In-Service Inspections of Multiple Systems Under Availability Requirement

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Abstract
System failures are usually observed during regular maintenance or inspection and this is especially the case for systems in standby or storage, which is common for safety critical systems. A periodic inspection policy is usually adopted. During the inspection, a lot of information is gained about the status of the system. Such information should be used in deciding upon the time for the next inspection. Hence sequential inspection is more appropriate, especially when the aging property of the system is unknown, and has to be estimated with the information from inspection. In this paper, a model is developed and sequential inspection strategies are studied in this situation. The focus is on the case when there are multiple systems inspected at the same, but discrete times. We also do not assume a known distribution of the system lifetime, and the estimation of that is incorporated into the analysis and decision-making. An availability criterion is considered and numerical example is provided to illustrate the procedure.
Keywords: Multiple systems; Availability; In-service inspections.

1 Introduction

A system can either be in an operational state or a down state. When the system is in a down state, which represents the failure condition of the deteriorating process, this event is named as a failure of the system. However, the state of the system may actually be usually unknown unless it is inspected. This is typical for systems in storage and for systems that can still perform a limited function after failure of some of its components. Although continuous monitoring and inspection is possible, periodic or discrete time inspection is usually employed, due to the cost and other practical constrictions. Here the term of periodic inspection used does not necessarily mean that the system is inspected after every fixed period of time regularly, but rather that the system is inspected at some discrete points in time.

For systems for which failures are only detected at the time of inspection, it is important to be able to determine the optimal time of inspection. Fewer inspections will lead to lower availability upon demand, and frequent inspection will lead to higher cost. When there is an availability requirement, the problem is usually to develop an inspection policy that meets the availability requirements. The problem is formulated for periodic inspection to minimize the cost with respect to the time interval for inspection (Cerone, 1993; Ito and Nakagawa, 2000; Hariga, 1996; Vaurio, 1999). Most of the optimal policies are derived based on average cost or reliability that are valid only asymptotically. In general, the system reaching the limiting or steady-state, for example, requires a long period of usage and usually it is not clear when its asymptotic results are accurate enough.

Yang and Klutke (2000) studied some inspection schemes, in which they focused on steady-state availabilities for several models under some inspection schemes. The inspection policy defined is based on the availability of the system when it is required. It is given as follows.

Suppose an inspection is carried out at time t, and this shows that a down state (or failure) of the system has not yet occurred. We now have to schedule the next inspection. Let Y be the random time to failure of the system. Then we schedule the next inspection at time $\tau > t$, where τ satisfies

$$\Pr\{Y > \tau; Y > t\} = 1 - \alpha. \tag{1}$$

Equation (1) says that the next inspection is scheduled so that, with probability $1-\alpha$, the system is still working and free of a failure prior to inspection. Let $F_\theta(y)$ be the cumulative distribution function of the time to failure, where θ is a known parameter (in general, vector). Then the inspection times (τ_1, τ_2, ...) can be calculated recursively in the following way. It follows from (1) that

$$\frac{\overline{F}_\theta(\tau_{j+1})}{\overline{F}_\theta(\tau_j)} = 1 - \alpha, \quad j \geq 0, \tag{2}$$

where $\overline{F}_\theta(\tau) = 1 - F_\theta(\tau)$, $\tau_0=0$, $\overline{F}_\theta(\tau_0) = 1$. It can be shown that (2) is equivalent to the equation

$$1 - \frac{\overline{F}_\theta(\tau_{j+1})}{\overline{F}_\theta(\tau_j)} = \frac{\overline{F}_\theta(\tau_j) - \overline{F}_\theta(\tau_{j+1})}{\overline{F}_\theta(\tau_j)} = \frac{1 - F_\theta(\tau_j) - [1 - F_\theta(\tau_{j+1})]}{1 - F_\theta(\tau_j)} = \frac{F_\theta(\tau_{j+1}) - F_\theta(\tau_j)}{1 - F_\theta(\tau_j)} = \alpha, \; j \geq 0, \tag{3}$$

that is, in other words, the probability that the failure occurs in the time interval (τ_j, τ_{j+1}) without failure at time τ_j is always assumed α.

It follows from (2) that

$$\overline{F}_\theta(\tau_{j+1}) = (1 - \alpha)\overline{F}_\theta(\tau_j), \quad j \geq 0. \tag{4}$$

With $\tau_0=0$, τ_1, τ_2, ... can be calculated recursively from (4). So that:

$$\overline{F}_\theta(\tau_j) = (1-\alpha)^j, \quad j = 1, 2, 3, ..., \tag{5}$$

the time τ_j (j=1, 2, 3, ...) is given by

$$\tau_j = \overline{F}_\theta^{-1}[(1-\alpha)^j], \quad j = 1, 2, 3, ... \tag{6}$$

Let N be the random number of inspections until the failure occurs. Then

$$\Pr\{N \le j\} = \Pr\{Y \le \tau_j\} = F_\theta(\tau_j) \tag{7}$$

and

$$E\{N\} = \sum_{j=0}^{\infty} j \Pr\{N = j\} = \sum_{j=1}^{\infty} j [\Pr\{N > j-1\} - \Pr\{N > j\}]$$

$$= \sum_{j=0}^{\infty} P\{N > j\} = \sum_{j=0}^{\infty} \overline{F}_\theta(\tau_j) = \alpha^{-1}. \tag{8}$$

For example, if α=0.05 then, from (8), on average 20 inspections will be necessary. Thus,

$$\tau_1 = \sup\{t > 0: \Pr\{Y > t\} \ge 1-\alpha\}, \tag{9}$$

$$\tau_j = \sup\{t > \tau_{j-1}: \Pr\{Y > t;\ Y > \tau_{j-1}\} \ge 1-\alpha\},\ \ j \ge 2. \tag{10}$$

This inspection policy is named as "quantile-based inspection policy". It makes use of the information about the remaining life that is inherent in the sequence of previous inspection times. The value of $1-\alpha$ can be seen as "minimum availability required" during the next period when the system was still operational at last inspection time. However, it is assumed in (Yang and Klutke, 2000) that the distribution of system lifetime is known and the policy is used for a single system, that is, only one item is inspected. In many cases, there can be several similar systems operating simultaneously on one site or under the same environment. The distribution of system lifetime, in most cases, is also unknown.

Lam and Yeh (1994) discussed a sequential policy and compared it with some continuous strategies based on a finite-state continuous-time Markov model; see also (Yeh, 1997). Brint (2000) discussed the problem of sequential inspection sampling for maintained system and presented a Bayesian formulation. Chelbi and Ait-Kadi (2000) also considered some general inspection strategies. These papers all deal with the case of a single system.

In this paper, we develop a sequential inspection policy for the case when there are a number of systems on the site or in storage. The condition of the systems is inspected and failed systems are replaced or repaired in such a way that they become as good as new. The time to the next inspection is determined based on the results of previous inspections. Furthermore, the system lifetime distribution is not assumed to be known, but incorporated into the analysis and decision-making. Availability criterion is discussed and numerical example is provided to illustrate the procedure.

2 In-Service Inspections of Multiple Systems

We assume that there are m items (or systems) in the field and they are inspected at the same, but discrete times. The following assumptions are used: (i) the inspections are carried out at times τ_1, τ_2, ... ; and all m systems are inspected each time; (ii) failures are observed only by inspection, and replacement or perfect repair is carried out for failed systems; (iii) the inspection action does not intervene with the system if failure is not found; (iv) the inspection time and repair/replacement time are negligible; (v) the lifetimes of all systems have the same distribution with cdf $F_\theta(y)$.

2.1 Complete Information About $F_\theta(y)$

Let us assume that the parameter θ is known and $1-\alpha$ is the required availability or the probability that the system is still functioning (i.e., it is in an operational state). Let

$$Y_{j(k)} = \begin{cases} 1, & \text{if a failure of the } k\text{th system is detected at the } j\text{th inspection;} \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

When the reliability function $\overline{F}_\theta(\tau)$ of the kth system is known and there is only one system k, then it follows from (4) that the time to the next inspection is $\Delta_{j(k)}$ which is the solution of

$$\overline{F}_\theta(\tau_{j(k)} + \Delta_{j(k)}) = (1-\alpha)\overline{F}_\theta(\tau_{j(k)}), \quad j \geq 0, \tag{12}$$

assuming $Y_{1(k)} = \cdots = Y_{j(k)} = 0$, where $\tau_{j(k)}$ is the time of the jth inspection of the kth system. To determine the time to the next inspection of all the m items (or systems) it can be used the following criterion:

$$\Delta_j = \min_{1\leq k\leq m} \Delta_{j(k)}. \tag{13}$$

Then the time of the next inspection of all the m systems is given by

$$\tau_{j+1}=\tau_j+\Delta_j. \tag{14}$$

In this case, every system in the field satisfies the minimum availability not less than $1-\alpha$ when the system is operational on the last inspection time.

It will be noted that the time of the first inspection of all the m systems, τ_1, can be found from

$$\Pr\{\min_{1\le k\le m} Y_k \ge \tau_1\}=1-\alpha, \tag{15}$$

where Y_k is a random variable representing a lifetime of the kth system, $k\in\{1, \ldots, m\}$.

2.2 Incomplete Information About $F_\theta(y)$

Let us assume that the parameter θ is not known, but there is a sample of observations $\mathbf{X}^n=(X_1, X_2, \ldots, X_n)$ from $F_\theta(x)$ as the results of tests conducted on the similar systems. Let Y be the random time to failure of the system and $F_\theta(y) \equiv F_\theta(x)$. After the jth inspection at time τ_j, the state of each system is observed and the state vector is denoted by $\mathbf{Y}_j=(Y_{j(1)},\ldots,Y_{j(m)})$. We summarize the information as

$$\mathbf{X}^{n+jm}=(\mathbf{X}^n, \mathbf{Y}_1, \ldots, \mathbf{Y}_j). \tag{16}$$

Let $W_{j+1}^k= W(Y_k, \tau_{j(k)},\mathbf{X}^{n+jm})$ be an ancillary statistic (Nechval, 1982, 1984; Nechval et al., 2000, 2001a, 2001b, 2002) whose distribution does not depend on θ, where Y_k is the random time to failure of system k. The inspection times ($\tau_{1(k)}$, $\tau_{2(k)}$, ...) for only one system k can be calculated recursively on the basis of relation

$$\Pr\{Y_k>\tau_{j+1(k)};Y_k>\tau_{j(k)},\mathbf{X}^{n+jm}=\mathbf{x}^{n+jm}\}=\frac{\Pr\{Y_k>\tau_{j+1(k)};\mathbf{X}^{n+jm}=\mathbf{x}^{n+jm}\}}{\Pr\{Y_k>\tau_{j(k)};\mathbf{X}^{n+jm}=\mathbf{x}^{n+jm}\}}$$

$$=\frac{\Pr\{W_{j+1}^k>w_{j+1}^k;\mathbf{X}^{n+jm}=\mathbf{x}^{n+jm}\}}{\Pr\{W_j^k>w_j^k;\mathbf{X}^{n+jm}=\mathbf{x}^{n+jm}\}}=1-\alpha, \quad j\ge 0, \tag{17}$$

where

$$W_j^k= W(Y_k, \tau_{j-1(k)},\mathbf{X}^{n+jm}), \tag{18}$$

$$w_j^k = w(\tau_{j(k)}, \tau_{j-1(k)}, \mathbf{x}^{n+jm}), \quad w_{j+1}^k = w(\tau_{j+1(k)}, \tau_{j(k)}, \mathbf{x}^{n+jm}), \tag{19}$$

$\Pr\{W_0^k > w_0^k; \mathbf{X}^n = \mathbf{x}^n\}=1$. In this case, for only one system k, the time to the next inspection is $\Delta_{j(k)}$ which is the solution of (17) assuming $Y_{1(k)} = \cdots = Y_{j(k)} = 0$ and $\tau_{j+1(k)} = \tau_{j(k)} + \Delta_{j(k)}$. To determine the time to the next inspection of all the m systems, it can be used the following criterion:

$$\Delta_j = \min_{1 \le k \le m} \Delta_{j(k)}. \tag{20}$$

Then the time of the next inspection of all the m systems is given by

$$\tau_{j+1} = \tau_j + \Delta_j. \tag{21}$$

In this case, every system in the field satisfies the minimum availability not less than $1-\alpha$ when the system is operational on the last inspection time.

It will be noted that the time to the first inspection, τ_1, can be found from

$$\Pr\{\min_{1 \le k \le m} Y_k > \tau_1; \mathbf{X}^n = \mathbf{x}^n\} = \Pr\{W > w; \mathbf{X}^n = \mathbf{x}^n\} = (1-\alpha), \tag{22}$$

where

$$W = W(\min_{1 \le k \le m} Y_k, \mathbf{X}^n), \quad w = w(\tau_1, \mathbf{x}^n). \tag{23}$$

3 Example

We consider in this example the problem of estimating the time to the first inspection (or warranty period) for a number of aircraft structure components, before which no visually detectable cracks in materials occur, based on the results of previous warranty period tests on the structure components in question. If in a fleet of m aircraft there are m of the same individual structure components, operating independently, the length of time until the first visually detectable crack of the minimum size $a^{\#}$ initially formed in any of these components is of basic interest, and provides a measure of assurance concerning the operation of the components in question. This leads to the consideration of the following problem. Suppose we have observations of times $X_1, ..., X_i, ..., X_n$ to initiation of the visually detectable crack of the size $a^{\#}$ as the results of tests conducted on the components; suppose also that there are m components of the same kind to be put into future use, with times $Y_1, ..., Y_k, ..., Y_m$ to visually detectable crack initiation. Then we want to be able to estimate, on the basis of $X_1, ..., X_i, ..., X_n$, the shortest time $Y_{(1)}$ among the times $Y_1, ..., Y_k, ..., Y_m$ to initiation of the visually detectable crack of the size

$a^{\#}$. In other words, it is desirable to construct lower simultaneous prediction limit (or the time to the first inspection), τ_1, which is exceeded with probability $1-\alpha$ by observations or functions of observations of future sample consisting of m units. In this example, the problem of estimating $Y_{(1)}$, the smallest of m observations from the underlying distribution, based on an observed sample of n observations from the same distribution, is considered. A solution is proposed for constructing a lower simultaneous prediction limit, τ_1, for $Y_{(1)}$. The results have direct application in reliability theory, where the time until the first failure in a group of m items in service provides a measure of assurance regarding the operation of the items.

Here we consider also the problem of sequential inspections. Attention is restricted to invariant families of distributions. The technique used here emphasizes pivotal quantities relevant for obtaining ancillary statistics. It is a special case of the method of invariant embedding of sample statistics into a performance index (Nechval, 1982, 1984; Nechval et al., 2000, 2001a, 2001b, 2002) applicable whenever the statistical problem is invariant under a group of transformations that acts transitively on the parameter space (i.e. in problems where there is a unique best invariant procedure). The analysis of problem considered here is easily seen to be invariant under location and scale changes.

3.1 Data Model

Let $\mathbf{Z}^n=(Z_1, Z_2, \ldots, Z_n)$ be a random sample of observations

$$Z_i = \frac{\ln[a_i(\tau_{j(i)})/a_i(\tau_{j-1(i)})]}{\tau_{j(i)} - \tau_{j-1(i)}}, \quad i=1(1)n; \ j\geq 1, \tag{24}$$

from fatigue tests on a particular type of structural components of aircraft, where $a_i(\tau_{j(i)})$ is the size of the crack which was detected at the time of flight hours $\tau_{j(i)}$ in the ith component. It is assumed that cracks start growing from the time the aircraft entered service, i.e. $\tau_0=0$, and $a_i(\tau_{0(i)})\equiv a(\tau_0)$ (an initial crack size) is approximately between 0.02 and 0.05 mm that was found through quantitative fractography for typical aircraft metallic materials (Barter et al., 2005). Choosing a typical value for initial crack size (e.g., 0.02 mm) is more conservative than choosing an extreme value (e.g., 0.05 mm). This implies that if the lead cracks can be attributed to unusually large initiating discontinuities then the available life increases.

Let us assume that Z_i, $\forall i=1(1)n$, follows the Weibull distribution with the cumulative distribution function

$$F_\theta(z_i) = \begin{cases} 1-\exp[-(z_i/\sigma)^\delta], & z_i > 0, \\ 0, & \text{otherwise,} \end{cases} \tag{25}$$

where $\theta=(\sigma,\delta)$, the parameters σ and δ ($\sigma>0$, $\delta>0$) are not known. For instance, consider the data of fatigue tests on a particular type of structural components of aircraft IL-86 given in Table 1. The data are for a complete sample of size $n = 7$.

Table 1: The data of fatigue tests on a particular type of structural components of aircraft IL-86 for $a(\tau_0)$=0.02 mm initial discontinuity

Component i	Flight hours ($\times 10^4$) $\tau_{1(i)}$	$a_i(\tau_{1(i)})\equiv a^{\#}$ (mm)	$a(\tau_0)$ (mm)	Z_i
1	4.6	2.0	0.02	1.001124
2	5.3	2.0	0.02	0.8689
3	5.7	2.0	0.02	0.807925
4	6.2	2.0	0.02	0.742769
5	6.4	2.0	0.02	0.719558
6	6.9	2.0	0.02	0.667416
7	7.9	2.0	0.02	0.582933

3.1.1 Goodness-of-Fit Testing

We assess the statistical significance of departures from the Weibull model by performing empirical distribution function goodness-of-fit test. We use the S statistic (Kapur and Lamberson, 1977). For complete datasets, the S statistic is given by

$$S=\frac{\sum_{i=[n/2]+1}^{n-1}\left(\frac{\ln(z_{(i+1)}/z_{(i)})}{M_i}\right)}{\sum_{i=1}^{n-1}\left(\frac{\ln(z_{(i+1)}/z_{(i)})}{M_i}\right)}=\frac{\sum_{i=4}^{6}\left(\frac{\ln(z_{(i+1)}/z_{(i)})}{M_i}\right)}{\sum_{i=1}^{6}\left(\frac{\ln(z_{(i+1)}/z_{(i)})}{M_i}\right)}=0.68, \tag{26}$$

The values of M_i are given in Table 13 (Kapur and Lamberson, 1977). The rejection region for the α level of significance is $\{S>S_{n;1-\alpha}\}$.The percentage points for $S_{n;1-\alpha}$ were given by Kapur and Lamberson (1977). For this example,

$$S=0.68 < S_{n=7;\,1-\alpha=0.95}=0.8. \tag{27}$$

Thus, there is not evidence to rule out the Weibull model.

It is well known (Nechval et al., 2003) that if $\hat{\delta}$, $\hat{\sigma}$ are maximum likelihood estimates for δ, σ from a sample of size n, then the quantities

$$W_i = \hat{\delta}\ln\left(\frac{Z_i}{\hat{\sigma}}\right), \quad i=1, 2, \ldots, n-2, \tag{28}$$

form a set of $n-2$ functionally independent ancillary statistics.

3.2 Time to the First Inspection

Let us assume that we deal with a fleet of the m aircraft, which are entered service. Then it can be shown that for the single structural component of each of the m aircraft with $a(\tau_0)\in(0.02, 0.05)$ at the time $\tau_0=0$, the first inspection time (or warranty period), τ_1, is given by

$$\tau_1 = \frac{\ln[a^{\#}/a(\tau_0)]}{\hat{\sigma}\exp(w/\hat{\delta})} \tag{29}$$

with w which satisfies

$$\Pr\{Y_{(1)} > \tau_1; \mathbf{Z}^n = \mathbf{z}^n\} = \Pr\{W < w; \mathbf{Z}^n = \mathbf{z}^n\} = 1 - \Pr\{W > w; \mathbf{Z}^n = \mathbf{z}^n\} = 1-\alpha, \tag{30}$$

where

$$W = \hat{\delta}\ln\left(\frac{1}{\hat{\sigma}}\frac{\ln[a^{\#}/a(\tau_0)]}{Y_{(1)}}\right) \tag{31}$$

is an ancillary statistic whose distribution does not depend on the unknown parameters δ and σ (Nechval et al., 2003), $a^{\#}$ is the initial size of the visually detectable crack, $Y_{(1)}$ is the minimum time to initiation of this crack in the above components, $\hat{\sigma}$ and $\hat{\delta}$ are the MLE's of σ and δ, respectively, and can be found (by a fixed point iteration) from solution of

$$\hat{\sigma} = \left(\frac{1}{n}\sum_{i=1}^{n} z_i^{\hat{\delta}}\right)^{1/\hat{\delta}}, \tag{32}$$

$$\hat{\delta} = \left[\left(\sum_{i=1}^{n} z_i^{\hat{\delta}}\ln z_i\right)\left(\sum_{i=1}^{n} z_i^{\hat{\delta}}\right)^{-1} - \frac{1}{n}\sum_{i=1}^{n}\ln z_i\right]^{-1}. \tag{33}$$

3.3 Sequential Inspections

Let us suppose now that an inspection is carried out at time τ_j, and this shows that the visually detectable crack of the initial size $a^{\#}$ has not yet occurred. We now have to schedule the next inspection. Then the inspection times can be calculated recursively as

$$\tau_{j+1} = \tau_j + \min_{1\le k\le m}\left(\frac{\ln[a^{\#}/a_k(\tau_j)]}{\widehat{\sigma}_j \exp(w_{j+1}^k/\widehat{\delta}_j)} \right), \tag{34}$$

where $a_k(\tau_j)$ is the crack size which was found (say, through quantitative fractography) at the time of flight hours τ_j in the kth component, $k\in\{1, \ldots, \mathrm{m}\}$,

$$a(\tau_0) < \max_{1\le k\le m} a_k(\tau_j) < a^{\#}, \tag{35}$$

w_{j+1}^k is determined from

$$\Pr\{Y_k > \tau_{j+1}; Y_k > \tau_j, \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\} = \frac{\Pr\{Y_k > \tau_{j+1}; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}}{\Pr\{Y_k > \tau_j; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}}$$

$$= \frac{\Pr\{W_{j+1}^k < w_{j+1}^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}}{\Pr\{W_j^k < w_j^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}} = \frac{1-\Pr\{W_{j+1}^k > w_{j+1}^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}}{1-\Pr\{W_j^k > w_j^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}} = 1-\alpha, \tag{36}$$

$$\mathbf{Z}^{n+jm} = (Z_1, \ldots, Z_n, Z_1(1), \ldots, Z_m(1), \ldots, Z_1(r), \ldots, Z_m(r), \ldots, Z_1(j), \ldots, Z_m(j))$$

$$\equiv (Z_1, \ldots, Z_{n+jm}), \tag{37}$$

$$Z_k(r) = \frac{\ln[a_k(\tau_r)/a_k(\tau_{r-1})]}{\tau_r - \tau_{r-1}}, \quad k=1(1)m, \quad r=1(1)j, \tag{38}$$

$$W_j^k = \widehat{\delta}_j \ln\left(\frac{1}{\widehat{\sigma}_j} \frac{\ln[a^{\#}/a(\tau_{j-1})]}{Y_k - \tau_{j-1}} \right), \quad w_j^k = \widehat{\delta}_j \ln\left(\frac{1}{\widehat{\sigma}_j} \frac{\ln\left[a^{\#}/a_k(\tau_{j-1})\right]}{\tau_j - \tau_{j-1}} \right), \tag{39}$$

$$\Pr\{W_j^k > w_j^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}$$

$$= \frac{\int_0^\infty s^{n+jm-2} e^{s\hat{\delta}_j \sum_{i=1}^{n+jm} \ln(z_i/\hat{\sigma}_j)} \left(e^{sw_j^k} + \sum_{i=1}^{n+jm} e^{s\hat{\delta}_j \ln(z_i/\hat{\sigma}_j)} \right)^{-(n+jm)} ds}{\int_0^\infty s^{n+jm-2} e^{s\hat{\delta}_j \sum_{i=1}^{n+jm} \ln(z_i/\hat{\sigma}_j)} \left(\sum_{i=1}^{n+jm} e^{s\hat{\delta}_j \ln(z_i/\hat{\sigma}_j)} \right)^{-(n+jm)} ds}, \tag{40}$$

$$W_{j+1}^k = \hat{\delta}_j \ln\left(\frac{1}{\hat{\sigma}_j} \frac{\ln[a^\# / a(\tau_j)]}{Y_k - \tau_j} \right), \quad w_{j+1}^k = \hat{\delta}_j \ln\left(\frac{1}{\hat{\sigma}_j} \frac{\ln[a^\# / a_k(\tau_j)]}{\tau_{j+1} - \tau_j} \right), \tag{41}$$

$$\Pr\{W_{j+1}^k > w_{j+1}^k; \mathbf{Z}^{n+jm} = \mathbf{z}^{n+jm}\}$$

$$= \frac{\int_0^\infty s^{n+jm-2} e^{s\hat{\delta}_j \sum_{i=1}^{n+jm} \ln(z_i/\hat{\sigma}_j)} \left(e^{sw_{j+1}^k} + \sum_{i=1}^{n+jm} e^{s\hat{\delta}_j \ln(z_i/\hat{\sigma}_j)} \right)^{-(n+jm)} ds}{\int_0^\infty s^{n+jm-2} e^{s\hat{\delta}_j \sum_{i=1}^{n+jm} \ln(z_i/\hat{\sigma}_j)} \left(\sum_{i=1}^{n+jm} e^{s\hat{\delta}_j \ln(z_i/\hat{\sigma}_j)} \right)^{-(n+jm)} ds}, \tag{42}$$

$\hat{\sigma}_j$ and $\hat{\delta}_j$ which are the MLE's of σ and δ, respectively, can be found (by a fixed point iteration) from solution of

$$\hat{\sigma}_j = \left(\frac{1}{n+jm} \sum_{i=1}^{n+jm} z_i^{\hat{\delta}_j} \right)^{1/\hat{\delta}_j}, \tag{43}$$

$$\hat{\delta}_j = \left[\left(\sum_{i=1}^{n+jm} z_i^{\hat{\delta}_j} \ln z_i \right) \left(\sum_{i=1}^{n+jm} z_i^{\hat{\delta}_j} \right)^{-1} - \frac{1}{n+jm} \sum_{i=1}^{n+jm} \ln z_i \right]^{-1}. \tag{44}$$

4 Conclusions

In this paper the model in (Yang and Klutke, 2000) is extended to the case of multiple systems. Under availability requirement, sequential inspections are obtained to ensure that the availability is at the required level. Sequential inspections are important, especially during the set-up and installation stage. Frequent inspection leads to a high cost and infrequent inspection will lead to low availability of the system upon demand. Although the cost might be an issue in this type of analysis, the focus here is to meet the availability requirement with an appropriate time to next inspection.

The sequential inspection procedure and decision making procedure studied in this paper allows an appropriate level of availability to be reached with minimum cost as well. Furthermore, we do not assume the distribution of system lifetime to be completely known which is usually the case. The information from the inspection can be used to determine the parameters of system lifetime distribution. Hence, such a combined estimation and decision-making analysis is important and useful in practice.

Although the procedure proposed in this paper can be implemented easily, there are several interesting questions that could be raised. Since the estimation for parameters of the lifetime distribution is required at the beginning, one could investigate different estimation methods and also investigate the effect of the estimation error. A common assumption is the independence of the failures and the case of dependence is of interesting. Modelling of the dependence is generally difficult, as specific models describing the degree of dependence will be needed.

Acknowledgements

The authors wish to acknowledge partial support of this research by the Latvian Council of Science and the National Institute of Mathematics and Informatics of Latvia under Grant No. 02.0918 and Grant No. 01.0031.

The authors thank the referees for their comments and suggestions.

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