The Universe from Nothing: A Mathematical Lattice of Empty Sets

Michel BOUNIAS[1]

and Volodymyr KRASNOHOLOVETS[2]

[1]*BioMathematics Unit, d'Avignon University and INRA, France*
[2]*Institute for Basic Research, 90 East Winds Court, Palm Harbor, FL 34683, USA*

Abstract.
Major principles of mathematical constitution of space and the principles of construction of physical space are presented. The existence of a Boolean lattice with fractal properties originating from nonwellfounded properties of the empty set is demonstrated. Space-time emerges as an ordered sequence of mappings of closed 3-D Poincaré sections of a topological 4-space-time provided by the lattice of primary empty cells. The fractal kernel stands for a particle and the reduction of its volume is compensated by morphic changes of a finite number of surrounding cells. Quanta of distances and quanta of fractality are demonstrated. Deformation attributes associated to mass determine the inert mass and the gravitational effects, but fractal deformations of cells are responsible for such characteristics as spin and charge.

Key words: space structure, topological distances, space-time differential element, fractality, particles

1 Introduction

In the area of sub atomic physics experimental sciences can only outline general peculiarities of hidden features of the observable world. That is why basic difficulties in the construction of a unified pattern of the physical world fall on theoreticians' shoulders. A great number of approaches to the description of the microworld have been proposed and a majority of the approaches have established a new branch in mathematical physics, so-called "physical mathematics". Nevertheless, despite the striking progress in present-day research, from particle physics to cosmology, many fundamental questions remain unsolved and often contradictory (Krasnoholovets, 2003a). A principal characteristic feature of modern theories remains an exceptional abstraction: They are still based on notions, which themselves must be clarified. For instance: What is the wave ψ-function in quantum mechanics, what is the nature of quantum fields used in quantum field theories, how can one understand 10 dimensions in string theories, are there Higgs bosons in the real physical space or they exist only in abstract physical constructions, what is the electric charge?, etc.

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Contrary to physical mathematics, Bounias (2000a), starting from pure mathematics, namely, the mathematical theory of demonstration, showed that any property of any given object should be consistent with the characteristics of the corresponding embedding space. In other words, the study of any object requires a preliminary knowledge of its properties, or its structure. M. Bounias (Bounias and Krasnoholovets, 2003a, b, c) carried out a tremendous work reconsidering fundamental concepts regarding such fundamental notions as measure, distance, dimension and fractality, which were developing in classical mathematics within the whole 20^{th} century. Bounias (2001) showed that using the biological brain's system, due to its property of self-decided anticipatory mental imaging, one could overpass mathematical limits in computed systems. It will be shown below how this makes eventually possible a scanning of an unknown universe by a part of itself represented by an internal observer.

Our approach is aimed at searching for distances that would be compatible with both the involved topologies and the scanning of objects not yet known in the studied spaces. No such configuration is believed to be an exception in a general case. Generalized conceptions of distances and dimensionality evaluation are introduced, together with their conditions of validity and range of application to topological spaces. It is argued (Bounias and Krasnoholovets, 2003a) that the empty set forms a Boolean lattice with fractal properties and that the lattice provides a substrate with both discrete and continuous properties. Space-time emerges as an ordered sequence of mappings of closed 3-D Poincaré sections of a topological 4-space-time ensured by the lattice of primary empty cells. A combination rule determining oriented sequences with continuity of set-distance function produces a particular kind of 'space-time'-like structure that favors the aggregation of such deformations into fractal forms standing for massive objects. The role of the fractality in deformations of cells, which generates such fundamental characteristics as charges and electric and magnetic properties, is investigated in detail.

2. Preliminaries

2.1. Measure, Distances and Dimensions

One of the problems faced by modelling unknown worlds could be called "the syndrome of polynomial adjustment". In effect, given an experimental curve representing the behavior of a system whose real mechanism is unknown, one can generally perform a statistical adjustment with using a polynomial system like $M = \sum_{(i=0 \to N)} a_i \cdot x^i$. Then, using a tool to test for the fitting of the $N + 1$ parameters to observational data will require increasingly accurate adjustment, so as to convincingly reflect the natural phenomenon within some boundaries. However, if the real equation is mathematically incompatible with the polynomial, there will remain some irreducible parts in the fitting attempts.

2.1.1. Measure

The concept of measure usually involves such particular features as the existence of mappings and the indexation of collections of subsets on natural integers. Classically, a measure is a comparison of the measured object with some unit taken as a standard.

The "unit used as a standard", this is the part played by a gauge (J). Again, a gauge is a function defined on all bounded sets of the considered space, usually having non-zero real values, such that (Tricot, 1999): (i) a singleton has measure naught: $\forall x,\ J(\{x\}) = 0$; (ii) (J) is continued with respect to the Hausdorff distance; (iii) (J) is growing: $E \subset F \Rightarrow \quad J(E) \subset J(F)$; (iv) (J) is linear: $F(r \cdot E) :\ r \cdot J(E)$. This implies that the concept of distance is defined: usually, a diameter, a size, or a deviation are currently used, and it should be pointed that such distances need to be applied on totally ordered sets. Even the Caratheodory measure (μ^*) poses some conditions that again involve a common gauge to be used: (i) $A \subset B \Rightarrow \mu^*(A) \leqslant \mu^*(B)$; (ii) for a sequence of subsets (Ei) : $\mu^* \cup (Ri) \leqslant \sum \mu^*(Ri)$; (iii) $\angle(A, B)$, $A \cap B = \emptyset$: $\mu^*(A \cup B) = \mu^*(A) + \mu^*(B)$; (iv) $\mu^*(E) = \mu^*(A \cap E) + \mu^*(\complement_E A \cap E)$.

The Jordan and Lebesgue measures involve respective mappings (I) and (m^*) on spaces, which must be provided with $\cap$, $\cup$ and $\complement$. In spaces of the $\mathbb{R}^n$ type, tessellation by balls are involved (Bounias and Bonaly, 1996), which again demands a distance to be available for the measure of diameters of intervals.

A set of measure naught has been defined by Borel (1912) first as a linear set (E) such that, given a number (e) as small as needed, all points of E can be contained in intervals whose sum is lower than (e). Applying Borel intervals imposes that appropriate embedding spaces are available for allowing these intervals to exist

2.1.2. Distances

Following Borel, the length of an interval F = [a,b] is :

$$L(F) = (b - a) - \sum_n L(C_n) \tag{1}$$

where C_n are the adjoined, i.e. the open intervals inserted in the fundamental segment. Since the Hausdorff distances, as well as most of classical ones, is not necessarily compatible with topological properties of the concerned spaces, Borel provided an alternative definition of a set with measure naught: the set (E) should be Vitali-covered by a sequence of intervals (U_n) such that: (i) each point of E belongs to a infinite number of these intervals; (ii) the sum of the diameters of these intervals is finite.

The intervals can be replaced by topological balls and therefore the evaluation of their diameter still needs an appropriate general definition of a distance.

A more general approach (Weisstein, 1999) involves a path $\varphi(x, y)$ such that $\varphi(0) = x$ and $\varphi(1) = y$. For the case of sets A and B in a partly ordered space, the symmetric difference $\Delta(A, B) = C_{A \cup B}(A \cap B)$ has been proved to be a true distance also holding for more than two sets (Bounias and Bonaly, 1996; Bounias 1999). However, if $A \cap B = \emptyset$, this distance remains $\Delta = A \cup B$, regardless of the situation of A and B within an embedding space E such that $(A, B) \subset E$. A solution to this problem has been derived in (Bounias and Krasnoholovets, 2003a).

2.1.3. Space Dimensions

One important point is the following: in a given set of which members structure is not previously known, a major problem is the distinction between unordered N-uples and ordered N-uples. This is essential for the assessment of the actual dimension of a space.

Fractal to Topological Dimension. Given a fundamental segment (AB) and intervals $Li = [Ai, A(i+1)]$, a generator is composed of the union of several such intervals: $G = \cup_{(i \in [1,n])} (Li)$. Let the similarity coefficients be defined for each interval by $\rho i = \text{dist}(Ai, A(i+1))/\text{dist}(AB)$.

The similarity exponent of Bouligand is (e) such that for a generator with n parts

$$\sum_{(i \in [1,n])} (\rho i)^e = 1. \tag{2}$$

When all intervals have (at least nearly) the same size, then the various dimension approaches according to Bouligand, Minkowsky, Hausdorff and Besicovitch are reflected in the resulting relation:

$$n \cdot (\rho)^e = 1, \tag{3a}$$

that is,

$$e = \text{Log}\, n / \text{Log}\, \rho. \tag{3b}$$

When e is an integer, it reflects a topological dimension, since this means that a fundamental space E can be tessellated with an entire number of identical balls B exhibiting a similarity with E, upon coefficient ρ.

Parts, Ordered N-uples and Simplexes. The following structure is called a "descent"

$$E^{(n)} = \left\{ e^{(n)}, (E^{(n+1)}) \right\} \tag{4}$$

where (E) denotes a set or part of set, and (e) is a kernel. A descent is finite if none of its parts is infinitely iterated. A second kind set, F = {a, (b, (c, (Z)))}, is ordinary if its descent is finite and extraordinary if its descents include some infinite part. A fractal feature in an extraordinary second kind set is recognizable.

Let the members of the above set E be ordered into a structure of the type of F, e.g. $F' = \{a, (b, (c, (d, e, f)))\}$. Usually, a part (a,b) or {a,b} is not ordered until it can be written in the form (ab) = {{a}, {a,b}}. Stepping from a part (a,b,c,) to a N-uple (abc) needs that singletons are available in replicates.

A simplex is the smaller collection of points that allows the set to reach a maximum dimension. In a general acceptation, it should be noted that the singletons of the set are called vertices and ordered N-uples are (N-1)-faces A^{N-1}.

One question emerging now with respect to the purpose of this study is the following: given a set of N points, how to evaluate the dimension of the space embedding these points?

3. Distances and Dimensions Revisited

3.1. The Relativity of a General Form of Measure and Distance

Our approach aims at searching for distances that would be compatible with both the involved topologies and the scanning of objects not yet known in the studied spaces. No such configuration is believed to be an exception in a general case.

3.1.1. General Case of a not Necessarily Ordered Topological Distance

Proposition. A generalized distance between spaces A,B within their common embedding space E is provided by the intersection of a path-set $\varphi(\mathrm{A},\mathrm{B})$ joining each member of A to each member of B with the complementary of A and B in E, such that $\varphi(\mathrm{A},\mathrm{B})$ is a continued sequence of a function f of a gauge (J) belonging to the ultrafilter of topologies on {E, A, B,...}.

The path $\varphi\,(A,B)$ is a set composed as follows:

(i) $\varphi(\mathrm{A},\mathrm{B}) = \bigcup_{\mathrm{a}\in\mathrm{A},\,\mathrm{b}\in\mathrm{B}} \varphi\,(\mathrm{a},\mathrm{b})$, all defined on a sequence interval $[0, f^{\mathrm{n}}(\mathrm{x})]$, $\mathrm{x} \in \mathrm{E}$. The relative distance of A and B in E, noted $\Lambda_{\mathrm{E}}\,(\mathrm{A},\mathrm{B})$ is contained in $\varphi(\mathrm{A},\mathrm{B})$:

$$\Lambda_{\mathrm{E}}\,(\mathrm{A},\mathrm{B}) \subseteq \varphi\,(\mathrm{A},\mathrm{B})\,. \tag{5}$$

Denote by E° the interior of E, then

$$\min\{\varphi\,(\mathrm{A},\mathrm{B})\ \cap \mathrm{E}^{\circ}\} \text{ is a geodesic of space E connecting A to B,} \tag{6}$$

$$\max\{\varphi\,(\mathrm{A},\mathrm{B})\ \cap \mathrm{C}_{\mathrm{E}^{\circ}}\,(\mathrm{A}\cup\mathrm{B})\} \text{ is a tessellation of E out of A and B.} \tag{7}$$

It is noteworthy that relation (6) refers to $\dim\Lambda_{\mathrm{E}} = \dim\varphi$, while in relation (7) the dimension of the probe is that of the scanned sets;

(ii) $\varphi\,(\mathrm{A},\mathrm{B})\ \cap \mathrm{E}^{\circ}$ is a growing function defined for any Jordan point, which is a characteristic of a Gauge. In addition, the operator $\Lambda_{\mathrm{E}}\,(\mathrm{A},\mathrm{B})$ meets the characteristics of a Frechet metrics, since the proximity of two points a and b can be mapped into the set of natural integers and even to the set of rational numbers: for that, it suffices that two members $\varphi\,(f^{\mathrm{n}}(\mathrm{x}), f^{\mathrm{n}}(\mathrm{y}))$ are identified with a ordered pair $\{\varphi\,(f^{\mathrm{n}}(\mathrm{x})), \{\varphi\,(f^{\mathrm{n}}(\mathrm{x}), f^{\mathrm{n}}(\mathrm{y}))\}\}$;

(iii) suppose that one path $\varphi(\mathrm{A},\mathrm{B})$ meets an empty space $\{\emptyset\}$. Then a discontinuity occurs and there exists some i, such that $\phi\left(f^{\mathrm{i}}(\underline{\mathrm{b}})\right) = \emptyset$. If all $\varphi(\mathrm{A},\mathrm{B})$ meets

$\{\emptyset\}$, then no distance is measurable. As a corollary, for any singleton $\{x\}$, one has $\varphi(f(\{x\})) = \emptyset$. The above properties meet other characteristics of a gauge: first, given closed sets $\{A,B,C, ...\}= E$, then a path set $\varphi(E, E\complement E)$ exploring the distance of E to the closure ($\complement E$) of E meets only open subsets, so that $\varphi(E, E\complement E) = \emptyset$; second, this is consistent with a property of the Hausdorff distance:

$$\max\{(A,B) \subset E | \Lambda_E(A,B) |\} \mapsto \quad \mathrm{diam}_{\mathbf{Hausdorff}}(E) \tag{8}$$

in all cases;

(iv) $\Lambda_E(A,B) = \Lambda_E(B,A)$ and $\Lambda_E(\{x,y\}) = \emptyset \Leftrightarrow x = y$. If the triangular inequality condition is fulfilled, then $\Lambda_E(A,B)$ will meet all of the properties of a mathematical distance.

3.1.2. The Case of Topological Spaces

Proposition. A space can be subdivided in two main classes: objects and distances.

The set-distance is the symmetric difference between sets: it has been proved that it owns all the properties of a true distance (Bounias and Bonaly, 1996) and that it can be extended to manifolds of sets (Bounias, 1997). In a topologically closed space, these distances are the open complementary of closed intersections called the "instant". Since the intersection of closed sets is closed and the intersection of sets with nonequal dimensions is always closed, as was shown previously, the instants rather stand for closed structures. Since the latter have been shown to reflect physical-like properties, they denote objects. Then, the distances as being their complementaries will constitute the alternative class: thus, a physical-like space may be globally subdivided into objects and distances as full components. The properties of the set-distance allow an important theorem to be now stated.

Theorem. Any topological space is metrizable as provided with the set-distance (Δ) as a natural metrics. All topological spaces are kinds of metric spaces called "delta-metric spaces".

The symmetric distance fulfills the triangular inequality, including in its generalized form, it is empty if $A = B = ...$, and it is always positive otherwise. It is symmetric for two sets and commutative for more than two sets. Its norm is provided by the following relation $||\Delta(A)|| = \Delta(A, \emptyset)$.

Therefore, any topology provides the set-distance which can be called a topological distance and a topological space is always provided with a self mapping of any of its parts into any one metrics: thus any topological space is metrizable. Reciprocally, given the set-distance, since it is constructed on the complementary of the intersection of sets in their union, it is compatible with existence of a topology. Thus a topological space is always a "delta-metric" space.

Distance $\Delta(A,B)$ is a kind of an intrinsic case $[\Lambda_{(A,B)}(A,B)]$ of $\Lambda_E(A,B)$, while $\Lambda_E(A,B)$ is called a "separating distance". The separating distance also stands for a topological metrics. Hence, if a physical space is a topological space, it will always be measurable. In other words, only in this case the physical space can be ensured with a metrics.

3.1.3. Dimensionality

A collection of scientific observation through experimental devices produce images of some reality, and these images are further mapped into mental images into the experimentalist's brain (Bounias, 2000a). The information from the explored space thus stands like parts of an apparatus being spread on the worker's table, in view of a further reconstitution of the original object. We propose to call this situation an informational display, likely composed of elements with dimension lower than or equal to the dimension of the real object.

In the construction of the set $\mathbb{N}$ of natural integers, von Neumann provided an equipotent from using replicates of $\emptyset$:

$$0 = \emptyset,\ 1 = \{\emptyset\},\ 2 = \{\emptyset, \{\emptyset\}\},\ 3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, ...$$

A von Neumann set is Mirimanoff-first kind since it is isomorphic with none of its parts. However this construction, associated with the application of Morgan's laws to $(\emptyset)$, allowed the empty set to be attributed an infinite descent of infinite descents and thus to be classified as a member of the hypersets family (Bounias and Bonaly, 1997).

The following propositions can be proved:

(i) in an ordered pair $\{\{x\}, \{x, y\}\}$, the paired part $\{x, y\}$ is unordered;

(ii) an abstract set can be provided with at least two kinds of orders: one with respect to identification of a max or a min, and one with respect to ordered N-uples. These two order relations become equivalent upon additional conditions on the nature of involved singletons. Thus a fully information measure should provide a picture of a space allowing its set component to be depicted in terms of ordered N-uples $(N = 0 \to n,\ n \subset \mathbb{N})$.

Regarding the definition of diameter (8), the following relation can be proposed:

$$\mathrm{diam}_f\ (E) = \{(x, y) \in E\ :\ \max f^{i}\ (\mathrm{Id}\ (x)) \cap \max f^{i}\ (\mathrm{Id}\ (y))\} \tag{9}$$

that gives a (N-2) members parameter (here E is a non-ordered set, Id the identity self map of E, and f the difference self map of E).

A set distance Δ is provided by the symmetric difference or by a n-D Borel measure. A diameter is evaluated on E as the following limit:

$$\mathrm{diam}\ (E) = \{\inf A \to \inf E, \sup A \to \min E, \inf B \to \max E, \sup B \to \sup E\Big|_{\lim \mathrm{dist}_E\ (A,B)}\}. \tag{10}$$

These approaches allow the measure of the size of tessellating balls as well as that of tessellated spaces. A diagonal-like part of an abstract space can be identified with and logically derived as, a diameter.

Each time a measure if obtained from a system, this means that no absolutely empty part is present as an adjoined segment on the trajectory of the exploring path. Thus, no space accessible to some sort of measure is strictly empty in a both mathematical and physical sense, which supports the validity of the quest of quantum mechanics for a structure of the void.

3.1.4. The Dimension of an Abstract Space: Tessellating With Simplex k-faces

A major goal in physical exploration will be to discern among detected objects which ones are equivalent with abstract ordered N-uples within their embedding space. A first of further coming problems is that in a space composed of members identified with such abstract components, it may not be found tessellating balls all having identical diameter. Thus a measure should be used as a probe for the evaluation of the coefficient of size ratio (ρ) needed for the calculation of a dimension.

The following principles should be hold:

(i) a 3-object has dimension 2 iff given A^1_{max} its longer side fulfills the condition, for the triangular strict inequality, where M denotes an appropriate measure

$$M\left(A^1_{max}\right) \; < M\left(A^1_2\right) + M\left(A^1_3\right); \tag{11}$$

(ii) let space X be decomposed in into the union of balls represented by D-faces A^D proved to have dimension $\dim\left(A^D\right) = D$ by relation (11) and size $M\left(A^1\right)$ for a 1-face. Such a D-face is a D-simplex Sj whose size, as a ball, is evaluated by $M\left(A^1_{max}\right)^D = Sj^D$. Let $\mathcal{N}$ the number of such balls that can be filled in a space H, so that

$$\bigcup_{i=1}^{\mathcal{N}} \{S^D_j\} \subseteq \left(H \approx L^d_{max}\right) \tag{12}$$

with H being identified with a ball whose size would be evaluated by L^d, L the size of a 1-face of H, and d the dimension of H. Then, if $\forall S_j$, $S_j \approx S_o$, the dimension of H is:

$$d\,(H) \approx \left(D \cdot \mathrm{Log} S_o + \mathrm{Log}\mathcal{N}\right) / \mathrm{Log}\, L^1_{max}. \tag{13}$$

Relation (13) stands for a kind of interior measure in the Jordan's sense. In contrast, if one poses that the reunion of balls covers the space H, then, d(H) rather represents the capacity dimension, which remains an evaluation of a fractal property.

In the analysis of a abstract space $H = L^x$, of which dimension x is unknown, the identification of which members can be identified with N-uples supposedly coming from a putative Cartesian product of members of H, e.g. $G \subset H : \angle a \in G,\ \ (aaa...a)_h \in G^h$, is allowed by an anticipatory process (because by definition the later enables the construction of the power set of parts ($\mathcal{P}^h$) of G).

4. A Universe as a Mathematical Lattice

4.1. Defining a Probationary Space

The universe from nothing. How is it possible? The question is in the center of attention of Marcer at al. (2003). In our works, we have looked at the problem starting from an idea of a probationary space. The probationary space (Bounias, 2001) is defined as a space fulfilling exactly the conditions required for a property to hold, in terms of: (i) identification of set components (ii) identification of combinations of rules (iii) identification of the reasoning system. All these components are necessary to provide the whole system with decidability. Since lack of mathematical decidability inevitably flaws also any physical model derived from a mathematical background, our aim is to access as close as possible to these imposed conditions in a description of both a possible and a knowable universe, in order to refine in consequence some current physical postulates.

The above considerations have raised a set of conditions needed for some knowledge to be gettable about a previously unknown space, upon pathwise exploration from a perceiving system A to a target space B within an embedding space B. In the absence of preliminary postulate about existence of so-called "matter" and related concepts, it has been demonstrated that the existence of the empty set is a necessary and sufficient condition for the existence of abstract mathematical spaces (W^n) endowed with topological dimensions (n) as great as needed (Bounias and Bonaly, 1997). Hence, the empty set appears as a set without members though containing empty parts. Indeed, members do not appear, because they are nothing ($\emptyset$) and at the same time they are empty elements $\emptyset$.

4.2. The Founding Element

It is generally assumed that some set does exist. This strong postulate has been reduced to a weaker form reduced to the axiom of the existence of the empty set (Bounias and Bonaly, 1997). It has been shown that providing the empty set ($\emptyset$) with ($\in$, $\subset$) as the combination rules, that is also with the property of complementarity ($\complement$), results in the definition of a magma allowing a consistent application of the first Morgan's law without violating the axiom of foundation iff the empty set is seen as a hyperset, that is a nonwellfounded set. A further support of this conclusion emerges from the fact that several paradoxes or inconsistencies about the empty set properties are solved (Bounias and Bonaly, 1997).

4.3. The Founding Lattice

Theorem and Lemmas. The magma $\emptyset^{\emptyset} = \{\emptyset, \complement\}$ constructed with the empty hyperset and the axiom of availability is a fractal lattice (writing $\emptyset^{\emptyset}$ denotes that the magma reflects the set of all self-mappings of ($\emptyset$), which emphasizes the forthcoming results).

The space constructed with the empty set cells of $E_\emptyset$ is a Boolean lattice. Indeed, let $\cup(\emptyset) = \mathrm{S}$ denote a simple partition of $(\emptyset)$. The combination rules $\cup$ and $\cap$ provided with commutativity, associativity and absorption are holding. In effect: $\emptyset \cup \emptyset = \emptyset$, $\emptyset \cap \emptyset = \emptyset$ and thus necessarily $\emptyset \cup (\emptyset \cap \emptyset) = \emptyset$, $\quad \emptyset \cap (\emptyset \cup \emptyset) = \emptyset$. Thus space $\mathrm{P}\{(\emptyset), (\cup, \cap)\}$ is a lattice. The null member is $\emptyset$ and the universal member is $2^\emptyset$ that should be denoted by $\aleph_\emptyset$. Since in addition, by founding property $\complement_\emptyset(\emptyset) = \emptyset$, and the space of $(\emptyset)$ is distributive, then $\mathrm{S}(\emptyset)$ is a Boolean lattice.

$\mathrm{S}(\emptyset)$ is provided with a topology of discrete space. The lattice $\mathrm{S}(\emptyset)$ owns a topology. The space $\mathrm{S}(\emptyset)$ is Hausdorff separated. Units $(\emptyset)$ formed with parts thus constitute a topology $(\mathcal{T}_\emptyset)$ of discrete space.

The magma of empty hyperset is endowed with self-similar ratios. The von Neumann notation associated with the axiom of availability, applying on $(\emptyset)$, provides existence of sets $\left(\mathrm{N}^\emptyset\right)$ and $\left(\mathrm{Q}^\emptyset\right)$ equipotent to the natural and the rational numbers (Bounias and Bonaly, 1997). Let us consider a Cartesian product $\mathrm{E_n} \times \mathrm{E_n}$ of a section of $\left(\mathrm{Q}^\emptyset\right)$ of n integers. The amplitude of the available intervals range from 0 to n, with two particular cases: interval [0, 1] and any of the minimal intervals [1/n-1, 1/n]. Interval [0, 1/n(n - 1)] is contained in [0, 1]. Hence, since empty sets constitute the founding cells of the lattice $S(\emptyset)$, the lattice is tessellated with cells (or balls) with homothetic-like ratios of at least r= n(n - 1). The absence of unfilled areas is supported by introduction of the "set with no parts" (Bounias and Krasnoholovets, 2003b).

Such a lattice of tessellation balls has been called the "tessellattice" (Bounias and Krasnoholovets, 2003a).

The magma of empty hyperset is a fractal tessellattice. Indeed, $(\emptyset) \cup (\emptyset) = (\emptyset, \emptyset)$ and $(\emptyset) \cap (\emptyset) = \emptyset$, moreover, the magma $\left(\emptyset^\emptyset\right) = \{\emptyset, \complement\}$ represents the generator of the final structure, since $(\emptyset)$ acts as the "initiator polygon", and complementarity as the rule of construction. These three properties stand for the major features that characterize a fractal object. Finally, the axiom of the existence of the empty set, added with the axiom of availability in turn provide existence to a lattice $\mathrm{S}(\emptyset)$, which constitutes a discrete fractal Hausdorff space.

4.4. Existence and Nature of Space-time

A lattice of empty sets can provide existence to at least a physical-like space.

Let $\emptyset$ denote the empty set as a case of the whole structure and $\{\emptyset\}$ denotes some of its parts. The set of parts of $\emptyset$ contains parts equipotent to sets of integers, of rational and of real numbers, and owns the power of continuum. Then, looking at the infering spaces $(\mathrm{W^n})$, $(\mathrm{W^m})$, ..., we obtain

$$\{(\mathrm{W^n}) \cap (\mathrm{W^m})\}_{\mathrm{m > n}} = (\Theta^\mathrm{n}), \text{ which is a closed space.} \tag{14}$$

These spaces provide collections of discrete manifolds whose interior is endowed with the power of continuum. Consider a particular case (Θ^4) and the set of its parts $\mathcal{P}(\Theta^n)$; then any of intersections of subspaces $(E^d)_{d<4}$ provides a d-space in which the Jordan-Veblen theorem allows closed members to get the status of both invesigated objects and perceiving objects. This stands for observability, which is a condition for a space to be in some sort observable, i.e., physical-like.

In any (Θ^4) space, the ordered sequences of closed intersections $\{(E^d)_{d<4}\}$, with respect to mappings of members of $\{(E^d)_{d<4}\}_i$ into $\{(E^d)_{d<4}\}_j$, provides an orientation accounting for the physical arrow of time (Bounias, 2000). Thus the following proposition: A manifold of potential physical universes is provided by the (Θ^n) category of closed spaces.

Our space-time is one of the mathematically optimum ones, together with the alternative series of $\{(W^3) \cap (W^m)\}_{m>3}$.

5. Principles of Construction of Physical Space

Is space independent from matter or matter is deformation of space? These discrepancies essentially come from the fact that probationary spaces supporting a number of explicit or implicit assessments have not been clearly identified.

At cosmological scales, the relativity theory places referential in an undefined space, with undefined gauges for substrate for the transfer of information and the support of interactions. Here, distances are postulated without reference to objects: the geometry of space is separated from matter (though the matter is able to influence the space).

At quantum scales, a probability that objects are present in a certain volume is calculated. But again, nothing is assessed about what are these objects, and what is their embedding medium in which such "volumes" can be found. Furthermore, whether these objects are of a nature similar or different to the nature of their embedding medium has not been addressed. In this case, objects are postulated without reference to distances.

We have treated some founding principles about the definition of the space of magmas (i.e., the sets, combination rules and structures) in which a given proposition can be valid. Such a space, when identified, is called a probationary space (Bounias, 2001). Here, it will be presented the formalism which leads from existence of abstract (e.g. purely mathematical) spaces to the justification of a distinction between parts of a physical space, which can be said empty, and parts that can be considered as filled with particles. This question is thus dealing with a possible origin of matter and its distribution, and changes in this distribution gives raise with motion, i.e., with physics.

The values of the constants of electromagnetic, weak and strong interactions as functions of distances between interacting particles converge to the same at a scale about 10^{-30} m. We may assume that it is this point at which the cleavage of space takes place, or in other words, the given scale corresponds to a violation of space homogeneity. The model proceeds from the assumption that all quantum theories (quantum mechanics, electrodynamics, chromodynamics, etc.) are in fact only phenomenological. Accordingly, for the understanding processes occurring in the real microworld, one needs a submicroscopic approach that in turn should be available for all peculiarities of the microstructures of the real space. In other terms, gauges for the analysis of all components of the observable universe should belong to an ultrafilter (Bounias and Krasnoholovets, 2003a). The study about the model of inertons (Krasnoholovets, 1997, 2002, 2004) has suggested that a founding cellular structure of space shares discrete and continuous properties, which is also shown to be consistent with the abstract theory of the foundations of existence of a physical space (Bounias and Bonaly, 1997).

5.1. Foundations of Space-time

A possible application of fractal geometry to the description of space-time has already been demonstrated by Nottale (1997, 2003). He studied relativity in terms of fractality basing his research on the Mandelbrot's concept of fractal geometry. Nottale introduced a scale-relativity formalism, which allowed him to propose a special quantization of the universe. In his theory, scale-relativity is derived from applications of fractals introduced as follows. The fractal dimension D is defined from the variation with resolution of the main fractal variable, i.e., the length l of fractal curve plays a role of a fractal curvilinear coordinate. He also introduced the topological dimension D_T determining it as $D_T = 1$ for a curve, 2 for a surface, etc. The scale dimension then was determined as $\delta = D - D_T$. If δ is constant, the above relationship gives a power-law resolution dependence $l = l_0(\ell/\epsilon)^\delta$. Such a simple scale-invariant law was identified with a Galilean's kind of scale-relativistic law.

Generally speaking, Nottale's approach leads to the conclusion that a trajectory of any physical system diverges due to the inner stochastic nature that is caused by the fractal laws. In this approach fractality is associated with the length of a curve as such.

In the present work we show that the fractal geometry can be derived from complete other mathematical principles, which becomes possible on the basis of reconsidered fundamental notions of space, measure, and length, which allow us to introduce deeper first principles for the foundation of fractal geometry.

Our approach follows a hypothesis (Bonaly, 1992) that a characteristic of a physical space is that it should be in some way observable. This implies that an object called the "observer" should be able to interact with other objects, "observed". We will call them the "perceiver" and the "perceived" objects, respectively. Bonaly's conjecture implied that perceived objects should be topologically closed, otherwise they would offer no frontier to allow a probe to reflect their shape.

The existence of closed topological structures and a proof was given that the intersection of two spaces having nonequal dimensions owns its accumulation points and is therefore closed. In other words, the intersection of two connected spaces with nonequal dimensions is topologically closed. This allowed the representation of the fundamental metrics of space-time by a convolution product where the embedding part

$$U4 = \int \left(\int_{dS} (d\vec{x}\, d\vec{y}\, d\vec{z}) \right) * d\psi(w) \tag{15}$$

where dS is the element of space-time and $d\psi\,(w)$ the function accounting for the extension of 3-D coordinates to the 4^{th} dimension through convolution $(*)$ with the volume of space.

5.1.1. Space-time as a Topologically Discrete Structure

The mapping of two Poincaré sections is assessed by using a natural metrics of topological spaces. Let $\Delta\,(A, B, C, ...)$ the generalized set distance as the extended symmetric difference of a family of closed spaces,

$$\Delta(A_i)_{i \in N} = \underset{\cup\{Ai\}}{\complement} \bigcup_{i \neq j} (A_i \cap A_j). \tag{16}$$

The complementary of Δ in a closed space is closed. It is also closed even if it involves open components with *nonequal dimensions*. In this system $m\,\langle\{A_i\}\rangle = \Delta$ has been associated with the instant, i.e., the state of objects in a timeless Poincaré section (Bounias, 1997). Since distances Δ are the complementaries of objects, the system stands as a manifold of open and closed subparts.

The set-distance provides a set with the finer topology and the set-distance of nonidentical parts provides a set with an ultrafilter. Regarding a topology or a filter founded on any additional property $(\perp)$, this property is not necessarily provided to a Δ-filter. The topology and filter induced by Δ are thus respectively the finer topology and an ultrafilter.

The mappings of both distances and instants from one to another section can be described by a function called the "moment of junction", since it has the global structure of a momentum. Let an indicatrix function $1(x)$ be defined by the correspondence of x with some $c\,(x)$ in $S\,(i+1)$, i.e. $1(x) = 0$ or 1. Then a function $f_{(E_i, E_{i+1})}$, or shortly f_E accounts for a distribution of the indicatrix functions of all points out of the maximum number of possibilities, which would be 2^E for the set of pairs of set E (see Bounias and Krasnoholovets, 2003b). The proportion of points involved in the mappings of parts of Ei into $E\,(i+1)$ equals $f_E^E\,(E) = f_E\,(E)\,/2^E, \quad 0 < f_E^E\,(E) < 1$.

Two species of the moment of junction are represented by the composition $(\perp)$ of $f_E^E\,(E)$ with either the set-distance of the instant. Hence

$$MJ_\Delta = \Delta\,(E) \perp f_E^E\,(E)\,, \tag{17a}$$

$$MJ_m = m\,\langle E\rangle \perp f_E^E\,(E)\,. \tag{17b}$$

Generally, $MJ_\Delta \neq MJ_m$. As a composition of variables with their distribution, (17a,b) actually represent a form of momentum.

5.1.2. Space-time as Fulfilling a Nonlinear Convolution Relation

The "moments of junctions" (MJ) mapping an instant (a 3-D section of the embedding 4-space) to the next one apply to both the open (the distances) and their complementaries the closed (the reference objects) in the embedding spaces. But points standing for physical objects able to move in a physical space may be contained in both of there reference structures.

Then, it appears that two kinds of mappings are composed with one another.

A 'space-time'-like sequence of Poincaré sections is a nonlinear convolution of morphisms. The demonstration involves two kinds of mappings:

(i) mapping ($\mathcal{M}$) connects a frame of reference to the next one: here, the same organization of the reference frame-spaces must be found in two consecutive instants of our spacetime, otherwise, no change in the position of the contained objects could be correctly characterized. However, there may be some deformations of the sequence of reference frames. Mappings ($\mathcal{M}$) denote the corresponding category of morphisms;

(ii) mapping ($\mathcal{J}$) connects the objects of one reference cell to the corresponding next one. Mappings ($\mathcal{J}$) thus behave as indicatrix functions of the situation of objects within the frames.

Each section, or timeless instant of our space-time, is described by a composition ($\circ$) of these two kinds of morphisms: "space-time instant" $= (\mathcal{M} \circ \mathcal{J})$. Besides, stepping from one to the next instant is finally represented by a mapping T, such that the composition $(\mathcal{M} \circ \mathcal{J})$ at iterate (k) is mapped into a composition $(\mathcal{M} \perp \mathcal{J})$ at iterate (k+i): $(\mathcal{M} \perp \mathcal{J})_{k+i} = \mathbf{T}^{\perp} (\mathcal{M} \circ \mathcal{J})_k$. Hence, mapping $(\mathbf{T}^{\perp})$ appears like a relation that maps a function F_{i+k} into F'_{j+k}. Such a relation represent a case of the general convolution, which is a nonlinear and multidimensional form of the convolution product. Mapping for the case of a integrable space gives

$$F'(X) = \int \alpha (X' - X) \, F(X) \, dX'. \tag{18}$$

This relation exhibits a great similarity with a distribution of functions in the Schwartz sense or a convolution product

$$\int_E f(X - u) F(u) d(u) = (f * F)(X). \tag{19}$$

Thus the connection from the abstract universe of mathematical spaces and the physical universe of our observable space-time is provided by a convolution of morphisms, which supports the conjecture of relation (15).

5.2. Relative Scales in the Empty-set Lattice

5.2.1. Quantum Levels at Relative Scales

Inside any of the above spaces, properties at microscale are provided by properties of the spaces whose members are empty set units.

Particular levels of a measure of these units can be discerned. It has been demonstrated (Bounias and Krasnoholovets, 2003b) that the Cartesian product of a finite beginning section of the integer numbers provides a variety of nonequal empty intervals. This means that a finite set of rational numbers inferring from a Cartesian product of a finite beginning section of integer numbers establishes a discrete scale of relative sizes. Indeed, intervals are constructed from the corresponding mappings. For example, with D=2, the smaller ratios available are $1/n$ and $1/(n-1)$, so that their distance is the smaller interval is $1/n(n-1)$. Smaller intervals (σ) in $(E_n)^3$ obey the order of increasing sizes:

$$\forall n > 1:$$
$$(\sigma_{(i)}) = 1/n^2(n-1) < (\sigma_{(ii)}) = 1/n(n-1)^2 < (\sigma_{(iii)}) = n-1)/n^2(n-1). \quad (20)$$

The maximal of the ratios of larger (n) to smaller $(1/n^2\,(n-1))$ segments is

$$\max\,(\sigma)\,/\min\,(\sigma) = n^3\,(n-1)\,. \quad (21)$$

A scaling progression covering integer subdivisions (n) divides a fundamental segment ($n = 1$) by 2, then each subsegment by 3, etc. Therefore the size of structures is a function of iterations (n). At each step (ν_j) the ratio of size in dimension D is $(\Pi\nu_j)^D$, so that the maximal, as follows from (21), is

$$\rho \propto \{(\Pi\nu_j)^D((\Pi\nu_j) - 1\}_{j=1\to n}. \quad (22)$$

Values (22) can be written as $\rho_j = a_j.10^{x_j}$ where in base 10 one takes a_j. belonging to the neighborhood of unity, i.e., $a_j. \in\,]1[$, and look at the corresponding integer exponents x_j as the order of sizes of structures constructed from the lattice $\ell = (\Pi\nu_j)^D$. Regarding distances ($D = 1$) to areas and volumes ($D = 2$ and 3), Eq. (22) consistently provides the orders of physical scales, from microscopic to cosmic, as has been demonstrated by Bounias and Krasnoholovets (2003b).

6. Matter Generated in a Lattice Universe

Space represented by the lattice $F\,(U) \cup (W) \cup (\not{\emptyset})$, where $(\not{\emptyset})$ is the set with neither members nor parts, accounts for relativistic space and quantic void, because (i) the concept of distance and the concept of time have been defined on it and (ii) this space holds for a quantum void since it provides a discrete topology, with quantum scales and it contains no "solid" object that would stand for a given provision of physical matter.

The sequence of mappings of one into another structure of reference (e.g. elementary cells) represents an oscillation of any cell volume along the arrow of physical time.

However, there is a transformation of a cell involving some iterated internal similarity, which precludes the conservation of homeomorphisms. If N similar figures with similarity ratios 1/r are obtained, the Bouligand exponent (e) is given by

$$N \cdot (1/r)^{e} = 1 \tag{23}$$

and the image cell gets a dimensional change from d to $d' = \ln(N) / \ln(r) = e > 1$. In this case the putatively homeomorphic part of the image cell is no longer a continued figure and the transformed cell no longer owns the property of a reference cell.

This transformation stands for the formation of a "particle" also called "particled cell", or more appropriately "particled ball", because it is a kind of topological ball $B\,[\emptyset,\, r\,(\emptyset)]$. Thus a particled ball is represented by a nonhomeomorphic transformation in a continuous deformation of space elementary cells.

6.1. Quanta of Fractality

A minimum fractal structure is provided by a self-similar figure whose combination rule includes an initiator and a generator, and for which the similarity dimension exponent is higher than unity.

Let an initial figure (A) be subdivided into r subfigures at the first iteration and let (r+a) be the number of subfigures constructed on the original one. Since each i^{th} iteration involves the subvolume v_i, in the simplest case we anticipate $v_i = v_{i-1} \cdot (1/r)^3$. A fractal decomposition consists in the distribution of the members of the set of fractal subfigures $\Gamma \supset \{\sum_{(i=1\to\infty)} \{(r+a)^{i} \cdot v_{i-1} \cdot (1/r)^{3}\}\}$ constructed on one figure, among a number of connected figures (C_1, C_2, ..., C_k) similar to the initial figure (A). If k reaches infinity, then all subfigures of A are distributed and (A) is no longer a fractal.

6.2. Interactions Involving Exchanges of Structures

The motion of any particle must be accompanied by the corresponding cloud of space excitations, this is the major thesis of submicroscopic mechanics (Krasnoholovets, 1997, 2002, 2003). These excitations called "inertons" appear due to the friction of the moving particle, which the latter experiences at its motion in the tessellattice tightly packed with cells (or balls, or superparticles).

Let a ball (A) containing a fractal subpart on it. Deformations can be transferred from one to another ball with conservation of the total volume of the full lattice (which is constituted by a higher scale empty set). If a fractal deformation is subjected to motion, it will collide with surrounding degenerate balls. Such collisions will result in fractal decompositions at the expense of (A) whose exponent

(e_A) will decrease, and to the profit of degenerate cells. This is a typical scattering from friction.

The remaining of fractality decreases from the kernel (i.e. the area adjacent to the original particled deformation) to the edge of the inerton cloud. At the edge, depending on the local resistance of the lattice, the decomposition (denoted as the n^{th} iteration) can result in $(e_n) = 1$. Therefore, while central inertons exhibit decreasing higher boundaries, edge inertons are bounded by a rupture of the remaining fractality.

6.3. Mass

A particled ball as described above provides formalism describing the elementary particles proposed by Krasnoholovets (1997; 2000). In this respect, mass is represented by a fractal reduction of volume of a ball, while just a reduction of volume as in degenerate cells is not sufficient to provide mass. The mass m_A of a particled ball A is a function of the fractal-related decrease of the volume of the ball

$$m_A \propto \left(1/V^{part}\right) \cdot (e_\nu - 1)_{\ e_\nu \geqslant 1} \tag{24}$$

where (e) is the Bouligand exponent, and (e-1) the gain in dimensionality given by the fractal iteration; the index ν denotes the possibly fractal concavities affecting the particled ball. Just a volume decrease is not sufficient for providing a ball with mass, since a dimensional increase is a necessary condition.

6.4. Particles and Inertons

Two interaction phenomena have been considered (Bounias and Krasnoholovets, 2003b): first, the elasticity (γ) of the lattice favours an exchange of fragments of the fractal structure between the particled ball and the surrounding degenerate balls. In a first approach, the resulting oscillation has been considered homogeneous. Second, if the particled ball has been given a velocity, its fractal deformations collide with neighbour degenerate balls and exchanges of fractal fragments occur.

The velocity of the transfer of deformations is faster for non-fractal deformations and slower for fractal ones, at slowering rates varying as the residual fractal exponent (e_i). The motion of the system constituted by a particled ball and its inerton cloud provides the basis for the de Broglie and Compton wavelength.

The system composed with the particle and its inertons cloud is not likely to be of homogeneous shape. This property will be accounted for in spin-related properties of observable matter (see also Krasnoholovets, 2000).

The fractality of particle-giving deformations gathers its space parameters $(\varphi_i)_i$ and velocities (v) into a self-similarity expression that provides a space-to-time connection. Indeed, let $(\varphi_\circ)$ and $(v_\circ)$ be the reference values. Then the similarity ratios are $\rho(\varphi) = (\varphi_i)/(\varphi_\circ)$ and $\rho(v) = v/v_\circ$, therefore,

$$\rho\,(\varphi)^e + \rho\,(v)^e = 1. \tag{25}$$

Once again, the right hand side of eq. (25) includes only space and space-time parameters.

Then for distances (l) and masses (m) we obtain using equation (25)

$$(\mathrm{l}/\mathrm{l}_\circ)^2 + (\mathrm{v}/\mathrm{v}_\circ)^2 = 1 \quad \Leftrightarrow \quad \mathrm{l} = \mathrm{l}_\circ \cdot \sqrt{1-(\mathrm{v}/\mathrm{v}_\circ)^2}, \tag{26}$$

$$(m_\circ/m)^2 + (\mathrm{v}/\mathrm{v}_\circ)^2 = 1 \quad \Leftrightarrow \quad m = m_\circ/\sqrt{1-(\mathrm{v}/\mathrm{v}_\circ)^2}), \tag{27}$$

The Lagrangian ($\mathcal{L}$) should obey a similar law and ($\mathcal{L}/\mathcal{L}_\circ$) should fulfill relation (25) as a form of $\rho\ (\varphi)^{\mathrm{e}}$. Then, $(\mathcal{L}/\mathcal{L}_\circ)^2 + (\mathrm{v}/\mathrm{v}_\circ)^2 = 1$ and analogously we can take $\mathcal{L}_\circ = -m\mathrm{v}_\circ^2$, thus finally $\mathcal{L} = -m_\circ \mathrm{v}_\circ^2 \cdot \left(1-(\mathrm{v}/\mathrm{v}_\circ)^2\right)^{1/2}$.

By analogy with special relativity, m, l and v, are the parameters of a moving object, while $v_\circ \equiv c$ is the speed of light.

The existence of inertons was successfully verified experimentally (see, e.g. Krasnoholovets, 2002, 2003; Bounias and Krasnoholovets, 2003b). Thus inertons are basic excitations of the tessellattice, or in other words, they are field particles, which provide for the quantum mechanical interaction, the gravitational interaction and the inertial interaction (this one is caused by so-called forces of inertia and the centrifugal force, i.e., in this case inertons appear due to resistance to the motion of any object on the side of the space).

7. Structural Classes of Particled Cells

7.1. Particle-like Components

Denote by $\theta = \{\rho, \mathrm{a}, \mathrm{I}\}$ a quantum of fractality where $\mathrm{I} = \sum_{\mathrm{i}=1\to\infty}\{1/2^{\mathrm{i}}\}$ is the initiator, ρ is the self-similarity ratio and a is the additional number of subfigures inserted in the $(1/\rho)$ fragments of the initial figure. The corresponding fractal structure is denoted as (Γ) that can be decomposed in a sequence of elementary components $\{\mathrm{C}_1, \mathrm{C}_2, \ \ldots, \mathrm{C}_\mathrm{k}, \ldots\}$. If all these elementary deformations are gathered on one single ball, then this ball contains all the quantum of fractality, though its dimension is not changed. It is therefore non-massive as it stands and its motion is determined by the velocity of transfer of non-massive deformations, that is the maximum permitted by the elasticity of the space lattice. Since the deformations are ordered and distributed in one particular structure, it owns a stability through mappings of Poincaré sections. Such particles are likely to correspond to bosons, i.e., to pseudo particles representing transfer of packs of deformations in an isolated form.

Hence, photon-like corpuscles will carry the equivalent of various quanta of fractality $\{\vartheta_\mathrm{i}\}_\mathrm{i}$, i.e., their equivalent in mass in a decomposed form. This represents as many deformations of the lattice, and finally of equivalent in energy.

7.2. Charges

These are opposite kinds of particle deformations. When a quantum of fractal deformations collapses into one single ball, two adjacent balls exhibit opposite forms: one in the sense of convexity and the other in the sense of concavity on the surface of the ball. Hence, there occurs a pair instead of a single object. The paired structures hold the same fractal dimension, and they will retain the same masses if they get the same volumes. This is realized if the member of the pair whose deformation is in the convex sense looses an equivalent volume in a nonfractal form.

The progression of such structures in the degenerate space will generate several kinds of inerton cloud equivalent, depending on convexity trends (Ξ) and symmetry properties (Ψ) of the corresponding structures. The properties generated by $Q = \{\Xi, \Psi\}$ can be called "charge effects" (regarding further effects including the magnetic properties see Bounias and Krasnoholovets, 2003a; Krasnoholovets, 2003).

7.3. Families of Massive Particles

Any single ball carrying a group $\{\Gamma i\} i$ of quantum fractals will represent a class of massive particles. Depending on both the number and the mode of association of these fractal quanta, various symmetries will result and provide these classes with specific properties.

Simpler particles made from one single quantum of fractality $\{\rho, \mathrm{a}\}$ would likely correspond with lepton-like structures, such that (here $N i$ are odd numbers) inequation

$$\mathrm{L_i} = (\{\mathrm{N_i} \cdot (\rho_\mathrm{i})^{\mathrm{e_i}}\})\,. \tag{28}$$

Hadron-like families will thus be represented by the following common structures

$$\mathrm{H_{i,k}} = (\{\mathrm{a_i}, \rho_\mathrm{i}, \mathrm{e_i}\}_{\mathrm{i,k}}). \tag{29}$$

7.4. Spins for Balls

Fermion-like Cases. Moving massive balls have been shown to carry a cloud of deformations transferred to degenerate balls of the surrounding space, with periodic exchange between this inertons cloud and the original particle. The spatial period of this pulse has been identified with the de Broglie wavelength. Hence, the center of mass (y) of the system composed of the particle and the inerton cloud permanently undergoes a movement forwards and backwards along the trajectory of the system. Two canonical positions are possible, with respect to the particle: (i) (y) is centered on the particled ball, and (ii) y is no longer centered. The probability of state of is thus $\mathrm{P}(y) = 1/2$.

Boson-like Cases. Consider a ball carrying quanta of masses in the decomposed form: then, such a system is opposed the minimum resistance by the surrounding degenerate balls, which are of the same nature, excepted that their individual densities of deformation are much smaller. Therefore, boson-like particles do not generate a cloud similar to that of a massive particle, and their center of mass (y) owns only one main state: thus P (y) = 1.

Spin Module. The state of the center of mass is assessed by the expected moment of junction ⟨MJ⟩ of its components, so that the spin-like system is described by P (x, y) $\langle m \rangle$, standing for $s\hbar/2$, i.e., is the classical spin module expression (see also Krasnoholovets, 2000).

8. Expansion of the Universe

In any one Poincaré section, representing a timeless instantaneous state (an instant) of universe, the lattice of space is represented by a stacking of balls with nonidentical shape.

Proposition. Elementary balls exhibit increasing volumes from the center to the periphery of a 3-D stacking.

Three arguments concur to the same proposition:

(i) oscillating deformations in excess in one cell can be partly compensated by transfer to neighboring balls, like an equivalent to the inerton cloud surrounding a particled ball. However, in central parts, the volume available is limited by the density of the stacking, and this limit is likely decreasing while going to the outer coats of the lattice. Can this volume be defined mathematically? This is a very specific question and the answer to it should also take into account the shape and symmetry of cells in the place of space studied.

In a simple estimation, we denote by (a) the radius of the canonical (smallest) volume that can be transferred from a ball to another. Assuming that each cell forwards a volume (a) to another situated closer to the periphery, in the stacking, then the radius of a ball in the n^{th} coat is approximated by:

$$\mathrm{r}_{\mathrm{n}} = \mathrm{r}_1 + (\mathrm{n} - 1)\,\mathrm{a}; \tag{30}$$

(ii) while the above considerations a valid for a particless lattice, if the lattice is filled with particled balls, then there results a kind of pressure due to the inerton clouds;

(iii) in contrast with the finiteness of volume to be compensatively distributed in the surrounding cells, the area of a particled cell is virtually infinite, and the needed area cannot be compensated by a finite number of the surrounding cells. Thus an influence of any particle is likely to be found up to the most remote parts of the lattice.

9. Conclusion

In this study, we have tried to pose as few postulates as possible examining principles of constitution of space. Generalized concepts of distances and dimensionality evaluation are proposed, together with their conditions of validity and range of application to topological space. The mathematical lattice of empty sets (the tessellattice) provides a substratum with both discrete and continuous properties, from which the existence of a physical universe is derived. This is a new theory of space whose physical predictions easily suppress the opposition of both quantum and relativistic approaches, because submicroscopic mechanics of particles derived from the theory easily results in the Schrödinger's and Dirac's formalisms (Krasnoholovets, 2000) and the gravitation phenomenon is deduced from this submicroscopic mechanics as well, which will be shown in further research.

Discrete properties of the lattice allow the prediction of scales on which microscopic to cosmic structures should occur. Deformations of primarily cells by exchange of empty set cells allow a cell to be mapped into an image cell in the next section as far as mapped cells remain homeomorphic. If a deformation involves a fractal deformation to objects, the change in the deformation of the cell takes place and the homeomorphism is not conserved. The fractal kernel stands for a particle and the reduction of its volume, which is associated with the appearance of mass, is compensated by morphic changes of a finite number of surrounding cells. Quanta of distances and quanta of fractality have been demonstrated. It is shown that the interaction of a particle-like deformation with the surrounding lattice results in a fractal decomposition process, which strongly supports previously postulated clouds of inertons as associated to moving particles. Families of actual particles and field particles have been analyzed.

The theoretical reasoning presented sheds light on the general problem of hypothetical "origin" of the universe. The research conducted brings out the sheer inconsistency of the Big Bang concept that is still treated today as the only one possible for cosmology. There was no any Big Bang in the remote past. The universe is not an empty space or a vague vacuum, but it is a substrate that eternally exists in the form of the tessellattice and this is the simple truth, because the plain evidence of facts is superior to all declarations.

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