Heterarchical Cognitive Maps : Anticipatory System in Virtual Maze

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Abstract
It has been recognized that there are two kinds of cognitive maps, route map and survey map. Researchers recently have focused on the integration of both maps. The attempt to integrate them, however, entails a problem; which type is prior to the other. This reveals an infinite regression. We propose that the paradoxical modality resulting from the integration can be expressed as a heterarchy, a dynamical hierarchical system. In the name of heterarchy, we stress negotiation among levels. If there is a discrepancy between levels, the expression of a level and interaction are destined to contain intrinsic indefiniteness. It reveals the negotiation. Route maps and survey maps are not integrated, but negotiated, and those form a heterarchy. We conducted a particular experiment in which negotiations between route maps and survey maps were enhanced.
Keywords: Heterarchy, Cognitive map, Virtual maze, Program length, Complexity.

1 Introduction

Anticipatory system [1] is very universal and can be found in searching activities of humans and animals. They search for targets by forming cognitive maps. Cognitive maps are models of the space in which they exist. Cognitive maps are formed only through searching processes and are formed based on their estimation of the length and the shape of their trajectories. However, the estimation cannot be validated without an already existing map. The vicious circle common to anticipatory systems [2] is latent in cognitive mapping, which shall be mentioned in detail later.

The concept of cognitive mapping originates in Tolman. His experiments of rats learning in mazes have shown clearly that they not only learn the sequences of movements (right, left and right…), but also they learn the relation of objects' positions in the maze [3]. Shemyakin, a pioneer, has studied the same capability in humans. He pointed out that there are two types of representation of large-scale space: route map type and survey map type [4]. Route maps are series-representations of the actual routes formed when following the route mentally. Survey maps are overall-representations of the relative positions of objects. Shemyakin has asserted that the representation of large-scale space develops from a route map type to a survey map type during childhood. Furthermore, Hart & Moore have proposed three systems of reference called the egocentric system of reference, fixed system of reference, and abstract system of reference [5]. The egocentric system of reference can only use the searching person as a landmark for positioning objects in space and it seems to correspond to the Shemyakin's route map. The fixed system of reference orientates the searching person and the objects, by using specific objects in space as landmarks. The abstract system of reference makes it possible to grasp the whole space by assigning a coordinate system, and the abstract system of reference seems to correspond to the Shemyakin's survey map. After asserting that children's representations of large-scale space develop from the egocentric systems of reference to the fixed systems of reference, then finally to the abstract systems of reference, Hart & Moore have gone on to say that adults' studying processes also have an order similar to children's development processes. Siegel & White have pointed out this similarity too, but the process they proposed is different from Hart & Moore's in that knowledge of each landmark comes first [6]. However, both are in agreement at the point that acquirement of a survey map is the final stage of this process. On the other hand, Pick & Montello have indicated that the integration of two or more reference systems is indispensable [7]. In actual search, a survey map needs to draw how one should move concretely: a route map. If a survey map exists independently of a route map, the situation will become the same as in the case where one is in an unknown land with a map drawn on paper. In this case, the searching person has to solve the problem of how to use the map. Recently, Ruddle and Osmann have conducted cognitive experiments by using virtual mazes on desktops [8,9]. They, however, have not answered the problem of integration of reference systems directly. (Refer to reference [10] about research on cognitive maps)

The integration of reference systems is indispensable even in the easiest case where a person walks straight. For example, a person walked straight from point A and as a result he passed through point B and arrived at point C. Point A, point B, and point C are then arranged linearly on the person's survey map. But, how does the person know "walking straight" primarily? It is not based only on his sense of balance or the flow of his view, in other words a route map. Supposing it is based only on these, his arrival point will be separated far from his target due to the inclination of the ground or the blinking of his eyes. It is possible to walk straight only because a survey map is referred to continuously. If one asks which one is prior to the other, route map or survey map, one will find an infinite regression. Primarily, two kinds of maps are not self-contained respectively and do not exist independently. Since it is thought that each can exist independently, the problem 'which is prior' occurs. A route map and a survey map are two sides of the same concept, a cognitive map. The concept of cognitive map can exist only by rubbing of these two sides. Therefore, the problem of priority does not make sense.

Generally, a concept consists of the pair, Intent and Extent. Intent is a set of attributes and Extent is a set of objects. Consider the concept 'Even Numbers' for example. The Intent of this concept is 'natural numbers which can be divided by 2' and the Extent is '2,4,6…'. It is usually supposed that only one of them can fully define the concept. In other words, we assume that Intent and Extent can be translated into each other. However, this assumption cannot bear thorough philosophical scepticism [11]. This is because it is impossible to enumerate all even numbers, and there are natural numbers which have never been applied the operation of 'dividing by 2'. Intent is not self-contained and neither is Extent. Therefore, you have to accept that the concept manages to exist only as a negotiation between Intent and Extent. However, this negotiation cannot be formalized, because the domain and the co-domain are not definable, although the form of this negotiation should be expressed as a function. We propose that this modality is expressed as heterarchy, a dynamical hierarchy system [12-14]. By using the word 'heterarchy', we will understand anticipatory systems from the viewpoint of stressing negotiation among levels.

Cognitive maps are in fact heterarchy. A route map and a survey map make a heterarchical system such as Intent and Extent. It is important that a single level is not self-contained. An easy method of making this clear is actively introducing discrepancies into levels. The same method can be seen in ECCA and slime mold computer proposed by Aono [15]. In this study, we introduced discrepancies in a cognitive experiment by dividing and rotating a virtual maze, in which participants searched for targets.

The outline of the experiment we conducted is the following. Participants searched for targets in a virtual maze, which was partitioned into 4 blocks that rotated independently without informing the participants. Suppose that object X is located in block A and object Y and object Z are located in block B. A participant experiences a situation where he cannot reach object Y from object X in the same way he did before. In this sense, this maze has discrepancies. This maze cannot have complete route maps and complete survey maps. But, the location of block A and block B dose not change. And the relative positions of object Y and object Z, which are in the same block, do not change either. Therefore, it becomes an adaptive strategy that a participant grasps the global position relation between blocks as a survey map and the local position relation in blocks as a route map. However, gluing local route maps never create a complete global survey map. Thus, a survey map and a route map should negotiate with each other. In this sense, they make a heterarchical system.

In this study, we recorded participants' behaviours in the virtual maze that we described above, and calculated their 'complexity' that we will define in the following chapters.

2 Experiment

2.1 Participants

Participants were 20 students (average age: 24.85 years) at Kobe University in Japan. They were divided into 2 groups at random. Each group included 10 participants and 2 different types of stimuli (A or B) were given to each group.

2.2 Stimuli

An overall view of the virtual maze is shown in Figure 1. We define '20 steps' as the length from one corner to the other. A square drawn in solid line on the bottom left corner of Figure 1 was a range participants could view on a desktop. There are buttons corresponding to 'go ahead', 'turn right' and 'turn left' respectively on a keyboard. Pressing the button 'go ahead' moved the area a participant could view at the speed of 10 steps per second. The direction of the movement was upward on the desktop at any time. Pressing the button 'turn right' or 'turn left', the view was rotated by 90 degrees. In the maze, there were 16 landmarks that could be distinguished by the designs. The maze could be seen only as for as 7×7-steps square. However, 4 special landmarks seen were within 40×40-steps square. In either square, the participant was located in the center. Therefore, the landmarks appear as floating in the dark.

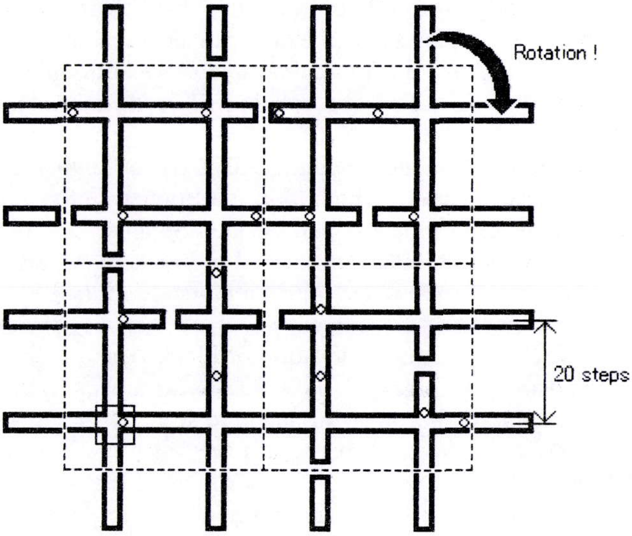

Figure 1: Overall view of the virtual maze.

In Group A, the maze was partitioned into 4 blocks that rotated independently with some frequency. More specifically, when a participant walked 200 steps, the block that was selected at random rotated by degree that was randomly chosen from 0, 90, 180, or 270 degrees. The participant's location and its direction of movement rotated with the block if the participant was in the block chosen. On the contrary, in Group B, there were no rotations of the blocks.

2.3 Procedure

Procedures of Group A and B were the same except for the stimuli.

Firstly, a training task was carried out. In this task, a participant searched for landmarks as many as possible by movement of 2000 steps. The participant was allowed to take a memo about the locations of landmarks by a restricted method.

Secondly, a main task was carried out. In this task, the participant searched for 4 specified landmarks that were located in different blocks from each other. When all of the 4 landmarks were found, the participant searched for another 4 landmarks specified.

It was repeated until the participant walked 4000 steps. The participant was allowed to use the memo he made in training task but he was not allowed to update the memo.

In the above experimental environment, the experiment was carried out and locations of participants in the maze were recorded whenever he moved 1 step.

3 Analysis

3.1 Analytic Method

First, trajectories of participants' search in the maze were expressed as bit-sequences. According to a relative move direction in the maze, a 2-bits number is assigned every 21 steps (Figure 2). Since the length of trajectory in a main task is 4000 steps, a bit-sequence with a length of $4000 \div 21 \times 2 \cong 380$ bits was obtained by this conversion.

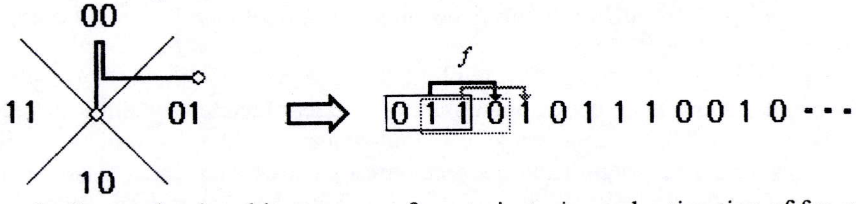

Figure 2: Conversion into bit-sequences from trajectories and estimation of functions.

Next, a function that computes this bit-sequence is estimated. The function was determined so it can fill the following formula as much as possible,

$$f(x_{i-3}, x_{i-2}, x_{i-1}) = x_i \tag{1}$$

where x_i was the i-th bit in the bit-sequence and f is the function. In the case that the value of x_i is different, even if the value of $(x_{i-1}, x_{i-2}, x_{i-3})$ is the same, the value of x_i which exists more in the bit-sequence is adopted. As for $(x_{i-1}, x_{i-2}, x_{i-3})$ that dose not exists in the bit-sequence, the value of x_i was 0.

For example, the following bit-sequence provides $f(0,0,1) = 1$ because the first 4 bits of the bit-sequence are 0011.

$$0011010110101 \tag{2}$$

Similarly, $f(0,1,1) = 0$, $f(1,1,0) = 1$ and $f(0,1,0) = 1$. Since 1010 appears twice and 1011 appears once in the bit-sequence, we adopt $f(1,0,1) = 0$. And we define $f(0,0,0) = 0$, $f(1,0,0) = 0$ and $f(1,1,1) = 0$ because of the absences of 000, 100 and 111 in the bit-sequence. Consequently, the function is determined as shown in the next table.

Table 1: The function estimated from bit-sequence (2).

$x_{i-1}\ x_{i-2}\ x_{i-3}$	000	001	010	011	100	101	110	111
x_i	0	1	1	0	0	0	1	0

In general, functions from several bits to one bit are called the Boolean functions, which can be expressed by Boolean polynomials. The function for Table 1 is the following formula,

$$f(x,y,z)=(x^c \cap y^c \cap z)\cup(x^c \cap y \cap z^c)\cup(x \cap y \cap z^c) \qquad (3)$$

where we use x, y, z instead of $x_{i-1}, x_{i-2}, x_{i-3}$. We define two kinds of indexes that express the grade of complexity of the functions. And we compute the indexes.

The first index is the minimum length of the program that is equivalent to the function. We define the number of the terms of the 'simplest equivalent' of the function as the length. Quine-McCluskey method gives simplest equivalents of arbitrary Boolean functions. We can get the following formula as the simplest equivalent of formula (3) by this method.

$$f(x,y,z)=(x^c \cap y^c \cap z)\cup(y \cap z^c) \qquad (4)$$

Therefore, the value of the first index is 2 in this example. We will call this index 'Program-length'. Wolfram has applied the same analysis method to cellular automata [16].

The second index is the average length of the bit-sequences that the function generates. For all initial conditions, bit-sequences are generated recursively until it becomes periodic. And, we calculate an average of the bit-sequence's lengths (number of digits). Bit-sequences which function (Table 1) generates are shown in Table 2. The average of the length in Table 2 is 3. So the value of the second index is 3 in this example. We will call this index 'Average-period'.

Table 2: Bit-sequences generated by the function (Table 1).

Initial condition	Bit-sequence	Length
000	0	1
001	10101	5
010	10	2
011	0101	4
100	00	2
101	010	3
110	101	3
111	0101	4

3.2 Analytic Result

In the example of the foregoing paragraph, functions from 3 bits to 1 bit were taken up since it is easy. In this study, we used functions from 6 bits to 2 bits. These functions can be expressed by 2 Boolean functions. The lengths of bit-sequences generated are at most $2^6 = 64$ bits and these functions have at most $2^6 \times 2 = 128$ terms. But, because of the symmetry of 0 and 1 in the whole functional space, the number of the terms is at most 64 as a matter of fact. In actual analysis, we calculated numbers of 'prime implicants (P.I.)', which are good approximations of the term's numbers of simplest equivalents [17].

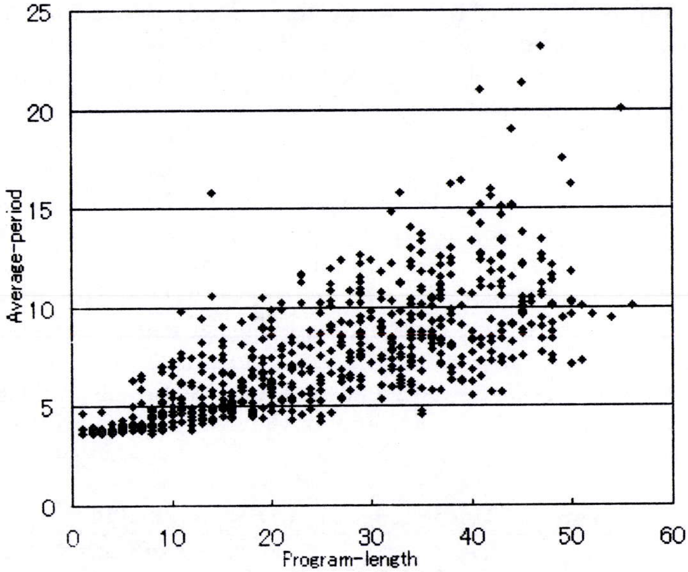

Figure 3: Plotting 630 functions chosen randomly.

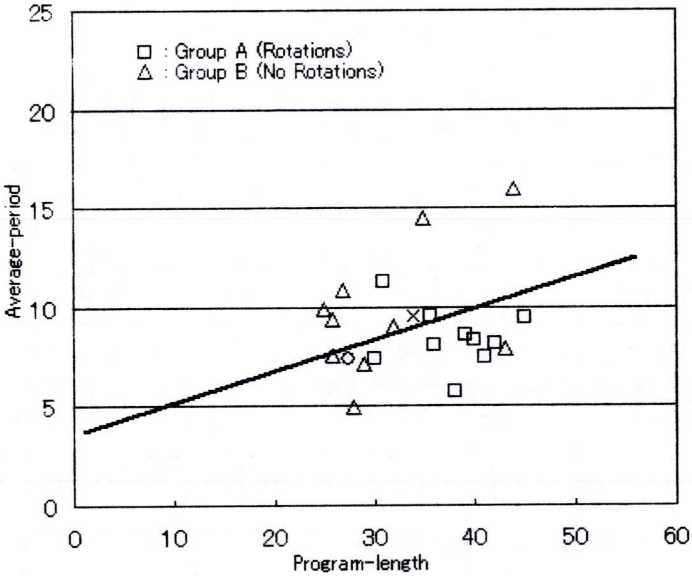

Figure 4: Plotting analytic results of Group A and B.

In Figure 3, values of 630 functions, which are randomly chosen from functions from 6 bits to 2 bits, are plotted on a set of axes. Program-lengths are plotted along the horizontal axis, and Average-periods on the vertical axis. Average period has positive correlation with Program-length, as it was expected. Plotting averages of Average-periods calculated to every Program-lengths gives a distribution on a straight line mostly. And, Using least-squares method provides a straight line, which is shown in Figure 4. In Figure 3, X mark is the result of analysing the shortest trajectory in the stimulus for Group A (Rotations). For Group B (No Rotations), an O mark is used instead of X. Both X and O are plotted near the line. Square marks and triangular marks are the analytic results of Group A and B, respectively. The square marks are mostly distributed under the line. On the contrary, the triangular marks are mostly distributed above the line. We define 'Gap' as the distance from a mark to the line with a sign (+ for marks above the line and – for ones under). Table 3 shows the averages of Gaps of Group A and ones of Group B. The averages of Gaps of Group B is bigger than Group A ($P<0.055$).

Table 3: Averages of Gaps of Group A and B.

	Averages of Gaps
Group A (Rotations)	-1.16
Group B (No Rotations)	+1.12

4 Discussion

What is the origin of complexity? It has been recognized that it is a mixture of homogeneous things and heterogeneous things in some ratio, such as "Edge of Chaos". Langton has presented a very clear and concrete view about the origin of complexity through the numerical analysis of Cellular Automata (CA) [18]. Lambda parameter, which he introduced, arranges all the rules of CA on a number line. One end of the line represents the most homogeneous rule and the other end represents the most heterogeneous rule. CA has rules that generate complex patterns. His computer simulation demonstrated that these rules are mapped at the boundary between the periodic area and the chaotic area. Lambda parameter can be regarded as a mixture ratio of homogeneous things and heterogeneous things. The space of the possible mixture ratio is assumed beforehand. Then all the rules are classified into periodic, chaotic and complex sub-spaces. The exterior of lambda parameter does not exist theoretically. This study objects to these views such as Langton's. The origin of complexity is not a mixture of homogeneous things and heterogeneous things in some ratio. The complexity could result from heterarchy.

In this study, experimental environment was set up so that a heterarchical aspect would be obvious. And the experimental results were analysed by the two kinds of indexes of complexity called Program-length and Average-period. Notice that these two indexes construct heterarchy too. We can consider that Program-length is the Intent and Average-period is the Extent of the concept 'complexity of function'. Usually, it is supposed that only one of them is enough because of consistency of Intent and Extent. On the contrary, if we abandon precise consistency and protect consequent and approximate consistency, it emerges that Intent and Extent negotiate with each other. The line of Figure 5 represents the consistency. Many results from Group A, in which we set heterarchical environment, are distributed to lower right side of the line. This result shows the inconsistency of Intent and Extent. Intuitively, the behaviours are simple but the programs are complicated. We propose to call this modality 'complex'. It is interesting that this result is the opposite of chaos, which should be distributed to the upper left side. (The above argument was illustrated in Figure 5.) The relation between efficiency of search and these results is under analysis now.

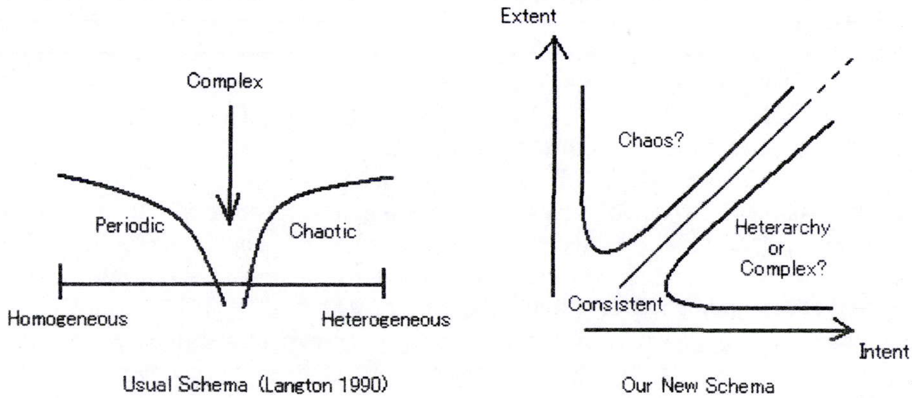

Figure 5: Usual schema vs. our new schema.

In physics, theorists usually premise that Intent and Extent are consistent like differential equations and trajectories. These two levels are completely theoretical. Discovery of chaotic dynamical system made theorists reconsider the consistency of differential equations and trajectories. However, very few attempts such as Matsuno's [19] have been made at constituting the interaction between both. On the other hand, in cognitive science, interaction between levels has been one of the main themes. These two levels are real. Each level exists independently of other levels, therefore the interaction between levels can be written as a function. In this schema, 'study' and 'development' should be expressed as the change of a function, and this will require another function (function which change functions). In heterarchy, two levels are neither theoretical nor real, because heterarchy includes discrepancy. Heterarchy is the viewpoint in which we accept the value of understanding even the most primitive phenomenon as negotiation between two levels. Negotiating continuously, heterarchy can be stable and open to 'study' and 'development'. Heterarchy is the methodology that constitutes negotiation between levels actively.

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