will be a very difficult task. Therefore, in this study, we used a combination of simulation and implicit stochastic method but with a successive approximation optimization model. Then, ANFIS based rules are used to formulate a release policy. Also, a novel feature is to show that ANFIS based fuzzy rules can represent reasonably well the nonlinear relations in the middle lakes. These nonlinear relations are in addition to the nonlinearity introduced in the system from lower and upper bounds on the system state variables. It is well known that ANFIS rules would work well only if the training data represent optimality. In order to derive this training data, we used successive approximation optimization. This is an implicit optimization method and hence is flexible in terms of possible objective functions and constraints.

On the other hand, it is a time consuming process unlike explicit stochastic optimizations carried out in Fletcher and Ponnambalam (1998). Notice here that since the decision making process here emulates the anticipation process of biological systems (in terms of linguistic representation of knowledge and learning capabilities), we might refer to our model as an anticipatory one. Our system under ANFIS would behave indeed as a set of nested predictive models with main task of representing faithfully the nonlinear mappings pertaining to the middle lakes dynamic representation.

2 Proposed Methodology

The proposed approach uses a combination of steps involving simulation, optimization, curve fitting, and ANFIS adaptive inferencing. Each of the steps will be explained briefly in the next few sections. The overall objective is that given the system objective function and constraints (i) generate a reasonably good optimal policy, (ii) generate optimal storage value and releases for each reservoir from simulation, (iii) determine release rules including the ANFIS based release rules, and (iv) evaluate the policy with simulation. These various steps are executed using the following procedures:

2-1 Successive Approximation Optimization

The objective of successive approximation is to optimize a single variable while keeping the other variables fixed and then carry out similar optimization until all the variables have been considered. Repeat this step until convergence occurs. In order to reduce computing time, the optimized variable is discretized between its lower and upper bounds. The performance function is evaluated for all the discrete values of the optimized variable. A cubic spline function is fitted with the performance value as the dependent variable and the optimized variable as the independent variable. The optimal value for the variable is determined by minimizing (or maximizing) this function. Because of the nature of this optimization method, the solution obtained is expected to be only suboptimal. The simulation stage, described next, is used to derive the performance function.

2-2 Simulation

Using either generated or historic inflows (which are random input variables), system constraints that include mass balance or continuity constraints, lower and upper bounds, and any nonlinear constraints relating releases and storages, the system dynamics is derived and simulated to produce the traces of system storages and releases. The initial state for the state variables can be either the historic value or, like most ergodic systems, can be any arbitrary value within allowed bounds. The objective function, described next, is calculated using these values. Simulation can be carried out using any release rules. In the result section, we use three different release rules and compare their performances.

2-3 Objective Function

The objective function can be either maximization of benefits or minimization of costs or minimization of sum of squared deviation of system variables from their targets. In this paper, the latter is used as following:

$$\min_{u_i^t} \sum_{i=1}^{5} \sum_{t=1}^{T} (s_i^t - ST_i^t)^2 + (u_i^t - UT_i^t)^2 \tag{1}$$

where, for a system with *5* reservoirs and *T* periods, u_i^t is the unknown deterministic release during time *t*, s_i^t is the storage volume for reservoir *i* at the beginning of time *t*, and ST_i^t is the desired target for the storage of reservoir at the beginning of time t, and similarly UT_i^t is the corresponding release target. However, this objective function, while commonly used in academic work, does not consider a very important practical objective, which is to have smoothness in releases and storages from period to period. Therefore, we modified the objective function in (1) to be as following.

$$\min_{u_i^t} \sum_{i=1}^{5} \sum_{t=1}^{T} \{(s_i^t - ST_i^t)^2 + (u_i^t - UT_i^t)^2 + \lambda [(s_i^t - s_i^{t-1})^2 + (u_i^t - u_i^{t-1})^2]\} \tag{2}$$

Depending on the value of λ, the chosen optimal releases will smoothen the differences in releases from period to period. See the results section for further discussion.

3 Fuzzy Logic Controllers

Systems featuring complexities and ambiguities have been understood and unconsciously addressed by humans since their early days of existence. In fact, humans have learned to make decisions even in the absence of clearly defined processes. This is carried out based on expertise and general knowledge acquired of the system. Some of humans' actions can be accomplished very effectively using a well-structured set of "IF-THEN" rules. Fuzzy set theory has been developed recently to mimic this powerful capability and to design systems that can deal effectively with complex processes. Elements of a fuzzy set are mapped to a universe of membership values using a function-theoretic form. The function maps elements of a fuzzy set into a real value belonging to the interval between 0 and 1. Fuzzy set theory is very useful in modelling complex and imprecise systems. Fuzzy logic controllers (FLC) are designed on the premises of fuzzy set theory. The typical FLC structure is composed of several modules and is shown in Figure 2.

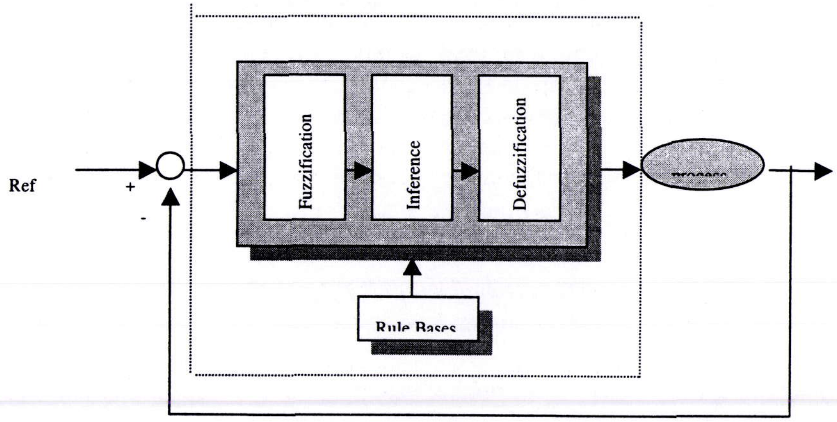

Fig. 2: Structure of FLC

The fuzzification process maps the input variables into a suitable range that corresponds to the universe of discourse used in the control rule base. Rule base is also called knowledge base that represents the experiences or the knowledge of experts about control targets. It is a set of linguistic statement in the form of "IF ...AND...AND...THEN...ELSE". These statements are normally called rules. The linguistic input variables to the FLC are typically error, error integral, and error rate of the system relative to some desired value. There are three different inference engine models, which are commonly used such as Mamdani inference model, Takagi-Sugeno inference model and Tuskamoto inference model. In Takagi-Sugeno model (Takagi and Sugeno, 1985), used in this study, the consequent of a rule is a function of input linguistic variables as follows:

$$R^1: \text{ IF } x \text{ is } A_1 \text{ AND } y \text{ is } B_1, \text{ THEN } z \text{ is } f_1(x, y)$$

$$R^2: \text{ IF } x \text{ is } A_2 \text{ AND } y \text{ is } B_2, \text{ THEN } z \text{ is } f_2(x, y)$$

The inferred values of the control action are $\alpha_1 f_1(x, y)$ and $\alpha_2 f_2(x, y)$. Then, the final inferred control action as:

$$z_0 = \frac{\alpha_1 f_1(x, y) + \alpha_2 f_2(x, y) + ... + \alpha_i f_i(x, y)}{\alpha_1 + \alpha_2 + ... + \alpha_i} \tag{3}$$

Since in this type of inference engine the final control action is crisp, we do not need to use defuzzification stage.

In our optimization process, we have used implicit stochastic method where the original deterministic optimization is solved repeatedly for each sample of the stochastic variables called the scenario. Although optimal solutions are available for each of these scenarios, it is not clear how to combine these results so that a simple releases policy can be determined. One of the simplest methods is to use multiple linear regression to relate the decision variable with the observed input variables. For highly nonlinear problems and for problems with high uncertainty, this method may not yield satisfactory results. Therefore, we propose a Fuzzy Inference System (FIS) to handle such a problem using fuzzy if-then rules as follows:

If reservoir storage is medium and inflow is low,
release = ***a*****reservoir storage +* ***b*****inflow +* ***c***

In this FIS, the premise variables are storage at the beginning of the period and the net inflow into the lake, and the consequent variable is the release from the corresponding lake. The parameters ***a, b***, and ***c*** must be estimated using available data or operator experience. Also, there is no systematic way to know what type and shape of membership functions of premise variables have the best performance in a defined FIS. An efficient way for doing this is using an artificial neural networks (ANNs) model trained by input-output data. This method is called Adaptive-Network-based Fuzzy Inference System (ANFIS) which uses a neural network with a hybrid back-propagation learning algorithm and least square method for tuning the membership functions of the premise variables and parameters estimation of consequent part.

3.1 Neural-Fuzzy Controllers (NFC)

ANNs perform two major functions, learning and recall. Learning is the process of adapting the connection weights or structure in an ANN to produce the desired output in response to a stimulus presented to the input buffer. Recall is the process of accepting an input stimulus and producing an output response in accordance with the network weight structure. Broadly speaking, there are two kinds of learning in ANNs, parameter learning that is concerned with updating of connecting weights and structure learning that focuses on the change in the network structure. The back-propagation learning algorithm (BPL) is one of the important historical developments in neural networks. This learning algorithm is applied to multi-layers feed-forward network consisting of processing neurons with continuous differentiable activation functions. For a given input-output pair (x^k, d^k), the BPL performs two phases of data flow. First, the input pattern x^k is propagated from the input layer to the output layer and, as a result of this forward flow of data, it produces the actual output y^k. Then the error signal, resulting from the difference d^k and y^k, are back-propagated from the output layer to the previous layers to update their weights.

Fuzzy system and neural networks are both numerical model-free estimators and dynamical systems. They share the ability to improve the intelligence of systems working in uncertain, imprecise, and noisy environments. Both have an advantage over traditional statistical estimation and adaptive control approaches to function estimation, without requiring a mathematical modeling on the system. At the same time, because of inside structure and methodology, there are also significant differences between them. Fuzzy logic and neural networks are complementary technologies. Neural networks extract information from systems to be learned or controlled while fuzzy logic techniques most often use verbal and linguistic information from experts. By combining the explicit knowledge representation of fuzzy logic with the learning power of neural networks, we get Neural–fuzzy networks.

NFC, based on a fusion of ideas from fuzzy control and neural networks, has the advantages of both neural networks (e.g., learning abilities, optimization abilities, and connectionist structures) and fuzzy control systems (e.g., human like "IF-THEN" rule thinking and ease of incorporating expert knowledge). In this way, we can bring the low-level learning and computational power of the neural networks to fuzzy control systems and also provide the high-level, human like "IF-THEN" rule thinking and reasoning of fuzzy control system to neural networks. ANFIS is a successful sample of NFCs, which is using Takagi-Sugeno fuzzy inference engine model with a six layers feed-forward network. Figure 3 shows the structure of ANFIS including two rules, two inputs x_1 (reservoir storage), x_2 (inflow into reservoir), and one output y (release from reservoir).

From the structure in Figure 3, we can see that an ANFIS is functionally equivalent to a fuzzy control system with Takagi-Sugeno model. ANFIS uses a hybrid learning algorithm. In this algorithm, the premise parameters are identified by the back-propagation and consequent parameters by least square method. In this paper, we will use an ANFIS model to determine the release policy when reservoir releases are not a constant value over the simulation horizon.

4 Case Study and Results

The underlying objective of Great Lakes Water Level Management is that regulation measures implemented should not cause undue hardship to any interest group, and that these measures should produce a net benefit for the people and resources of the Great Lakes-St. Lawrence River Basin. Five broad interest groups have been identified in the 1993 Reference Study. These are (1) Commercial Navigation (2) Riparian or shore property (3) Hydro-power (4) Recreational boating and (5) Environment. Value functions are associated with each interest group. These value functions vary not only with the particular interest group, but also with the season (month) of the year. These functions determine a value or penalty for a specific lake

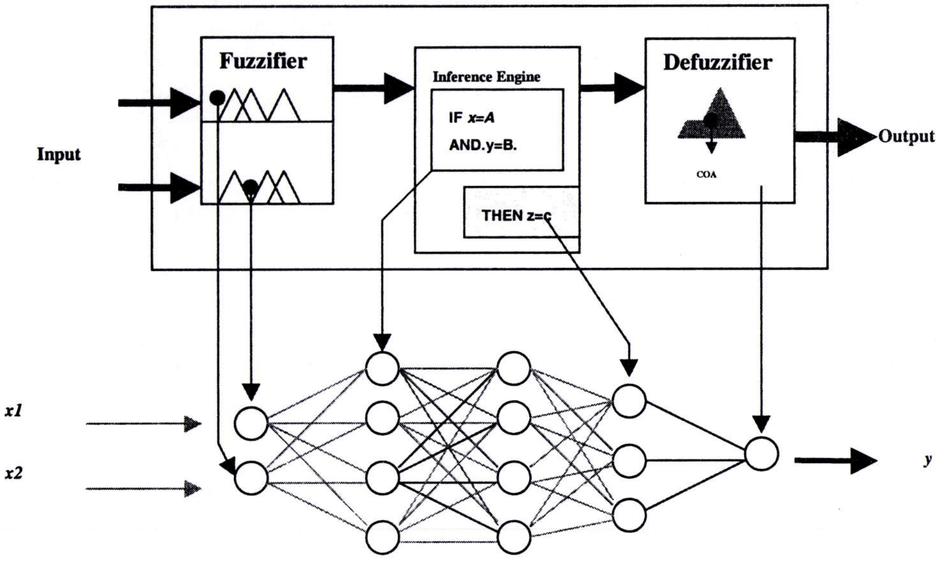

Fig. 3: ANFIS structure

water level or outflow based on the interest group's preference. These functions range from zero, the most preferred condition, to one, the least preferred condition. These functions are then combined in an optimization model in an attempt to serve all interests in combination. Sample Value Functions for Lake Erie are given in Lakes Levels Reference Study - International Joint Commission, Final Report - Annex 3 (1993). The scope of analysis is limited to the consideration of a single penalty function corresponding to the year 1989. For the value function provided, the storage levels and discharges with zero or the most desirable value is considered as target levels or discharges.

The formulation proposed in this study incorporate upper and lower storage limit in the dynamic equations for the multi-reservoir system. For this purpose, upper and lower storage limits for each of the Great Lakes are taken as the storage levels associated with the least desirable penalty, equal to one, on either side of the target value discussed above. The upper and lower limits for the outflows for each lake are obtained in a similar fashion. The storage and release targets are given in Tables 1 and 2. The objective of optimization in this study is the minimization of deviations of lake storages and releases from specified target values including the smoothness requirement described in eq. 2 considering only the releases of Superior. The results correspond to having either $\lambda=0$ or $\lambda=10$.

Water supplies to the Great Lakes are quoted in terms of "Net Basin Supplies" (NBS). Data for the period 1900 - 1989 was used for the analysis carried out in this study. In keeping with the existing level of regulation for the Great Lakes system, the present study assumes full regulation only for Lakes Superior and Ontario. Thus, the three middle lakes have natural (and nonlinear) flow conditions represented by equations

$$u_2^t - 0.0841168[(\frac{s_2^t}{480.8} + \frac{s_3^t}{4.6})/2 - 543.4]^2 (\frac{s_2^t}{480.8} - \frac{s_3^t}{4.6})^{0.5} = 0 \quad (4)$$

$$u_3^t - 0.1280849(\frac{s_3^t}{4.6} - 543.4)^2 (\frac{s_3^t}{4.6} - \frac{s_4^t}{105.15})^{0.5} = 0 \quad (5)$$

$$u_4^t - 0.2605000(\frac{s_4^t}{105.15} - 550.11)^{2.2} = 0 \quad (6)$$

4-1 Re-optimization in Simulation

Through equations 3,4, and 5, the simulation in each period uses a nonlinear optimization method to calculate the releases for the middle lakes. This step is called re-optimization because while solving for equations 3 to 5 and the mass balance equations for all the lakes and their corresponding lower and upper bounds for the storages, an objective function similar to eq. 1 is used where the release targets are from release rules instead of the original targets in eq. 1. In re-optimization, because the horizon is single period, the single period cost is minimizing while the targets that are provided for re-optimization comes from long-term optimization. We believe that this feature is useful to consider both short-term and long-term objectives simultaneously.

4-2 Release Rules

From simulation results of successive approximation optimization rule curves, it was noted that for months 1 to 11, the release rule curve was exactly satisfied, that is release targets were achieved in every year. But for month 12, the releases were different from the target. Therefore, in this particular system, it is necessary to derive ANFIS type rules only for the month 12 for the two reservoirs with control, namely Superior, the first one, and Ontario, the last and fifth one. However, the ANFIS premise variable for lake 5 (Lake Ontario) considered net inflow that included release from lake 4 which in turn required releases from the lakes St.Clair and Michigan-Huron. For this reason, ANFIS type rules were derived for all lakes for month 12. As we mentioned before, the premise variables are storage at the beginning of the period and the net inflow into the lake, and the consequent variable is the release from the corresponding lake.

4-3 Results

Table 1 presents the storage targets and Table 2 the release targets of all the reservoirs and release rules of the controllable two reservoirs (Superior and Ontario) obtained by the optimization model for $\lambda=0$ (SAO0) or $\lambda=10$ (SAO10). We can see that the release rules with $\lambda=10$ are somewhat closer to the release targets of the system. Table 3 compares the value of simulated objective function in the three above release rules. Compared to the ANFIS based rule SAO0, the objective function of the traditional target based rule is worse by 6.7% with an 18% increase in the smoothness part of the objective function. Compared to the ANFIS based rule SAO10, the ANFIS based rule SAO0 is significantly worse in the smoothness part of the objective function, which validates our model for the imposition of smoothness. But this comes with a 10.7% increased cost in the long run objective function value (eq. 1). Figures 4 and 5 present the storage level results corresponding to the simulation of the ANFIS based rule SAO10. The spread in the various storage values is mainly because of the huge inflow variance (not provided here) with of over 100% coefficient of variation (standard deviation/mean). The line in the figures depicts the storage targets given in Table 1.

Table 1: Monthly Storage targets

Storage Targets (10^5 tcf)	1	2	3	4	5	6	7
S1	2.0276	2.0270	2.0272	2.0281	2.0292	2.0295	2.0296
S2	2.7779	2.7785	2.7796	2.7810	2.7822	2.7828	2.7825
S3	0.0264	0.0264	0.0264	0.0264	0.0264	0.0264	0.0264
S4	0.5999	0.6000	0.6005	0.6007	0.6009	0.6008	0.6006
S5	0.1949	0.1956	0.1960	0.1965	0.1971	0.1971	0.1966

Table 1: Continued

8	9	10	11	12
2.0297	2.0301	2.0295	2.0286	2.0284
2.7818	2.7808	2.7797	2.7789	2.7783
0.0264	0.0264	0.0264	0.0264	0.0264
0.6003	0.6000	0.6000	0.6000	0.5998
0.1961	0.1960	0.1959	0.1953	0.1949

Table 2: Release Rule Description

Release (tcfs)	1	2	3	4	5	6	7
R1	83.60	84.00	84.00	84.00	92.60	93.20	98.40
R2	155.28	155.9	166.99	169.43	172.41	171.85	175.01
R3	191.78	190.17	203.41	208.42	207.45	208.54	209.84
R4	217.86	220.10	224.01	235.73	46.06	258.31	249.68
R5	223.73	240.94	243.07	41.26	269.36	302.29	310.00
SAO0 R1	228.68	183.77	116.80	61.03	88.95	27.60	27.50
SAO0 R5	92.35	168.14	173.77	219.27	269.82	330.00	338.33
SAO10 R1	61.03	49.82	49.82	72.20	73.58	97.52	101.90
SAO10 R5	92.35	168.14	173.77	175.68	269.82	330.00	338.33

Table 2: Continued

8	9	10	11	12
83.40	80.20	80.20	80.20	66.36
175.36	172.22	169.64	166.94	160.59
210.17	208.19	205.90	204.29	194.80
243.02	236.88	227.31	224.51	217.41
306.30	280.00	280.00	280.00	256.42
27.50	57.70	122.37	27.50	81.48*
233.62	230.56	314.67	280.00	240.69*
94.43	103.00	81.71	74.42	59.60*
233.62	230.56	314.67	279.37	228.13*

R1 – R5 are release targets. SAO0 R1 and R5 are successive approximation optimization release for $\lambda=0$. SAO10 R1 and R5 are successive approximation optimiazation release for $\lambda=10$. * Mean values from simulation using ANFIS

Table 3: Objective function values

Description of Rule	Original objective function (Eqn.1)	Smoothness part of obj. function in (Eqn. 2)
Release Target Rule	3.9404×10^5	6.9860×10^6
ANFIS ($\lambda=0$) [SAO0]	3.6925×10^5	5.9060×10^6
ANFIS ($\lambda=10$) [SAO10]	4.1349×10^5	4.3230×10^5

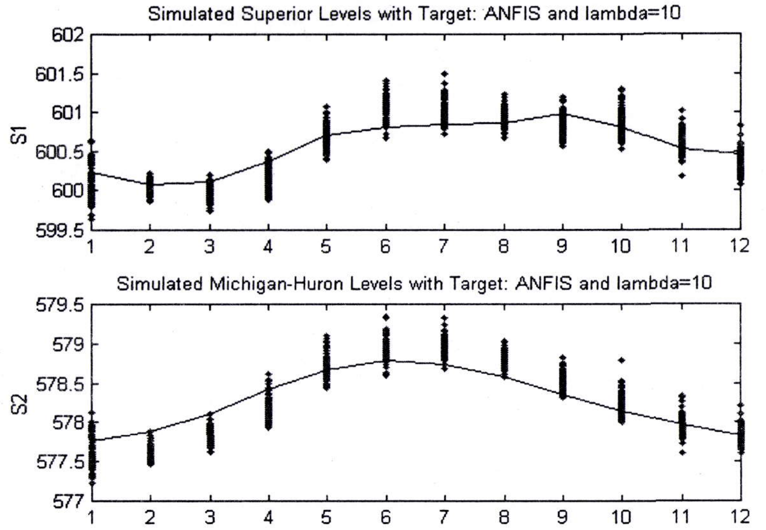

Fig. 4: Simulated Superior and Michigan-Huron storage levels

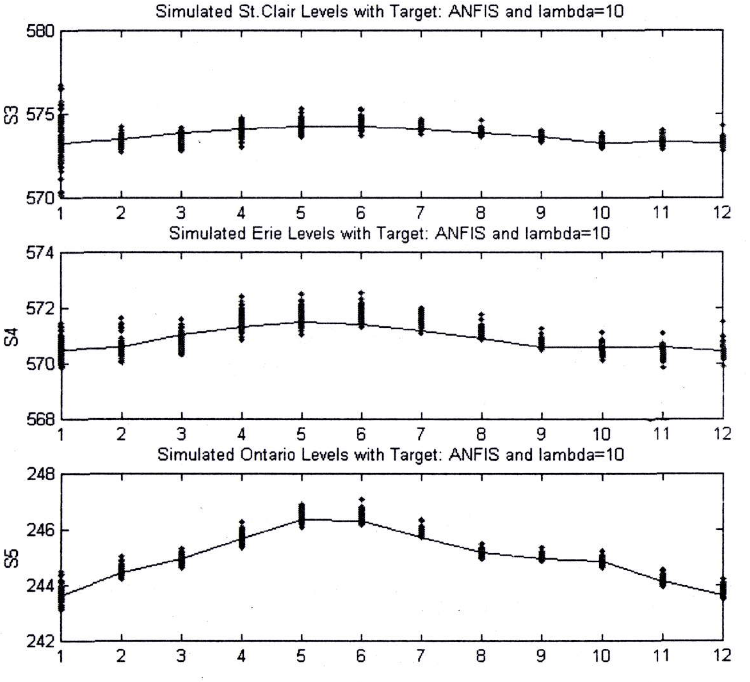

Fig. 5: Simulated St.Clair, Erie and Ontario storage levels

6 Conclusion

The objective of this study was to develop new operating rules or policies improving upon the current use of storage rules. This was achieved using two techniques, (i) using successive approximation optimization to generate a reasonably good set of training data and (ii) using ANFIS to develop state variable based release rule. However, in this study, ANFIS based rule was needed only for one month (December) and this is somewhat of a surprising result. The possible reason could be due to the simple release rule, which was just target based, in our successive approximation optimization stage. Future research would attempt to remedy this situation. The results of objective functions here demonstrate possible large benefits obtainable in the operation of the Great Lakes system using such operating policies.

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