A new Dynamic Model of the Universe Motion

Branko Novakovic, Dario Novakovic, and Alen Novakovic
FSB - University of Zagreb, Luciceva 5, P. O. B. 509, 10000 Zagreb, Croatia
Fax. (+385) 1 61 68 351, E-mail: branko.novakovic@fsb.hr

Abstract
The proposed model of the universe motion gives some new specific answers to the important questions of the cosmology: what occurred at the initial singularity?, how old is the universe?, how big is the universe? , and what is it's ultimate fate? This new approach is based on the new solution of the Einstein's field equations in a vacuum. This solution confirms that the so called cosmological constant, Λ, is not really constant, but a function of the gravitational radius. As the consequence, the acceleration equation of the universe motion shows that the universe acceleration can be attractive (negative) or repulsive (positive). The repulsive acceleration gives rise to the accelerating expansion of the universe at the present time. The change from the contracting phase into the expanding one, takes place at the minimal radius, $r = GM/2c^2$. This could be the solution of the initial singularity.
Keywords: Attractive and Repulsive Gravity, Universe Motion, Minimal Universe Radius, Expanding Universe, Oscillating (Cyclic) Universe.

1 Introduction

As it is well known, Einstein proposed a static model of the universe, based on the cosmological constant Λ [1]. Then, Edwin Hubble discovered in 1929, that our universe is expanding. Georges Lemaitre proposed his theory in 1931, that the universe is born from "praatom" and George Gamov introduced in 1948 the new theory about "praatom" , the Big Bang theory. In 1979 Alan Guth proposed the inflationary model of the universe, with explanation why it is considered geometrically flat [2]. Today the universe continues to expand, and at an accelerating rate [3]. Caldwell, Dave and Steinhardt proposed the quintessence mechanism [4]. In the Friedmann-Robertson-Walker cosmologies [5], both the cosmological constant and the quintessence approaches are added to the standard Cold Dark Matter (CDM) model, what is resulted with known ΛCDM and QCDM models, respectively. Parker and Raval proposed a new Vacuum Cold Dark Matter (VCDM) model [6]. Recently, Steinhard and Turok offered the Cyclic Universe model, as a radical new cosmological scenario[7]. The main problem of the all models of the universe motion is to answer (among the others) to the important questions of the cosmology [7]: what occurred at the initial singularity?, how old is the universe?, how big is the universe? and what is it's ultimate fate?

In this paper proposed model of the universe motion gives some new specific answers to the mentioned questions. This new approach is based on the new General Lorentz Transformation model (GLT-model) given in [8]. It includes two parameters α and α' as functions of space-time coordinates. For a gravitational potential field, the parameters α and α' have been identified by employing the gravitational red-shift experiment [8] and the Einstein's field equations in a vacuum [9]. By the solution of the Einstein's field equations in a vacuum, including Λ, it has been discovered that the so-called cosmological constant, Λ, is not really constant, but a function of the gravitational radius [10]. This function has the important influence to the parameters α and α'. As the consequence, the acceleration equation of the universe motion shows that the universe acceleration can be attractive (negative) or repulsive (positive). The repulsive acceleration gives rise to the accelerating expansion of the universe at the present time.

The acceleration equation of the universe motion tell us that the change from negative to positive acceleration (or vice-versa) takes place at the radius $r = GM/c^2$. Here M and G are gravitational mass and constant, and c is the speed of the light in a vacuum. At this point $r = GM/c^2$ the universe velocity has a maximum (in an expanding phase) or a minimum (a negative maximum, in a contracting phase). The change from the contracting phase into the expanding one, takes place at the minimal radius, $r_{min} = GM/2c^2$. At this point the universe velocity is equal to zero, but the acceleration is positive and maximal, what is the sufficient condition for starting with the expanding phase. The minimal radius is proportional to the mass, and can be equal to zero only if the mass is equal to zero. Thus, an initial singularity is possible if an initial mass is equal to zero.

On the other side, the change from the expanding phase into the contracting one is possible if the energy conservation constant k is less than one ($k < 1$). For that case, the proposed model describes the oscillating (cyclic) scenario of the universe motion. In the case where k is equal to one ($k = 1$), the proposed model describes the universe that will approach to the maximal velocity, and than the rate of the expansion, or velocity, will decreasing going to zero at an infinite radius. Maybe some kind of the perturbations (that are not included in this model) will stop this expansion and change it into the contraction. In the case where k is greater than one ($k > 1$), the situation is similar to the previous one, but the rate of the expansion, or velocity, will decreasing going to the constant velocity at an infinite radius. Employing the proposed model of the universe motion and using some of the observation data, one can estimate present values of the main parameters of the universe motion: velocity, mass, radius, acceleration, time and state. Some new aspects of the universe motion are presented in the references [12-18].

The presented model of the universe motion is not in the collision with the Big Bang theory, nor with the inflationary theory of the universe, if one accepts the assumption of the inflationary model [2]: the initial matter of the universe could have been a billion times smaller than a single proton. In that case, according to the proposed model of the universe motion, the initial (minimal) radius of the universe was very, very small, close to zero, but was not equal to zero. At the minimal (initial) radius of the universe a matter density, temperature and a repulsive acceleration were very, very high, but finite quantities. Maybe this could be the solution of the initial singularity.

2 Derivation of Velocity and Acceleration Equations of the Universe Motion

In this section a new approach to the description of the universe motion is proposed. This approach is quite different to the mentioned ones, and is based on the new general structure of the Lorentz transformations in the form of General Lorentz Transformation model (GLT-model) [8]. This structure includes two non-dimensional parameters α and α' as functions of space-time coordinates. Thus, for a gravitational potential field, the parameters α and α' have been identified by employing the gravitational red-shift experiment [8] and the Einstein's field equations in General Relativity [9,10].

It has been shown in the reference [10] that the general diagonal form of the line element of the GLT-model, in the spherical polar coordinates, can be described by the equation:

$$\underline{ds}^2 = -\alpha^2 c^2 dt^2 + \alpha^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{1}$$

where $\underline{ds}^2$ is a diagonal form of the line element. The general solution of that line element, in a gravitational potential field, is given by the relation [10]:

$$\alpha^2 = \left(1 - \frac{GM}{rc^2}\right)^2 = 1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2 = 1 - \frac{2GM}{rc^2} + \Lambda r^2, \ \Lambda = \left(\frac{GM}{r^2c^2}\right)^2, \tag{2}$$

where GM/c^2 is a constant of integration that also satisfies the gravitational red-shift experiment [8]. In the equation (2) G is a gravitational constant, M is a total gravitational mass, r is a gravitational radius and c is the speed of the light in a vacuum. Here, the parameter $\Lambda = f(r)$, has been identified together with derivation of the solution of the parameter α in the equation (1). If displacement four-vector dX is defined in frame O by the expression:

$$dX \to 0\,(cdt, dr, d\theta, d\phi) = \{dx^i\},\ i = 0,1,2,3, \tag{3}$$

then, the related covariant metric tensor of the line element (1) has the form [10]:

$$[g_{ij}] = diag[-\alpha^2, \alpha^{-2}, r^2, r^2 \sin^2\theta] = diag[g_{00}, g_{11}, g_{22}, g_{33}], \tag{4}$$

where diagonal components g_{ii} $i = 0,1,2,3$, are given by the relations:

$$g_{00} = -\left(1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2\right), \quad g_{11} = \left(1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2\right)^{-1}, \tag{5}$$

$$g_{22} = r^2, \quad g_{33} = r^2 \sin^2\theta \ .$$

The related determinant of the metric tensor (4) has the form:

$$det[g_{ij}] = -r^4 \sin^2\theta, \tag{6}$$

where for $r = 1$ and $\theta = \pi/2$, one finds out $\det(g_{ij}) = -1$, what we expected that should be.

In order to find out the velocity and acceleration equations of the universe motion one can start with the Lagrangean of the line element of the GLT-model (1):

$$L=\left[-\frac{ds^2}{c^2 d\tau^2}\right]^{1/2} = \frac{1}{c}\left[-g_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\right]^{1/2}, i,j=0,1,2,3, \tag{7}$$

where $d\tau$ is the differential of the proper time τ, and dx^i is a component of the displacement four-vector dX in (3). Applying the equations (1) to (5) and using (7) one obtains the Lagrangean in the following form:

$$L=\left(\begin{array}{c}\left(1-\frac{GM}{rc^2}\right)^2\left(\frac{dt}{d\tau}\right)^2-\frac{1}{c^2}\left(1-\frac{GM}{rc^2}\right)^{-2}\left(\frac{dr}{d\tau}\right)^2- \\ -\frac{r^2}{c^2}\left(\frac{d\theta}{d\tau}\right)^2-\frac{r^2}{c^2}\sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2\end{array}\right)^{1/2} \tag{8}$$

The related Euler – Lagrange equations are given by the relations:

$$\begin{gathered}\frac{\partial L}{\partial z^i}=\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{z}^i}\right), \quad i=0,1,2,3, \\ z^o=t, \quad z^1=r, \quad z^2=\theta, \quad z^3=\phi, \quad \dot{z}^i=\frac{dz^i}{d\tau}.\end{gathered} \tag{9}$$

Applying i = 0 to the relation (9) one obtains energy conservation equation:

$$\frac{\partial L}{\partial t}=0 \Rightarrow \frac{\partial L}{\partial \dot{t}}=const.=k \Rightarrow \left(1-\frac{GM}{rc^2}\right)^2\frac{dt}{d\tau}=k, \quad \frac{dk}{d\tau}=\dot{k}=0, \tag{10}$$

where k is an energy conservation constant. Applying i = 3 to the relation (9) one obtains angular momentum conservation equation:

$$\frac{\partial L}{\partial \phi}=0 \Rightarrow \frac{\partial L}{\partial \dot{\phi}}=const.=h \Rightarrow -\frac{r^2\dot{\phi}\sin^2\theta}{c^2}=h, \quad \frac{dh}{d\tau}=\dot{h}=0, \tag{11}$$

where h is an angular momentum conservation constant. In the case $\theta = \pi / 2$ (as in Newtonian theory) the angular momentum conservation equation (11) is transformed into the well-known relation:

$$\frac{\partial L}{\partial \phi}=0 \Rightarrow \frac{\partial L}{\partial \dot{\phi}}=const.=h \Rightarrow -\frac{r^2\dot{\phi}}{c^2}=h, \quad \frac{dh}{d\tau}=\dot{h}=0. \tag{12}$$

Now, substituting the relation:

$$\frac{dt}{d\tau}=k\left(1-\frac{GM}{rc^2}\right)^{-2}, \tag{13}$$

derived from (10), into the equation (8) and employing $\varepsilon = L$, with $\varepsilon = 1$ for a time-like geodesics and ε=0 for a null geodesics, one obtains the following relation:

$$\frac{1}{2}\left[\dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)\left(1 - \frac{GM}{rc^2}\right)^2\right] - \frac{GM\varepsilon^2}{r}\left(1 - \frac{GM}{2rc^2}\right) = \frac{c^2}{2}\left(k^2 - \varepsilon^2\right),$$

$$\dot{r} = \frac{dr}{d\tau}, \qquad \dot{\theta} = \frac{d\theta}{d\tau}, \qquad \dot{\phi} = \frac{d\phi}{d\tau} \quad . \tag{14}$$

This relation represents the generalized energy equation, which is the extended Schwartzschild's energy equation and more extended Newtonian energy equation.

If one employs $\theta = \pi / 2$ (as in Newtonian theory) and if in a weak gravitational field (like in our Solar system) one can neglected the following terms:

$$\frac{1}{2}\left(\frac{GM}{rc}\right)^2 \approx 0 \quad \Rightarrow \quad \left(\frac{GM}{rc^2}\right)^2 \approx 0 \ , \tag{15}$$

then the relation (14) can be transformed into the well-known Schwartzschild's energy equation:

$$\frac{1}{2}\left[\dot{r}^2 + r^2\dot{\phi}^2 \left(1 - \frac{2GM}{rc^2}\right)\right] - \frac{GM\varepsilon^2}{r} = \frac{c^2}{2}\left(k^2 - \varepsilon^2\right). \tag{16}$$

This equation can further be reduced to the Newtonian energy equation by neglecting the term $2GM/rc^2$:

$$\frac{1}{2}\left[\dot{r}^2 + r^2\dot{\phi}^2 \ \right] - \frac{GM\varepsilon^2}{r} = \frac{c^2}{2}\left(k^2 - \varepsilon^2\right). \tag{17}$$

Thus, the energy equation (14) includes both Schwartzschild's and Newtonian energy equations as special approximations in a gravitational field. The generalized energy equation (14) can be transformed into the new relation:

$$\frac{v^2}{2} - \frac{GM\varepsilon^2}{r}\left(1 - \frac{GM}{2rc^2}\right) = \frac{c^2}{2}\left(k^2 - \varepsilon^2\right),$$

$$v^2 = \left[\dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)\left(1 - \frac{GM}{rc^2}\right)^2\right], \tag{18}$$

where **v** is a velocity. Thus, the velocity equation can be derived from the first equation in (18):

$$v = \pm\left[\frac{2GM\varepsilon^2}{r}\left(1 - \frac{GM}{2rc^2}\right) + \left(k^2 - \varepsilon^2\right)c^2\right]^{1/2} . \tag{19}$$

The radial velocity ($\dot{r}$) equation can also be derived from (18), or directly from (14):

$$\dot{r} = \pm \left[\frac{2GM\varepsilon^2}{r} \left(1 - \frac{GM}{2rc^2} \right) + \left(k^2 - \varepsilon^2 \right) c^2 - r^2 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \right) \left(1 - \frac{GM}{rc^2} \right)^2 \right]^{1/2} . \tag{20}$$

From the observation we know that the universe motion is in the radial direction only. Therefore, one can substitute $\dot{\theta} = 0$ and $\dot{\phi} = 0$ into the relation (20). As the result we have the new radial velocity equation in the following form:

$$\dot{r} = v = \pm \left[\frac{2GM\varepsilon^2}{r} \left(1 - \frac{GM}{2rc^2} \right) + \left(k^2 - \varepsilon^2 \right) c^2 \right]^{1/2} . \tag{21}$$

From this point the radial velocity $\dot{r}$ will be denoted with v and will be called just velocity, for the simplicity. The hypothetical possibility that the universe is expanding with orbiting will be considered in the next paper by using the relation (20).

Now, which geodesic follows the universe motion? In the case of a null geodesic ($\varepsilon = 0$), the velocity equation (21) is transformed into the relation:

$$v = \pm kc . \tag{22}$$

This means that a velocity is constant because the parameters k and c are constants. Meanwhile, we know from the observations that our universe is expanding at an accelerating rate. Therefore, it is naturally assume that the universe motion follows time-like geodesic ($\varepsilon = 1$). In that case, and assuming that **M** and **r** are the universe mass and radius, respectively, the velocity equation (21) is transformed into the universe velocity relation:

$$v = \pm \left[\frac{2GM}{r} \left(1 - \frac{GM}{2rc^2} \right) + \left(k^2 - 1 \right) c^2 \right]^{1/2} . \tag{23}$$

The sign (+) is valid for an expanding phase, while the sign (-) is related to the contracting one. The velocity equation (23) has two zeros at the positions r_1 and r_2 :

$$r_1 = \frac{GM}{(1+k)c^2}, \qquad r_2 = \frac{GM}{(1-k)c^2} . \tag{24}$$

From the relations (24) one can see that the velocity (23) has two finite zeros only if the energy conservation constant k is les than one ($k < 1$). This corresponds to the cyclic scenario of the universe motion. If $k = 1$, then the velocity (23) has one finite and one infinite zero:

$$r_1 = \frac{GM}{2c^2}, \qquad r_2 = \infty . \tag{25}$$

This corresponds to the flat universe. Finally, if the energy conservation constant k is greater than one ($k > 1$), then the velocity (23) has only one real zero r_1 in (24), because the other one gives a negative r_2 that is not possible. This corresponds to the open scenario of the universe motion.

In order to determine the energy conservation constant k one can employ the first relation in (18). This relation can be transformed into the following form:

$$\varepsilon^2\left[1-\frac{2GM}{rc^2}\left(1-\frac{GM}{2rc^2}\right)\right]=k^2-\frac{v^2}{c^2} \ . \tag{26}$$

For the time-like geodesic ($\epsilon = 1$) the equation (26) has the form:

$$1 - \frac{\dfrac{2GM}{r}\left(1-\dfrac{GM}{2rc^2}\right)}{c^2} = k^2 - \frac{v^2}{c^2} \ . \tag{27}$$

Comparing the left and right side of the equation (27) one can conclude the following:

$$k^2 = 1 \quad \Rightarrow \quad k = \pm 1 \quad \Rightarrow \quad k = 1 \ , \tag{28}$$

because a negative value of k is not possible. For a null geodesic ($\epsilon = 0$) the equation (26) is transformed into the relation:

$$v^2 = k^2 c^2 \ . \tag{29}$$

Applying k = 1 to the equation (29) one obtains:

$$v = \pm c \ . \tag{30}$$

This relation corresponds to the light velocity in a vacuum. The solution of the energy conservation constant (k = 1) is in an agreement with the solution of the cosmological constant problem in [10]. Namely, in the reference [10], it has been discovered that the Ricci scalar of the Ricci tensor is equal to zero (R = 0). This means that our universe is flat. Thus, if the energy conservation constant k is equal to one (k = 1) then our universe is flat. Meanwhile, some kind of the very small quantum fluctuations (that are not included in the presented model) maybe can change the energy conservation constant k = 1 into k < 1. In that case we have the cyclic scenario of the universe motion.
Now, applying k = 1 to the equation (23) one obtains the universe velocity relation in the form:

$$v = \pm \left[\frac{2GM}{r}\left(1-\frac{GM}{2rc^2}\right)\right]^{1/2} . \tag{31}$$

On the other hand, applying k = 1 to the relations (24) one obtains the positions r_1 and r_2 of the two zeros of the universe velocity (31) in the form (25). This solution corresponds to the flat universe.

From the relations (25) one can recognize that the velocity of the universe motion (31) is equal to zero at the minimal or maximal radius:

$$r = r_{min} = \frac{GM}{2c^2}, \quad or \quad r = r_{max} = \infty, \quad \Rightarrow \quad v = 0 \ . \tag{32}$$

Further, the maximal velocity of the universe motion is equal to the speed of the light in a vacuum at the radius:

$$r = r_c = \frac{GM}{c^2} \quad \Rightarrow \quad \dot{r}_{max} = v_{max} = \pm\, c\,. \tag{33}$$

The related scalar potential of a gravitational potential field can be recognized from (14) and (18) in the following form:

$$V_p = -\,\frac{GM\varepsilon^2}{r}\left(1 - \frac{GM}{2rc^2}\right). \tag{34}$$

The corresponding radial acceleration equation, a_c, can be derived by employing the following relation [11]:

$$a_c = -\,\frac{\partial V_p}{\partial r}\,. \tag{35}$$

Now, applying (35) to the relation (34) we obtain the acceleration equation in the form:

$$a_c = -\,\frac{GM\varepsilon^2}{r^2}\left(1 - \frac{GM}{rc^2}\right). \tag{36}$$

Assuming that **M** and **r** are the universe mass and radius, respectively, and taking into account that $\varepsilon = 1$ and $\dot{\theta} = 0$ and $\dot{\phi} = 0$, the relation (36) is transformed into the acceleration equation of the universe motion:

$$a_c = \ddot{r} = -\,\frac{GM}{r^2}\left(1 - \frac{GM}{rc^2}\right). \tag{37}$$

Further, one can introduce the substitution of the gravitational mass by the relation:

$$M = \rho_m \frac{4r^3\pi}{3} = \rho\frac{4r^3\pi}{3c^2}\,, \tag{38}$$

where ρ_m is a mass density and ρ is the related energy density. Applying (38) to (37), we obtain the new form of the acceleration equation:

$$a_c = -\,\frac{4\pi G}{3c^2}\left[\rho - \frac{4\pi G r^2\rho^2}{3c^4}\right]r, \qquad a_c = -\,\frac{4\pi G}{3c^2}\left[\rho + 3p\right]r,$$

$$p = -\,\frac{4\pi G}{c^4}\left(\frac{r\rho}{3}\right)^2, \qquad p = p_r + p_\Lambda = \frac{\rho}{3} - \left(\frac{\rho}{3} + \frac{4\pi G}{c^4}\left(\frac{r\rho}{3}\right)^2\right), \tag{39}$$

$$p_r = \frac{\rho}{3}, \qquad p_\Lambda = -\frac{\rho}{3}\left(1 + \frac{4\pi G r^2\rho}{3c^4}\right).$$

In the relation (39), p_r is a radiation pressure, p_Λ is a pressure of a vacuum energy and p is a total pressure. From (39) one can see that the pressure p_Λ is negative and is greater than the radiation pressure p_r. Therefore, the total pressure p is negative, that gives rise to the expansion of the universe at the present time.

According to the Einstein's theory of the general relativity, the scale factor $a(\tau)$, which represents the expansion of the universe as a function of proper time, satisfies the differential equation:

$$\frac{d^2a}{d\tau^2} = -\frac{4\pi G}{3c^2}(\rho + 3p)a , \tag{40}$$

where ρ is the total energy density, and p is the total pressure. If the gravitational radius **r** in the second equation of (39) is replaced by a scale factor $a(\tau)$, then the second equation from (39) is transformed into the equation (40). This confirms that the derivation procedure of the acceleration equation (39) is correct.

Thus, the equations of the velocity and acceleration of the universe motion, that include the influences of the gravitational mass M, radiation pressure p_r and vacuum energy pressure p_Λ are described by (31) and (37), respectively. The equation of the state of the universe, w or w_m, can be derived by employing the relations (38) and (39):

$$w = \frac{p}{\rho} = -\frac{1}{3}\left(\frac{GM}{rc^2}\right), \quad w_m = \frac{p}{\rho_m} = -\frac{c^2}{3}\left(\frac{GM}{rc^2}\right) = -\frac{GM}{3r} , \tag{41}$$

where w is related to the energy density ρ, while w_m is related to the mass density ρ_m of the universe. From the relation (41) one can see that for $r_{min} = GM / 2c^2$ the universe state is $w = -2/3$, or $w_m = -2c^2/3$. On the other side, for $r \to \infty$, the universe state is $w = w_m = 0$. Thus, the universe states, w and w_m, are changing in the regions $(-2/3 \le w \le 0)$ and $(-2c^2/3 \le w_m \le 0)$, respectively.

3 Estimation of the Present Universe Mass, Radius, Acceleration, Velocity, Time and State

It is well-known that at the large scale the universe is isotropic and homogenous in the sense of the mater density, ρ_m:

$$\rho_m = \frac{3M_1}{4r_1^3\pi} = \frac{3M_2}{4r_2^3\pi} , \tag{42}$$

where M_1 and M_2 are the masses in the spheres of the radius r_1 and r_2, respectively. After including $r_i = v_i\, t$, $i = 1, 2$, in the equations (42), we obtain a very useful formula:

$$\frac{M_2}{M_1} = \left(\frac{v_2}{v_1}\right)^3 , \tag{43}$$

where v_i is the velocity at the radius r_i, $i = 1, 2$. Now, substituting $M_i = \rho_m 4r_i^3\pi/3$, $i = 1, 2$, into the relation (43), one can derive the well-known Hubble parameter equation:

$$H_B = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v}{r} = const. \tag{44}$$

From the observation we know that the universe velocity is increasing at the present time. Therefore, the present universe velocity should be between its initial and maximal velocity. Following the velocity equation (31) and the relations (25), (32) and (33), we know that the initial velocity is at the radius $r_o = r_{min} = GM/2c^2$, while the maximal velocity is at the radius $r_c = GM/c^2$. Now, taking into account known present mass density ρ_{mp} and the relation (42), one can calculate the expected maximal universe mass at the radius r_o and the expected minimal universe mass at the radius r_c :

$$M_{max} = \left(\frac{6c^6}{\pi G^3 \rho_{mp}} \right)^{1/2} , \quad M_{min} = \left(\frac{3c^6}{4\pi G^3 \rho_{mp}} \right)^{1/2} . \tag{45}$$

Employing the well known present mass density $\rho_{mp} = 1*10^{-26}$ kg / m^3 , we obtain from the equations (45):

$$M_{min} = 2.420962 * 10^{53} \ kg \leq M \leq 6.847514 * 10^{53} \ kg = M_{max} \ . \tag{46}$$

So, the real universe mass should be between these two values. In order to calculate the real universe mass one can employ the equations (42) to (44), and the velocity equation (31). Starting with the substitution $M = \rho_m 4r^3\pi / 3$ into the velocity equation (31) we obtain the new velocity equation in the form:

$$v^2 = \frac{8\pi G \rho_m}{3} r^2 \left(1 - \frac{2\pi G \rho_m}{3c^2} r^2 \right) . \tag{47}$$

Taking into account the Hubble parameter equation (44), this relation can be transformed into the new one:

$$\frac{v^2}{r^2} = H_B{}^2 = \frac{8\pi G \rho_m}{3} \left(1 - \frac{2\pi G \rho_m}{3c^2} r^2 \right) , \tag{48}$$

which can be employed for calculation of the present universe radius:

$$r = \left[\frac{3c^2}{2\pi G \rho_m} \left(1 - \frac{3H_B{}^2}{8\pi G \rho_m} \right) \right]^{1/2} . \tag{49}$$

This calculation is possible because parameters ρ_m and H_B are known from the observations. Very important limitations of the universe radius r and Hubble parameter H_B can be derived from (47) and (49), by using the conditions $\mathbf{v^2 \geq 0}$ and $\mathbf{r \geq 0}$, respectively:

$$r^2 \leq \frac{3c^2}{2\pi G \rho_m} , \qquad H_B{}^2 \leq \frac{8\pi G \rho_m}{3} . \tag{50}$$

Employing the well known present mass density $\rho_{mp} = 1*10^{-26}$ kg / m^3 , we obtain, from (50), the present limitations of the universe radius r, Hubble parameter H_B and the universe expanding velocity v_p at the radius 1Mpc:

$$r \leq 2.5384*10^{26}\ m\,, \qquad H_B \leq 2.3637*10^{-18}\ s^{-1}\,,$$
$$v_p = H_B * 1Mpc \quad , \quad \Rightarrow \quad v_p \leq 72.920\ km/s/1Mpc\ . \tag{51}$$

These limitations, as well as the limitation (46), are very important for determination of the present universe parameters. Now, knowing the present universe radius from (49) and mass density from the observations, one can calculate the universe mass, using the following formula:

$$M = \frac{4r^3\pi\rho_m}{3}\ . \tag{52}$$

The present universe velocity and acceleration can be than calculated by employing the equations (31) and (37), respectively:

$$v = \pm\left[\frac{2GM}{r}\left(1-\frac{GM}{2rc^2}\right)\right]^{1/2}, \qquad a_c = -\frac{GM}{r^2}\left(1-\frac{GM}{rc^2}\right)\ . \tag{53}$$

For derivation of the proper time equation of the universe one can employ the following relations:

$$\tau = \tau_p + \tau_e\,,\ \ d\tau_e = \frac{dr}{v},\ \ \tau_e = \frac{r^2}{3GM}\left(1+\frac{GM}{rc^2}\right)\left[\frac{2GM}{r}\left(1-\frac{GM}{2rc^2}\right)\right]^{1/2}, \tag{54}$$

where **v** is from (53), τ_e is the proper time of the current expanding phase, while τ_p is the proper time of the previous motion phase. At the initial radius $r_o = r_{min} = GM / 2c^2$, the proper time $\tau_e = 0$. The proper time τ_p depends on the previous motion phase. In the case of the cyclic scenario of the universe motion the proper time τ_p is the final time point of the previous cycle and the initial time point of the next one. Therefore, one can put $\tau_p = 0$. Meanwhile, if the previous phase of the universe motion is started with the Big Bang and followed by the inflationary scenario, then the time τ_p is the final time point of this motion phase and the initial time point of the current expanding phase. It is well known that the estimated age of the universe is about 15 billion years ($\tau \cong 15*10^9$ years). The recent estimation in [12] suggests that the universe is 13.7 billion years old. The margin of error is about 1 percent. Thus, taking into account time duration of the current expanding phase τ_e from (54), and the estimated age of the universe τ, one can calculate the time duration of the previous inflationary phase τ_p using the equation:

$$\tau_p = \tau - \tau_e\ . \tag{55}$$

For calculation of the present states of the universe, **w** or $\mathbf{w_m}$, one can employ the relation (41). Now, we are ready to calculate the present universe parameters as Λ, radius, mass, velocity, acceleration, pressure, time and state. Employing the equations (2), (32), (33), (39), (41), (49) and (52) to (55) one can estimate the present values of the main parameters of the universe motion. Thus, the universe parameters are calculated, in the table 1, for fixed mass density $\rho_m = 1*10^{-26}$ kg/m3 and different velocity v_p at 1 Mpc = $3.085*10^{22}$ m.

Table 1: The universe parameters as functions of a velocity v_p at the radius 1Mpc

v_p, km/s/1Mpc	40	45	50	55
H_B, s^{-1}	$1.2966*10^{-18}$	$1.4587*10^{-18}$	$1.6207*10^{-18}$	$1.7828*10^{-18}$
r, m	$2.1224*10^{26}$	$1.9974*10^{26}$	$1.8477*10^{26}$	$1.6667*10^{26}$
M, kg	$4.0027*10^{53}$	$3.3363*10^{53}$	$2.6410*10^{53}$	$1.9384*10^{53}$
v, m/s	$2.7519*10^{8}$	$2.9135*10^{8}$	$2.9946*10^{8}$	$2.9714*10^{8}$
a_c, m/s^2	$2.3612*10^{-10}$	$1.3301*10^{-10}$	$0.3082*10^{-10}$	$-0.6413*10^{-10}$
r_c, m	$2.9676*10^{26}$	$2.4735*10^{26}$	$1.9580*10^{26}$	$1.4371*10^{26}$
r_{min}, m	$1.4838*10^{26}$	$1.2368*10^{26}$	$0.9790*10^{26}$	$0.7186*10^{26}$
τ_e, y	$11.7651*10^{9}$	$12.3537*10^{9}$	$12.6300*10^{9}$	$12.5620*10^{9}$
τ_p, y	$1.9349*10^{9}$	$1.3463*10^{9}$	$1.0700*10^{9}$	$1.1380*10^{9}$
w, -	-0.4661	-0.4128	-0.3532	-0.2874
w_m, m^2/s^2	$-4.1947*10^{16}$	$-3.7151*10^{16}$	$-3.1791*10^{16}$	$-2.5668*10^{16}$
p, N/m^2	$-4.1947*10^{-10}$	$-3.7151*10^{-10}$	$-3.1791*10^{-10}$	$-2.5868*10^{-10}$
Λ, m^{-2}	$4.3401*10^{-53}$	$3.8439*10^{-53}$	$3.2894*10^{-53}$	$2.6765*10^{-53}$

In the table 1, **v** is the expanding velocity at the present universe radius **r**. For calculation of the parameter τ_p it has been employed the recent estimated age of the universe τ = **13.7** billion years. In the case where v_p = **45 km/s/1Mpc**, the estimated parameter τ_e from (54) tell us that the current expanding phase of the universe has been started about **12.3537** billions years ago. Of course, if the mass density ρ_m is different from $1*10^{-26}$ kg/m^3, then the proposed model of the universe motion will give different values compare to the table 1. From the table 1 we can see that the columns with v_p equal to 40; 45; and 50 satisfy the both limitations (46) and (51). Meanwhile, the last column, with v_p equal to 55, satisfies only the limitations (51).

4 Analysis of Acceleration and Velocity Equations of the Universe Motion

From the equations (37) and (53), one can see that the universe acceleration can be positive or negative. The change from negative to positive acceleration (and vice-versa) takes place at the radius $r = r_c = GM / c^2$. The universe acceleration is negative in the region $r > (GM/c^2)$, and positive in the region $r < (GM/c^2)$. At the present time our universe has got the positive acceleration that is the reason for its present accelerating expanding. The velocity equation of the universe motion is given by (31) and (53). In the case that the energy conservation constant k is equal to one (k =1), as it is shown in the section 2, the universe velocity has got two zeros. The first one is at the point $r = (GM/2c^2)$, and the second zero is obtained for $r \to \infty$. The universe velocity can also be positive or negative, depending on initial conditions, and has a maximum (or a minimum) at the point $r = r_c = (GM/c^2)$. The maximal velocity is equal to the speed of the light in vacuum, $v_{max} = c$, and is positive in the expanding phase, and negative in the contracting one. The change from the contracting phase into the expanding one, takes place at the minimal radius, $r = r_{min} = (GM/2c^2)$, where velocity is equal to zero, and the acceleration is maximal and positive. The minimal radius is proportional to the mass, and can be equal to zero only if the mass is equal to zero.

It seams that the change from the expanding phase into the contracting one, for the model with the energy conservation constant k equal to one (k =1), is not possible. It is because both the universe acceleration and velocity are tend to zero at an infinite radius ($r \rightarrow \infty$). For that case, the proposed model describes the universe that will approach to the maximal velocity, $v_{max} = c$, and than the rate of the expansion, or velocity, will decreasing going to zero at an infinite radius. Meanwhile, some kind of the very small quantum fluctuations (that are not included in the presented model) maybe can stop this expansion and change it into the contraction. If it will happen, then the contracting phase will start with the motion in the opposite direction. The velocity will slowly increase to the speed of the light in a vacuum at the radius $r = r_c = (GM/c^2)$, and than will decrease to the zero at the minimal radius $r_{min} = (GM/2c^2)$. At this point one cycle will finish and the second one will start with a new expansion phase. In the case $k > 1$, the universe will expand forever. On the other side, if the energy conservation constant $k < 1$, then the proposed model describes the cyclic scenario of the universe motion. The graphical presentation of the scalar potential V_p , eq. (34) with $\epsilon = 1$ and $v_p = 45$ km/s/Mpc, is shown in the Fig. 1.

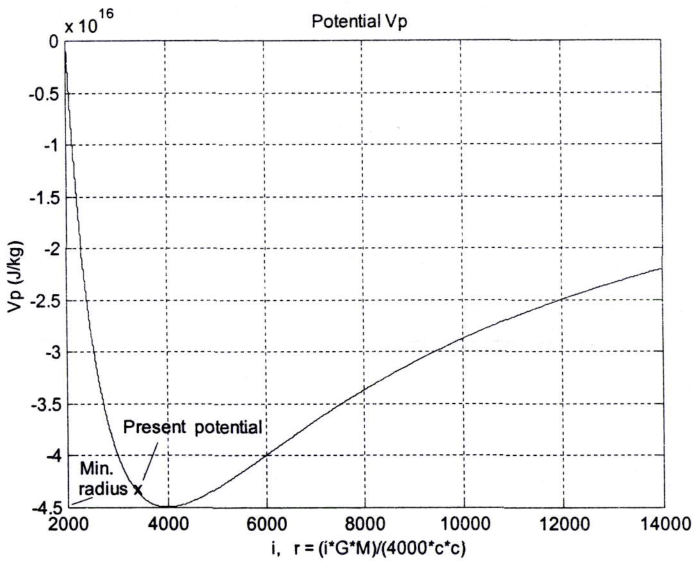

Figure 1: The scalar potential $\mathbf{V_p}$ as the source of the universe motion, eq. (34).

The universe acceleration, eq. (37), is presented in the Fig. 2. The simulation of the expanding and contracting velocities, eq. (31), together with the universe acceleration, eqs. (37), and including the energy conservation constant k = 1, is shown in the Fig. 3. Finally, the state of the universe, eq. (41), is presented in the Fig. 4.

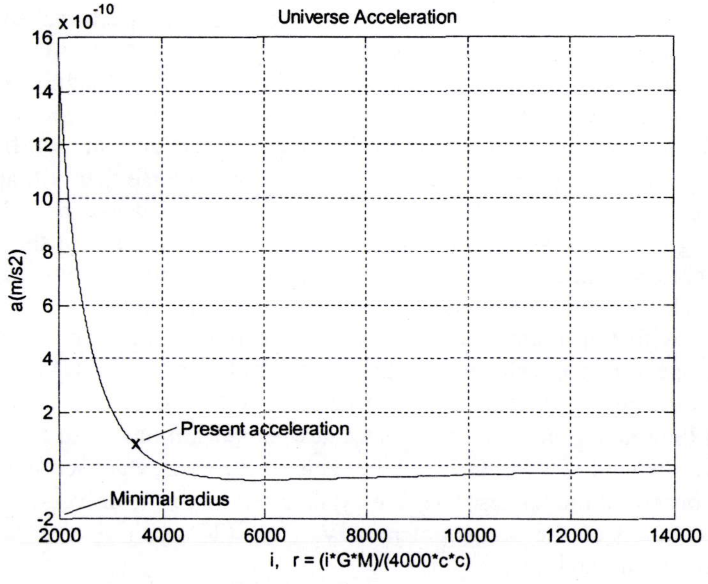

Figure 2: The universe acceleration, $\mathbf{a_c}$, eq. (37).

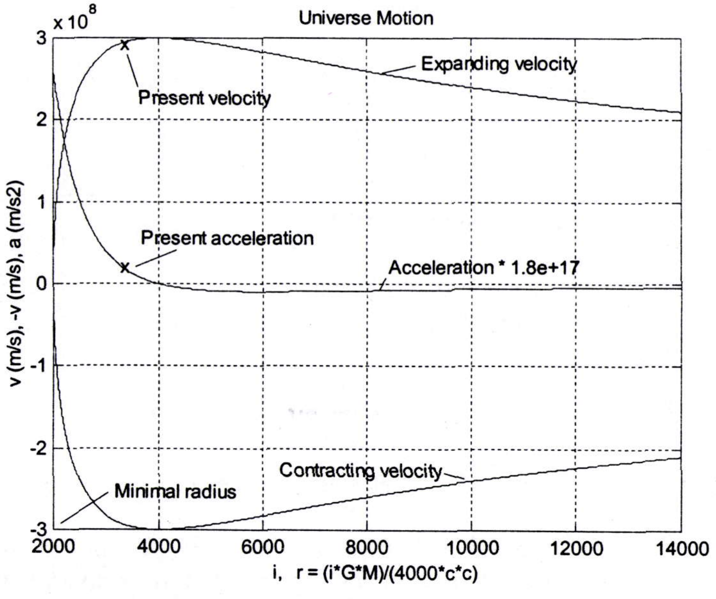

Figure 3: The universe velocities and acceleration, $\mathbf{v}$, eq. (31), $\mathbf{a_c}$, eq. (37).

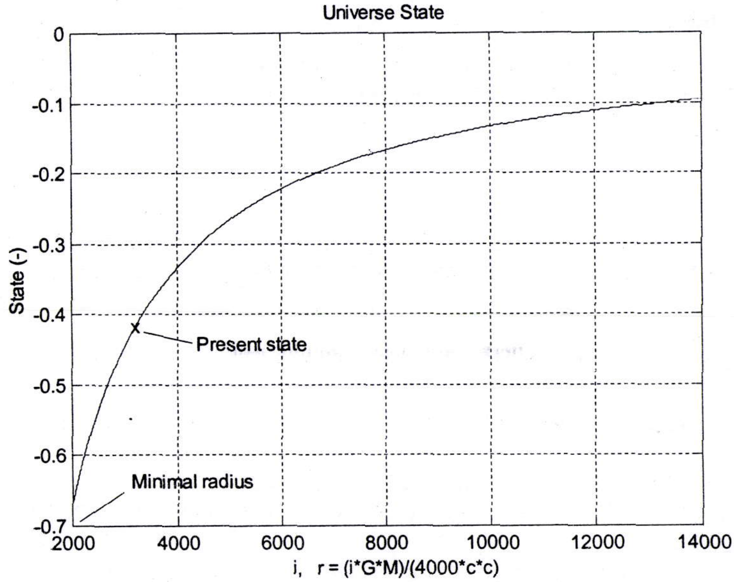

Figure 4: The state of the universe, **w** , eq. (41).

5 Conclusion

If the proposed model of the universe motion is correct, then it gives some new specific answers to the important questions of the cosmology [7]. Thus, the minimal radius of the universe, $r_{min} = GM/2c^2$, confirms that the initial singularity does not exist, if the initial mass is different from zero. Based on the recent observation data and the time equations (54) and (55) it is shown that the current expanding phase of the universe motion is started about 12.4 billions years ago. The present universe radius is about $1.99 * 10^{26}$ m. The acceleration equation of the universe motion shows that the universe acceleration can be attractive (negative) or repulsive (positive). This repulsive acceleration gives rise to the accelerating expansion of the universe at the present time.

The change from negative to positive acceleration (or vice-versa) takes place at the radius $r = GM/c^2$. The change from the contracting phase into the expanding one, takes place at the minimal radius, $r_{min} = GM/2c^2$, where velocity is equal to zero. If the energy conservation constant $k = 1$, then both the universe acceleration and velocity are tend to zero for $r \to \infty$. After approaching the maximal velocity, $v_{max} = c$, the expansion velocity will decreasing going to zero at an infinite radius. Maybe some kind of the small quantum fluctuations (that are not included in the presented model) will stop this expansion and change it into the contraction. If $k > 1$, then the situation is similar to the previous one, but the expansion velocity will decreasing going to the constant velocity at an infinite radius. Finally, if $k < 1$ then the proposed model describes the cyclic scenario of the universe motion. The further investigation of the energy conservation constant k will take the place in our future work.

Acknowledgements

This work was supported by grants from the National Scientific Foundation of Republic of Croatia. The authors wish to thank the anonymous reviewers for a variety of helpful comments and suggestions.

References

[1] Einstein A. (1955). The Meaning of Relativity. 5th ed., Princ. Univ. Press, Princet., N. J.

[2] Guth A. H. (1981). Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. Phys. Rev. **D23**, 347 (1981); New York Academy of Sciences Magazine, Jan./Febr., (2002).

[3] Perlmutter S. et al. (1998). Nature **391**, 51 (1998); Riess A. et al., Astron. J. **116** 1009 (1998).

[4] Caldwell R. R. et al. (1998). Phys. Rev. Lett. **80**, 1582 (1998), Caldwell R. R., astro-ph/9908168.

[5] Friedmann A. (1924). Über die Krümmung des Raumes . Z. Physik, **21**, p. 326, (1924).

[6] Parker L. and Raval A. (1999). Phys. Rev. D **60**, 063512 (1999) ; **60**, 123502 (1999) ; **62**, 083503 (2000).

[7] Steinhard P. J. and Turok N. (2002). http://feynman.princeton.edu/~steinh/ , hepth/0111030, Science, in the press (2002), hepth/0111098, Phys. Rev. **D**, in the press (2002).

[8] Novakovic B., Novakovic D. and Novakovic A. (2000). A New General Lorentz Transformation Model . AIP Conf. Proc., **CP 517**, CASYS'99-Third Int. Conf., ed. by D. M. Dubois, pp. 437-450, Liege, Belgium, 1999.

[9] Novakovic B., Novakovic D. and Novakovic A. (2001). A Metric Tensor of the New General Lorentz Transformation Model. CASYS Int. Journal of Comp. Anticipatory Systems, ed. by D. M. Dubois, Publ. by CHAOS, Liege, Belgium, Vol. 10, pp. 199-217.

[10] Novakovic B., Novakovic D. and Novakovic A. (2004). The Cosmological Constant Λ is not Really Constant but the Function of a Gravitational Radius. AIP Conf. Proc. in press (2004), ed. by D. M. Dubois, presented at CASYS'03, Sixth Int. Conf. On Comput. Anticip. Syst., Liege, Belgium, Aug. 11-16, 2003.

[11] Novakovic B., Novakovic D. and Novakovic A. (2002). A new Approach to Unification of Potential Fields Using GLT Model. CASYS Int. Journal of Comp. Anticipatory Systems, ed. by D. M. Dubois, Publ. by CHAOS, Liege, Belgium, Vol. 11, pp. 196-211.

[12] Sterling B. (2003). Surprise! Our Little Corner of the Universe is Even Smaller than we Thought . *www.wired.com/wired/archive/11.11/view.html?pg=4.*

[13] Kremer G. M. (2003). Cosmological Models Described by a Mixture of der Waals Fluid and Dark Energy. Phys. Rev. D **68**, 123507 (2003) (5 pages).

[14] Jorás S. E. and Stuchi T.J. (2003). Chaos in a Closed Friedmann-Robertson-Walker Universe: An Imaginary Approach. Phys. Rev. D **68**, 123525 (2003) (12 pages).

[15] Susperregi M. (2003). Dark Energy and Dark Matter from an Inhomogeneous Dilaton. Phys. Rev. D **68**, 123509 (2003) (9 pages).

[16] Castro P. G., Douspis M. and Ferreira P. G. (2003). Scale of Homogeneity of the Universe from WMAP. Phys. Rev. D **68**, 127301 (2003) (3 pages).

[17] Avelino P. P., Martins C. J. A. P., Santos C. and Shellard E. P. S. (2003). Topological Defects: A Problem for Cyclic Universe. Phys. Rev. D **68**, 123502 (2003) (15 pages).

[18] Khoury J., Steinhardt P. J. and Turok N. (2004). Designing Cyclic Universe Models. Phys. Rev. Lett. **92**, 031302 (2004).