Synchronized Chaotic Signals in Canonical State Models

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Abstract

The paper deals with examining of synchronized chaotic signals in canonical state models of piecewise-linear (PWL) systems [1]. The Pecora-Carroll drive-response concept and the inverse approach are considered [2]. The theory of the Pecora-Carroll drive-response concept is expanded in the way that the third-order canonical state models make up synchronizing subsystems and the second-order canonical state models make up synchronized subsystems.

Keywords: chaotic signal, canonical state model, synchronization

1 Introduction

In this paper we focus on the analysis of the first form synchronizing - (x_1, x_2) synchronized chaotic system of elementary canonical state models (ECSM) of PWL systems [1]. Have a look at the synchronizing subsystem (1.1) and the synchronized subsystem (1.2) where *h()* is a piecewise-linear function. Let us compare state matrices (1.1) and (1.2). Eigenvalues or equivalent eigenvalue parameters of the synchronizing and synchronized subsystems depend on each other. The synchronized subsystem cannot be designed itself because it is always a part of the synchronizing one.

Although we are limited by the above condition we can design a new extended synchronizing subsystem so that eigenvalues of both subsystems are independent. Let us consider the synchronizing subsystem in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} q_1' & -1 & a_{13} \\ q_2' & 0 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} p_1' - q_1' \\ p_2' - q_2' \\ b_3 \end{bmatrix} . h(\mathbf{w}^{\mathrm{T}}.\mathbf{x}) \tag{1.1}$$

and the synchronized subsystem in the form

$$\begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \end{bmatrix} = \begin{bmatrix} q_1' & -1 \\ q_2' & 0 \end{bmatrix} . \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} + \begin{bmatrix} p_1' - q_1' \\ p_2' - q_2' \end{bmatrix} h(x_1') . \tag{1.2}$$

If the vector $\mathbf{w}$ is given by $\mathbf{w}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & w_3 \end{bmatrix}$, the synchronized subsystem will be a part of the synchronizing subsystem.

2 Synthesis of synchronized chaotic systems

Let p_1, p_2, p_3 and q_1, q_2, q_3 be equivalent eigenvalue parameters of the synchronizing subsystem and p_1', p_2' , q_1', q_2' be equivalent eigenvalue parameters of the synchronized subsystem. Then evaluating characteristic polynomials of determinants $|\mathbf{sI} - \mathbf{A}_0|$ and $|\mathbf{sI} - \mathbf{A}|$ for the synchronizing part we can get equations for circuit parameters as follows

$$
\begin{aligned}
&a_{33} = q_1 - q_1',\\
&b_3.w_3 = (p_1 - p_1') - (q_1 - q_1'),\\
&a_{13}.a_{31} + a_{23}.a_{32} = q_1'.a_{33} + (q_2' - q_2),\\
&a_{23}.a_{31} + q_1'.a_{32}.a_{23} - q_2'.a_{32}.a_{13} = q_2'.a_{33} - q_3,\\
&a_{13}.b_3 + (p_1' - q_1').a_{31}.w_3 + (p_2' - q_2').a_{32}.w_3 = (p_1' - q_1').a_{33} + q_1'.b_3.w_3 +\\
&\qquad + (p_2' - p_2) + (q_2 - q_2'),\\
&(p_1'.q_2' - p_2'.q_1').a_{32}.w_3 - a_{23}.b_3 - (p_2' - q_2').a_{31}.w_3 + (p_2' - q_2').a_{32}.a_{13} -\\
&\qquad - (p_1' - q_1').a_{32}.a_{23} = (p_3 - q_3) - (p_2' - q_2').a_{33} - q_2'.b_3.w_3,
\end{aligned}
\tag{2.1}
$$

where $a_{13}, a_{23}, a_{31}, a_{32}, a_{33}, b_3$ and w_3 are unknown. To solve these equations we may introduce simplifying conditions

$$\tilde{q}_1 = q_1; \tilde{q}_2 = q_2; \tilde{p}_1 = p_1; \tilde{p}_2 = p_2, \tag{2.2}$$

then $a_{33} = 0$, $b_3.w_3 = 0$ and we get equations reduced by

$$
\begin{aligned}
&a_{13}.a_{31} + a_{32}.a_{23} = 0,\\
&a_{23}.a_{31} + q_1.a_{32}.a_{23} - q_2.a_{32}.a_{13} = -q_3,\\
&b_3.a_{13} + (p_1 - q_1).w_3.a_{31} + (p_2 - q_2).w_3.a_{32} = 0,\\
&(p_1.q_2 - p_2.q_1).w_3.a_{32} - b_3.a_{23} + (p_2 - q_2).w_3.a_{31} + (p_2 - q_2).a_{32}.a_{13} +\\
&\qquad + (q_1 - p_1).a_{23}.a_{32} = p_3 - q_3.
\end{aligned}
\tag{2.3}
$$

If $w_3 = 0$, we can obtain a solution. Then parameters $a_{\ 3}$, $a_{\ 3}$, a_3 , and a_3 are given by

$$
\begin{aligned}
&a_{13} = 0\\
&a_{31} = \frac{b_3.q_3}{p_3 - q_3}\\
&a_{23} = \frac{q_3 - p_3}{b_3}\\
&a_{32} = 0
\end{aligned}
\tag{2.4}
$$

Furthermore if $b_3 = 0$, we will obtain rather complicated solution. Parameters a_{13}, a_{23}, a_{31}, and a_{32} are given by

$$a_{13} = \frac{\left((p_1 - q_1)(p_2^2 + p_1 p_2 q_1 - p_2 q_1^2 - p_1^2 q_2 - 2p_2 q_2 + p_1 q_1 q_2 + q_2^2) q_3 w_3\right)}{\left((p_2^2 - p_1 p_2 q_1 + p_2 q_1^2 + p_1^2 q_2 - 2p_2 q_2 - p_1 q_1 q_2 + q_2^2)(-p_3 + q_3)\right)}$$

$$a_{23} = \frac{\left((p_2 - q_2)(-p_2^2 - p_1 p_2 q_1 + p_2 q_1^2 + p_1^2 q_2 + 2p_2 q_2 - p_1 q_1 q_2 - q_2^2) q_3 w_3\right)}{\left((p_2^2 - p_1 p_2 q_1 + p_2 q_1^2 + p_1^2 q_2 - 2p_2 q_2 - p_1 q_1 q_2 + q_2^2)(p_3 - q_3)\right)}$$

$$a_{31} = \frac{(p_2 - q_2)(-p_3 + q_3)}{(-p_2^2 - p_1 p_2 q_1 + p_2 q_1^2 + p_1^2 q_2 + 2p_2 q_2 - p_1 q_1 q_2 - q_2^2) w_3}$$

$$a_{32} = \frac{(-p_1 + q_1)(p_3 - q_3)}{(p_2^2 + p_1 p_2 q_1 - p_2 q_1^2 - p_1^2 q_2 - 2p_2 q_2 + p_1 q_1 q_2 + q_2^2) w_3} \tag{2.5}$$

The method that is proposed in this paper is the very common method how to design any synchronizing and synchronized subsystems. Studying the synchronized chaotic PWL systems, we can establish new elementary linear forms of synchronized second-order parts [8].

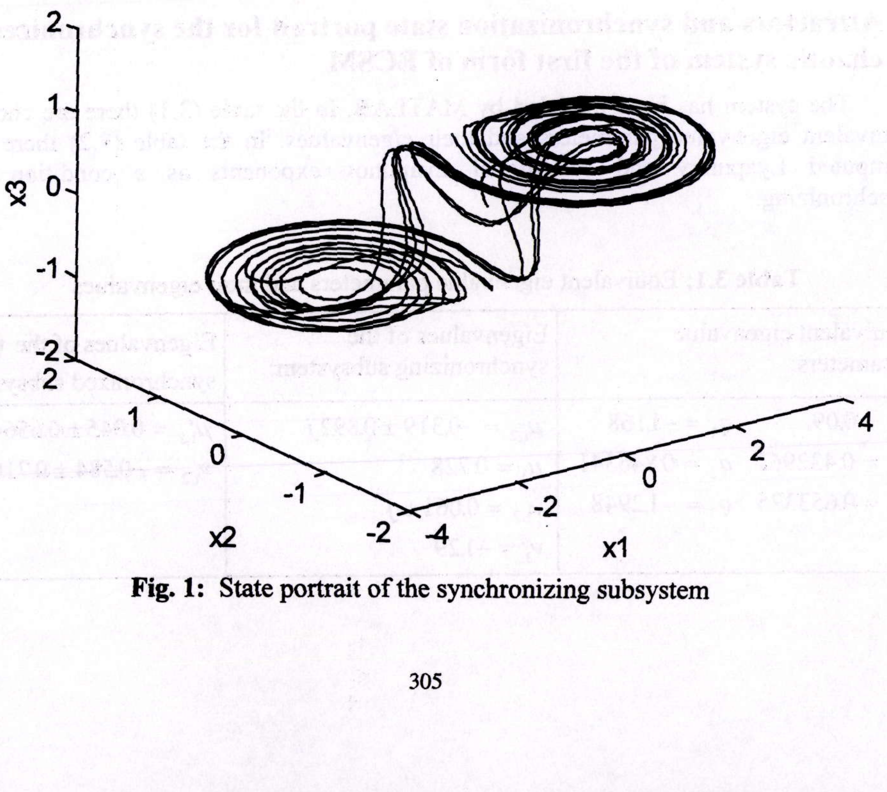

Fig. 1: State portrait of the synchronizing subsystem

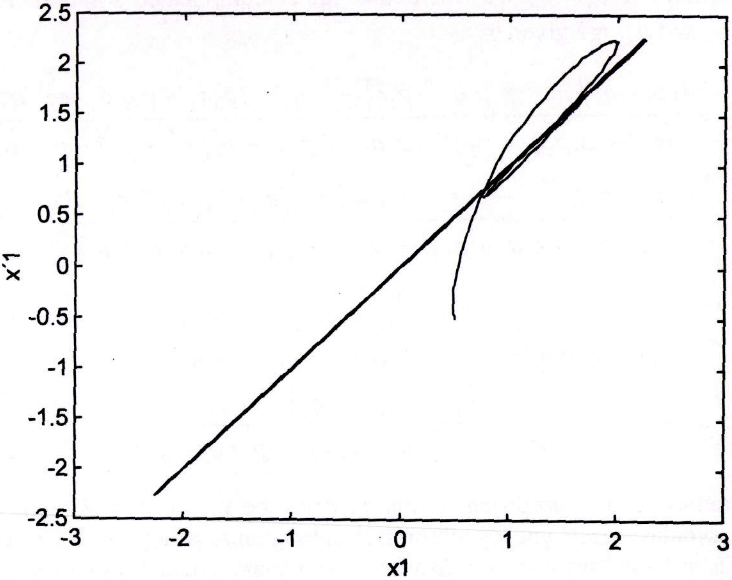

Fig. 2: Synchronization of $x_1 \Leftrightarrow x_1'$

3 Attractors and synchronization state portrait for the synchronized chaotic system of the first form of ECSM

The system has been modeled by MATLAB. In the table (3.1) there are chosen equivalent eigenvalue parameters and their eigenvalues. In the table (3.2) there are computed Lyapunov and conditional Lyapunov exponents as a condition for synchronizing.

Table 3.1: Equivalent eigenvalue parameters and their eigenvalues

Equivalent eigenvalue parameters:		Eigenvalues of the synchronizing subsystem:	Eigenvalues of the (x_1, x_2) synchronized subsystem:
$p_1 = 0.09$	$q_1 = -1.168$	$\mu_{1,2} = -0.319 \pm 0.892j$	$\mu'_{1,2} = 0.045 \pm 0.656457j$
$p_2 = 0.432961$	$q_2 = 0.846341$	$\mu_3 = 0.728$	$\nu'_{1,2} = -0.584 \pm 0.710834j$
$p_3 = 0.653325$	$q_3 = -1.2948$	$\nu_{1,2} = 0.061 \pm j$	
		$\nu_3 = -1.29$	

Table 3.2: Computed Lyapunov exponents and conditional Lyapunov exponents

Initial conditions:	Lyapunov Exponents of the synchronizing subsystem:	Conditional Lyapunov Exponents of the (x_1, x_2) synchronized subsystem:
$x_0 = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$ $x_0' = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}$	$\lambda_1 = 0.103803$ $\lambda_2 = 0$ $\lambda_3 = -0.942795$	$\lambda_1' = -0.403484$ $\lambda_2' = -0.414631$

4 Conclusion

In this paper the synthesis of synchronized chaotic systems of ECSM is proposed. We have also shown the synchronization state portrait. Our results are completed with Lyapunov and conditional Lyapunov exponents. The next research points to design a CNN structure having third-order cells of ECSM and synchronization behavior is going to be studied. This new CNN paradigm can also be exploited in many engineering applications (signal, image and information processing, etc.) as well as in modeling many biological systems.

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