Topos for Foundations of Quantum Gravity and Spectral Sequences induced by Non-Discrete Systems

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Abstract
The *theory of temporal topos* (*or t-topos*) gives a new definition for treating particle-wave *duality* as one entity, i.e., as a presheaf over a *Grothendieck site* (*generalized time* category). The theory *t-topos* also gives a new definition of an *entanglement* of particles providing a natural explanation of the *EPR-type non-locality,* which is much simpler than the well established definition of entanglement given in terms of the Hilbert space decompositions and Hilbert space associated with the global quantum system (See, e.g., [AMS] for the definition.). The notion of generalized time is also discussed in [R.S]. For quantum gravity, the theory called the *t.g. relativistic principles of t-topos* will be announced in [Topos'04] based on the current project, [E.P.T.T] and [P.M.S.T].
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1. Introduction

We will begin with the definition of the fundamental category of contravariant functors from a category with a Grothendieck topology.

Definition 1.1 Let S be a site, namely, a category with a Grothendieck topology and let $\hat{S}$ be the category of presheaves from S to the product category $\prod_{\alpha\in\Delta} C_\alpha$. That is, $\hat{S} = (\prod_{\alpha\in\Delta} C_\alpha)^{S^{opp}}$, where S^{opp} is the dual category of S. Then site S is said to be a *temporal site* when S is used in this context. Category $\hat{S}$ is said to be a *temporal topos* or simply a *t-topos*. We sometimes call an object of $\hat{S}$ an *entity*.

Remarks 1.2

(i) See [P.M.S.T] or [G-M] for Grothendieck topologies which is sufficient for our needs. For our earlier model of a generalized time category $\hat{T}$, [B-G-R] (In particular, see 9.1 and 9.2 of [B-G-R].) is better suited since a Grothendieck topology is defined on a set.

(ii) For an object F in $\hat{S}$, which we write as $F \in Ob(\hat{S})$ and for an object V in S, i.e., $V \in Ob(S)$, $F(V)$ is an object in $\prod_{\alpha\in\Delta} C_\alpha$. Namely,

$$F(V) = (F(V)_\alpha)_{\alpha\in\Delta},$$

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where $F(V)_\alpha$ is the α-th component of $F(V)$. We also say that $F(V)$ is the *manifestation* of F during the generalized time period V.

Definition 1.3 Let F be an object of $\hat{S}$. *The ur-state of F in* C_1 *during a generalized time period W,* i.e., an object W of the temporal site S, is defined by the pair $(F, W) = F(W)$, i.e., F is manifested during the generalized time period W. When a generalized time period is *not given*, F is said to be in a *pre-state* or in an *unmanifested state*. When an object W of the temporal site S is *not specified*, $F(W)$ is said to be in the *ur-wave state* of F and sometimes denoted as $\{F(W)\}_{W \in Ob(S)}$. For a specified object V, the object $F(V)$ is said to be in the *ur-particle state* of F *over the generalized time period* V. Considering uniform quantum (sub-Planck) decompositions, (as defined in Definitions 2.1 and 2.2) of F and also of a generalized time period W, when F is not observed by an object in $\hat{S}$, it is in the *quantum fluctuation state,* i.e., $\{ F_\lambda(V_\beta) \}$.

Definition 1.4 An *observation* of an object m of $\hat{S}$ by another object P of $\hat{S}$ in a non-discrete category C_α, $\alpha \in \Delta$, over a generalized time period V is a natural transformation ϕ over this specified V. Namely, the morphism in C_α

$$\phi_V : m(V) \longrightarrow P(V) \qquad (1.1)$$

is said to be an *observation* of m by P during the generalized time period V. If such a natural transformation ϕ exists over a generalized time period V, then m is said to be *observable* or *measurable by P during the generalized time period V*. When such a morphism as in (1.1) does not exist, m is said to be *non-observable* or *non-measurable* by P *during the generalized time period V*. We also say that m *interacts* with P if there exists such a natural transformation from m to P over some generalized time period.

Note 1.4.1 When an object m of $\hat{S}$ is not observed, not only m is in the ur-wave state, but also m is considered as the totality of decomposed objects of $\hat{S}$ which are to be evaluated at unspecified objects of S. (See Definitions 2.1 and 2.2.) It may be most appropriate to consider an unobserved object m to be simply presheaf "*m*," namely, such a state of m is in the *un-manifested state*. Compare this notion with the notion of *quantum fluctuation* in Definition 1.3.

Note 1.4.2 For a morphism from V to U in S, there is induced the morphism ρ_V^U from $m(U)$ to $m(V)$. When m is observed (or measured) by P during V, i.e., $\phi_V : m(V) \longrightarrow P(V)$, the composite morphism $\phi_V \circ \rho_V^U$ from $m(U)$ to $P(V)$ is obtained. However, according to Definition 1.4, this is not an observation of m by P since the composite morphism $\phi_V \circ \rho_V^U$ is not over the same generalized time period. Namely, even though m is observable by P over V, when m is in a different state, i.e., $m(U)$, m need not be observable by P in the sense of Definition 1.4. Morphism $\phi_V \circ \rho_V^U$ may be said to be an indirect observation.

Definition 1.5 Let C_1 be the microcosm discrete category such that elementary particles are examples of objects of C_1. Recall that a discrete category is a category having no morphisms except identity morphisms.

Note 1.6 For every particle in C_1, there exists an associated presheaf in $\hat{S}$. For example, let e̲ be an elementary particle in C_1, then there exists a presheaf *e* in $\hat{S}$ such that we have e̲ = *e(V)* for some *V* in *S*.

Remark 1.7 In C_1 there exists (locally) a usual linear time. Let m̲ be any object in C_1. For example, *m* can be an electron in C_1. When m̲ exists in C_1, the usual time *t* is associated. We have m = *m(V)* in C_1, where *m* is an object of $\hat{S}$ and *V* is an object of *S*. We will denote such an object m̲ in C_1 by m̲ = *m(t(V))* or $m(t_V)$ instead of $m(V_t)$. That is, we regard the usual time *t* depending upon the generalized time period *V* in *S*. Note that in C_1, not every object in *S* assigns *t*. Namely, for *m* in $\hat{S}$, not every *V* in *S* corresponds to an object $m(t_V)$ of C_1. In general, we say that a *state* of an object *F* in $\hat{S}$ in category C_α, $\alpha \in \Delta$, among the product category $\prod_{\alpha \in \Delta} C_\alpha$, is *determined* when an object in *S* is specified.

Definition 1.8 Let $m_1, m_2, ---, m_r$ be objects of $\hat{S}$. If the *r*-tuple $(m_1, m_2, ---, m_r)$ can be considered as one object of $\hat{S}$, then objects $m_1, m_2, ---, m_r$ are said to be *ur-entangled* (or *ur-correlated*). We also call $(m_1, m_2, ---, m_r)$ a *discrete system* consisting of entities $m_1, m_2, ---, m_r$ of $\hat{S}$ when there exist no morphisms among objects $m_1, m_2, ---, m_r$ over any generalized time period.

Definition 1.9 Let l_n, $n = 1, 2, ---, r$, be objects of $\hat{S}$. When there exists an object *U* of *S* such that there are morphisms among $\{l_n(U)\}$, $\{l_n\}_{1 \le n \le r}$ is said to be a *non-discrete entangled system* of objects of $\hat{S}$.

Section 2 Spectral Sequences for Entangled Systems

Definition 2.1 Let M be a particle in the macrocosm discrete category C_2. Then a finite sum of presheaves $\sum_{\lambda \in \Lambda} m_\lambda$ is said to be a *uniform quantum decomposition* of M with respect to a generalized time period *V* if each m_λ is an object of $\hat{S}$ so that $m_\lambda(V)$ is an object of C_1, and M consists of totality $\sum_{\lambda \in \Lambda} m_\lambda(V) = (\sum_{\lambda \in \Lambda} m_\lambda)(V) = \mathrm{M}$.

Definition 2.2 Let m̲ = *m(V)* be an object in C_1. Then a covering $\{V_\varepsilon \longrightarrow V\}$ is said to be a *uniform Planck decomposition* of *V with respect to m* if each V_ε is an object of *S* so that $m(V_\varepsilon)$ is a Planck scale object. Then V_ε is said to be of a *Planck generalized time period with respect to m.* Similarly, a finite sum $\sum_{\beta \in \Omega} m_\beta$ is said to be a *uniform Planck decomposition* of *m with respect to V* if each m_β is an object of $\hat{S}$ so that $m_\beta(V)$ is a Planck scale object, and *m(V)* consists of totality $\sum_{\beta \in \Omega} m_\beta(V)$. We denote the *Planck scale discrete category* as C_{Pl}. For ur-subplanck objects in terms of inverse limits, see [P.M.S.T].

Remark 2.3 First note, for example, when we consider the C_1-components of *m(V)* and *P(V)* such a morphism as ϕ_V in (1.1) must belong to a non-discrete category C_α. However, in the following, we simply say that ϕ_V is an observation of *m(V)* by *P(V)* in C_1. An $\hat{S}$-theoretic interpretation of an observation of an electron by an observer is the following. Let *e* be the presheaf in $\hat{S}$ corresponding to an electron e̲. Let *P* be an observer, i.e., an object of $\hat{S}$, and let V be an generalized time period. An observation of *e* by *P* is a natural transformation ϕ from *e* to *P* over a generalized time period. In order to give a meaning of an observation of *e* by *P* in C_2 during the period *V*, first consider a quantum uniform decomposition $\{V_i \longrightarrow V\}$ of V with respect to *e* so that $e(V_i)$ may be an object of the microcosm category C_1 for all V_i. Then the observation

$$\phi_V : e(V) \longrightarrow P(V) \tag{2.1}$$

in C_2 is interpreted as $\{e(V_i)\} \longrightarrow P(V)$. That is, $\{e(V_i)\}$ is the ur-wave state of *e* for the generalized time period *V*.

Remark 2.4 on Presheaf τ

We noted earlier that the physical time in C_1 depends upon generalized time. That is, one is tempted to hypothesize that τ is an object of $\hat{S}$ so that $\tau(V)$ is an object of C_1 and $\tau(V)$ is the physical local time in C_1. On the other hand, after a uniform sub-Planck decomposition of V for τ, say V_ε, $\tau(V_\varepsilon)$ may be an object of C_{Pl} where $\tau(V_\varepsilon)$ is a Planck scale physical time object in C_1. The triviality of $\tau(V_\varepsilon)$ in C_1 together with below Planck decompositions of objects is interpreted as a nature of *quantum fluctuation*.

Remark 2.5 on Presheaf κ

Let κ be the presheaf associated with space with dimension *d* in C_1. That is, for an object *V* of *S*, $\kappa(V)$ is physical space in C_1 of dimension *d*. Then decompose $\kappa(V)$ as $\kappa(V) = (\kappa(V)^3, \kappa(V)^{d-3})$ may be interpreted as $\kappa(V)^3$ is an object of C_1, and $\kappa(V)^{d-3}$ is an object of C_{Pl}.

Remark 2.6 on Entanglement of κ and τ

We may assume that associated presheaves κ and τ are entangled. Namely, the pair (κ, τ) is an object of $\hat{S}$. That is, for an object *V* of *S*, we have

$$(\kappa, \tau)(V) = (\kappa(V), \tau(V)). \tag{2.2}$$

Furthermore, another hypothesis on κ and τ is that κ and τ are sheaves. See [P.M.S.T] for details.

Note 2.7 on Entanglement and Dependency of space-time on Object

Let *e* and *e'* be associated presheaves to electrons e̲ and e̲'. Assume that e̲ and e̲' are entangled. Namely, **e** = *(e, e')* is an object of $\hat{S}$. Then for an object *V* of *S*, we have, by Definition 1.8

$$\mathbf{e}(V) = (e, e')(V) = (e(V), e'(V)). \tag{2.3}$$

Suppose that *e(V)* and *e'(V)* are physically distant apart in C_2. For this *V*, let $(\kappa, \tau)(V)$ be the local space-time in a neighborhood of *e(V)*. Then, the same $(\kappa, \tau)(V)$ can not be simultaneously the local space-time for *e'(V)*. Namely, the associated space-time presheaf (κ, τ) depends upon a particle (see [P.M.S.T]).

Definition 2.8 Let m^p be a sheaf belonging to the subcategory $\tilde{S}$ of sheaves of the temporal topos $\hat{S}$ where p = *0, 1, 2, ---, n*. Assume that $m^* = \{m^p\}_{i=0,1,2,--,n}$ are entangled, and that over a generalized time period *W*, $m^*(W) = \{m^p(W)\}_{i=0,1,2,--,n}$ is a non-discrete system. The system m^* may be said to be a *non-discrete entangled network.* By following composite morphisms in the system m^*, a *sequence of non-discrete entangled system* $m^\bullet = \{m^p\}_{i=0,1,2,--,n}$ *associated with the non-discrete entangled system* m^* is obtained. Then let $Cm^\bullet$ be the complexification of $m^\bullet$ in the sense of [Bel.'01].

Spectral Sequence Assertion 2.9 There exist doubly indexed cohomological spectral sequences with the abutment $R^n\Gamma(W,Cm^\bullet)$:

$$E_2^{p,q} = R^p\Gamma(W,\underline{H}^q(Cm^\bullet)) \Rightarrow R^n\Gamma(W,Cm^\bullet) \qquad (2.4.1)$$

and

$$E_1^{p,q} = R^q\Gamma(W,Cm^p) \Rightarrow R^n\Gamma(W,Cm^\bullet) \qquad (2.4.2)$$

where $\Gamma(W, -)$ is the global section functor from $\tilde{S}$ to the non-discrete category and $R^i\Gamma$, $i = p, q$, is the *i-th* derived functor of Γ.

Remarks 2.10 (i) All the needed cohomological notions are found in [G-M], [K]. (ii) Interpretations of (2.4.1) and (2.4.2) are that the state of the complexified entangled system can be computed by the cohomological state of an individual object, i.e., (2.4.1) and by the state of an individual object, i.e., (2.4.2).

Section 3 Associated Brain Sheaves (Applications)

In this section, the hypothesis is that a brain (more precisely the associated brain sheaf) is not only a presheaf but also a sheaf. This hypothesis is based on the fact that brain parts (subbrains) are capable of pasting local information data to obtain global information.

Speaking sheaf-theoretically, a (physical) brain B in category C_2 is regarded as the 2^{nd}-component of the associated sheaf *B* with B evaluated at a generalized time period *V*. This sheaf *B* is said to be the *associated brain sheaf* (See [Ro.'01], [Bel.'01], [Kol.'02] for the definition of an associated sheaf.). That is, since category C_2 is discrete, there exists an equality rather than an isomorphism. Namely we have

$$\mathrm{B} = B_2(V), (3.1)$$

where $B_2(V)$ indicates the second component of *B(V)* in the product category $\prod_{\alpha \in \Gamma} C_\alpha$.

We will focus on the category C_2 or C_1, where current imaging techniques in neuroscience take place and also on a non-discrete category C_α, where communication between local objects and global objects take place. We assume that objects of C_α are sets. As in [Kol.'02] and [Bel.'01], all the manifested, i.e., existing objects in the categories C_2 (or C_1) and C_α are the 2^{nd}- (or the 1^{st}-) and α^{th}- components of the object in the product category $\prod_{\alpha \in \Gamma} C_\alpha$. We will simplify our notation as follows. Let $B(V)$ be the associated sheaf evaluated at a generalized time period V which is an object in $\prod_{\alpha \in \Gamma} C_\alpha$. We will use the same notation $B(V)$ for the 2^{nd}-component object in C_2 and also for the α^{th}-component object in the non-discrete category C_α where information is taking place, rather than writing them in the component forms $B_2(V)$ and $B_\alpha(V)$. We will fix this category C_α. (It is a different issue to consider a functor from a non-discrete category to another non-discrete category. Such a functor is called an interpretation functor in [Tokyo '99].) In the sense of (3.1), $B(V)$ is the usual physical brain in C_2 existing over the generalized time period V, and $B(V)$ is the object possessing information in C_α during the generalized time period V. Various brain imaging methods for brain B are interpreted as measuring the images of *the images of functor B over generalized time periods.*

We write the brain sheaf B as a direct sum of subsheaves of B: $B = \sum_{j \in J} B_j$, J is a finite index set. An example of such a subdivision can be much finer than the well known subdivisions of a brain into frontal lobes, parietal lobes, temporal lobes, occipital lobes, e.g., into neurons in macro-level or even micro-level C_1 as entities. We consider a covering family of the generalized time period V i.e., $\{f_i: V_i \longrightarrow V\}$. Global information is an element of $B(V) = (\sum_{j \in J} B_j)(\{f_i: V_i \longrightarrow V\})$. (See [K] or [P.M.S.T] for the notion of a covering family. Note that we are assuming that an object is a set in category C_α.) One of the main goals is to formulate the mechanism for obtaining global information from local data as elements of $\{B_j(V_i)\}_{j \in J,\, i \in I}$ in category C_α.

Remark 3. 1 Before we state our main assertion, we will make general remarks on two kinds of functorial (restriction) morphisms. Let D' be an associated brain subsheaf of a brain sheaf D and let $W' \xrightarrow{g} W$ be a morphism in the site S. Then we have the functorial morphism from $D(W) \xrightarrow{Dg} D(W')$. On the other hand, the restriction morphism from $D(W)$ to $D'(W)$ is defined as the assignment from an element t_D of $D(W)$, which is a local datum, to the element $t_{D|D'}$ of $D'(W)$ that is the restricted brain activity to the sub-brain sheaf D' induced by t_D.

Let us return to our earlier situation. For $i \in I$ and $j \in J$, consider an element s_i^j of $B_j(V_i)$ in the category C_α.

Definition 3.2 The family of the subsheaves $\{B_j\}$ is said to *paste well* with respect to V_i if the following condition is satisfied.

(Condition): the images of the restriction maps for s_i^k in $B_k(V_i)$ and s_i^j in $B_j(V_i)$, k and $j \in J$, coincide as an element of $B_k(V_i) \times B_j(V_i)$, then there exists a

unique element s_i in $B(V_i) \overset{def}{=} \bigcup_{j \in J} B_j(V_i)$ such that the induced element of s_i on $B_j(V_i)$ equals s_i^j for all $j \in J$, where the induced morphism is as defined in Remark 3.1. For sheaf B_j, by the definition of a sheaf, the covering family $\{f_i : V_i \longrightarrow V\}$ always *pastes well* with respect to B_j in the sense as in [K], [P.M.S.T]. We are ready to state the main assertion showing the mechanism of a brain function as a sheaf: from given local data to global information. First, we will state the case where $\{B_j\}_{j \in J}$ is not entangled or partially entangled.

Main Assertion A As in the above, let decompose B as brain subsheaves as follows: $B = \sum_{j \in J} B_j$ as objects of $\tilde{S}$ and let us consider a covering family of V : $\{f_i : V_i \longrightarrow V\}$. Suppose that $\{B_j\}_{j \in J}$ (and $\{V_i\}_{i \in I}$ paste well with respect to every V_i (and every B_j). Then for given local data $\{s_i^j\}$ in $B_j(V_i)$, $i \in I$ and $j \in J$, there exists a global element s in *B(V)* whose restrictions to subsheaves and generalized time subperiods coincide with given local data.

Sketch of Proof We are given local data with respect to generalized time periods and with respect to brain subsheaves. That is, $\{s_i^j\}$ in $B_j(V_i)$, $i \in I$ and $j \in J$, are given. In the diagram below, first we will paste for subsheaves at V_i. For local data $\{s_i^j\}$ in $B_j(V_i)$ and $\{s_i^k\}$ in $B_k(V_i)$ in the third row, we get an element s_i in $B(V_i)$ which is the second object in the second row. Next, we will paste generalized time subperiods at B_j. For local data $\{s_i^j\}$ in $B_j(V_i)$ and $\{s_m^j\}$ in $B_j(V_m)$, we get an element s^j of $B_j(V)$. The restrictions from $B_j(V_m \times V_i)$ and from $B_k(V_m \times V_i)$ to $B_j(V_m \times V_i) \cap B_k(V_m \times V_i)$ give an element $s_{m,i}$ in $B(V_m \times V_i)$. The restrictions from $B_j(V_m) \cap B_k(V_m)$ and from $B_j(V_i) \cap B_k(V_i)$ to $B_j(V_m \cap V_i) \cap B_k(V_m \cap V_i)$ give an element of $B_j(V) \cap B_k(V)$. Since $\{s_i\}$ and $\{s^j\}$ restrict well, we get a unique element s in *B(V)*.

$B(V)$

$B(V_m)$ $\quad$ $B(V_i)$ $\quad$ $B_j(V)$ $\quad$ $B_k(V)$

$B(V_m \times V_i)$ $\quad$ $B_j(V_m)$ $\quad$ $B_j(V_i)$ $\quad$ $B_k(V_i)$ $\quad$ $B_j(V) \cap B_k(V)$

$B_j(V_m) \cap B_k(V_m)$ $\quad$ $B_j(V_m \times V_i)$ $\quad$ $B_j(V_i) \cap B_k(V_i)$ $\quad$ $B_k(V_m \times V_i)$

$B_j(V_m \times V_i) \cap B_k(V_m \times V_i)$

where the induced morphisms are not indicated in the above diagram.

Main Assertion B When $\{B_j\}_{j\in J}$ is totally entangled. The above commutative diagram becomes as simple as

$$\begin{array}{ccc} & B(V) & \\ B_j(V) & & B_k(V) \\ & B_j(V)\cap B_k(V) & \end{array}$$

where the induced morphisms are not indicated in the above diagram. If the image of the induced morphism of s_j from $B_j(V)$ to $B_j(V)\cap B_k(V)$ coincides with the image of the induced morphism of s_k from $B_k(V)$ to $B_j(V)\cap B_k(V)$, then there exists a global element s in $B(V)$ whose restrictions to $B_j(V)$ and $B_k(V)$ are s_j and s_k, respectively.

Remarks 3.3 (i) The statement of Main Assertion B is the dual statement of a presheaf to be a sheaf.

(ii) In our above discussion, we wrote a brain sheaf as a direct sum of subsheaves so that each subsheaf evaluated at a generalized time period is an object of macro category C_2. By considering the notions of quantum, or even Planck, uniform decomposition of a brain sheaf (See [E.P.T.T] for the definition of the decompositions.), one can carry out the similar construction as above where each object obtained by a subsheaf evaluated at a generalized time period is an object in micro category C_1.

Remark 3.4 For the case of Main Assertion B where entangled $\{B_j\}_{j\in J}$ are entangled, there are spectral sequences (2.4.1) and (2.4.2):

$$E_2^{p,q} = R^p\Gamma(V,\underline{H}^q(CB^{-\bullet})) \Rightarrow R^n(V,CB^{-\bullet})$$

and

$$E_1^{p,q} = R^q(V,CB^{-p}) \Rightarrow R^n(V,CB^{-\bullet})$$

where $B_i = B^{-i}$ is used in the above. The above spectral sequences indicate that an entangled system (the complex of associated entangled brain sheaves) may be computed from an individual part. Namely, an individual local state over V of an associated brain governs the global state over V.

4. Conclusion

By introducing the notion of a (pre-)sheaf, fundamental concepts and results in quantum physics are reformulated in terms of t-topos theory, especially those of non-locality and entanglement. While t-topos theory being developed in Section 1 and Section 2 as a possible quantum gravity theory, in Section 3, t-topos notion is applied to brain functions where a brain is regarded as a macro object component of the associated (brain) sheaf evaluated at a generalized time period of the temporal site. In this last section, the sub-brains' ability to paste given local data to get global information is phrased in terms of sheaf-category notions, i.e., t-topos theory.

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