Influence of the Transmission Elasticity on the Law of Motion of a Robot Arm.

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Abstract

In this paper we refer about the first results of a research, recently started, on the influence of the transmission elasticity on the law of motion of the end effector of an industrial robot.
Aim of this investigation is to asses if by an appropriate choise of the law of motion it is possible to obtain an end effector motion that is precise enough, even if the transmission is not very stiff.
The first results of a number of computer simulation show that the law of motion chosen can affect, aslo significantly, the movement precision of a manipulator arm in which the transmission can no more been considered as "rigid".

Keywords: robot, calibration, law of motion

1 Introduction

It must be observed that, in most cases, all the mechanical parts of an industrial robot arm have a relatively great stiffness; this is because the deformations, caused by forces that act on the arm during the motion, must be negligible. In addition, of course, no resonances between the forces and the natural frequencies must occur.
These high stiffnesses bring to very heavy robot arms and, this limits the maximum speeds and accelerations that can be reached.
In some cases it can be useful or necessary to construct robot arms or part of automatic tool machines that are lighter in weight. In some other cases it could be useful to utilize transmissions (e.g. timing belts) which are not very stiff. But, in all these cases the low stiffness can cause deformation that bring to unprecision of the end effector motion.
For these reasons it seemed to us interesting to start an investigation on the influence of the law of motion given by the actuators on the precision of a robot arm motion. The main aim is to obtain an end effector motion that is precise enough, even if the transmission is not very stiff, by finding an adequate law of motion for the actuators.

2 The Mathematical Model

The mathematical model that has been employed for this first investigation is a 2 d.o.f. (degree of freedom) planar manipulator, with two revolute joints, having axes parallel each to the other, moved by two motors with non rigid trasmission between motor and link, a scheme of which is reported in fig.1.

A prototype of this manipulator has been designed and constructed at D.I.M.E. [1] and it is planned it will be used for the experiment of this investigation.

The 2 d.o.f. planar manipulator has been schematized by means of the following equations of motion:

$$\begin{cases} \left(I_{O,1}+I_{O,2}\right)\ddot{\vartheta}_1'+\sigma_1\dot{\vartheta}_{r1}+k_1\vartheta_{r1}-\sigma_2\dot{\vartheta}_{r2}-k_2\vartheta_{r2}=\left(m_2\dot{\vartheta}'^{\,2}_2\frac{l_2}{2}\right)sin\vartheta'_2 l_1 \\ I_{A,2}\ddot{\vartheta}_2'+I_{G,2}\ddot{\vartheta}_1'+\sigma_2\dot{\vartheta}_{r2}+k_2\vartheta_{r2}=-m_2l_1l_2\frac{\dot{\vartheta}_1'^{2}}{2}sin\vartheta_2' \end{cases} \tag{1}$$

Where:
I = link inertial parameter
m = link mass
l = link lenght
G = link centre of gravity
σ = transmission damping
k = transmission stiffness
ϑ' = real angular position of the link;
ϑ = angular position of the motor
$\vartheta_r = \vartheta' - \vartheta$

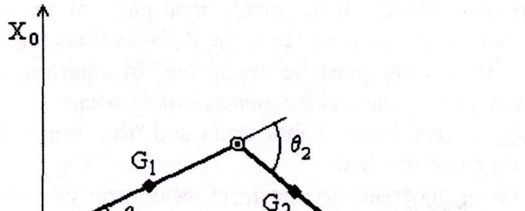


Fig.1: Scheme of the 2 d.o.f. planar manipulator

The system modelled by eq.1 has two natural frequencies that are given by the two roots ω of the equation that follows:

$$I_2(I_1 + I_{2r})\omega^4 - [k_2 I_{G2} + k_2(I_1 + I_{2r}) + k_1 I_2]\omega^2 + k_1 k_2 = 0 \tag{1a}$$

This gives to the system two natural angular frequencies ω_1 and ω_2 which depend on the angle ϑ_2 between the two links, as shown in fig.2.
Also the ratio σ/σ_{cr2} between the damping and the critical damping of the system depends on the arm configuration ϑ_2 as shown in fig.3.

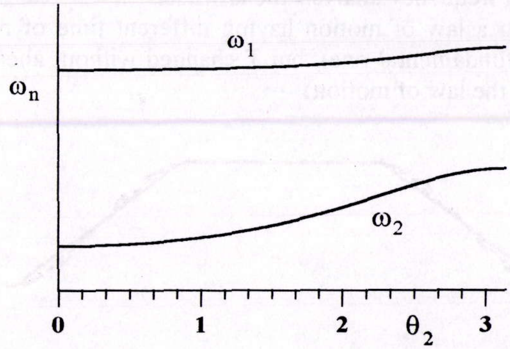


Fig.2: Natural frequencies as a function of the position θ_2 of the second link.

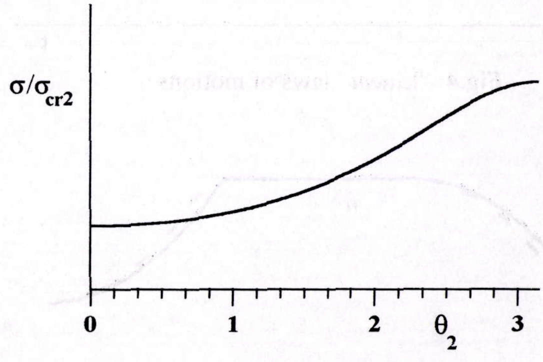


Fig.3: Damping ratio σ/σ_{cr2} versus θ_2

2.1 The Laws of Motion

In order to investigate the law of motion influence on the end effector path of a manipulator having non rigid transmissions between motor and link, we simulated, by solving eq. (1), the path described by the end effector for several laws of motion. The three laws of motion that have been first considered are shown in fig.4. They have all the same velocity and acceleration shape, but the jerk was changed; the areas of the diagram were obviously hold constant.

We will call this kind of law of motion "linear".

As most of the industrial robots can also move their links with a law of motion for which the law of the velocity during the accelerations is not linear but parabolic, we considered also the two laws of motion shown in fig.4 that bring to different jerks.

Each law of motion has been given in eq.(1) by means of 18 harmonic components that have been computed by a frequency analysis the law itself.

It was investigated using a law of motion having different time of motion T (that is different periods of the fundamental one), but T changed without altering the shape of the diagrams (by scaling the law of motion).

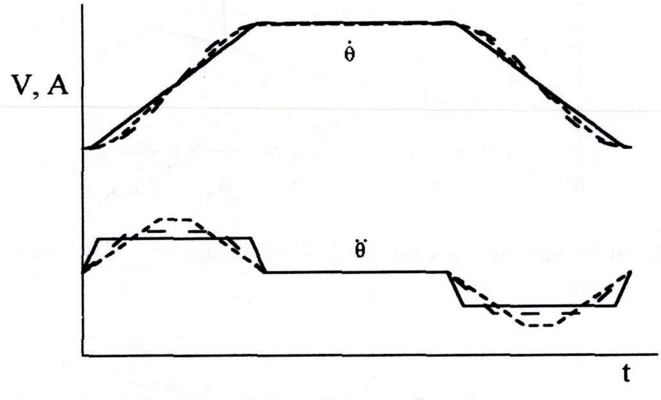


Fig.4 "Linear" laws of motions

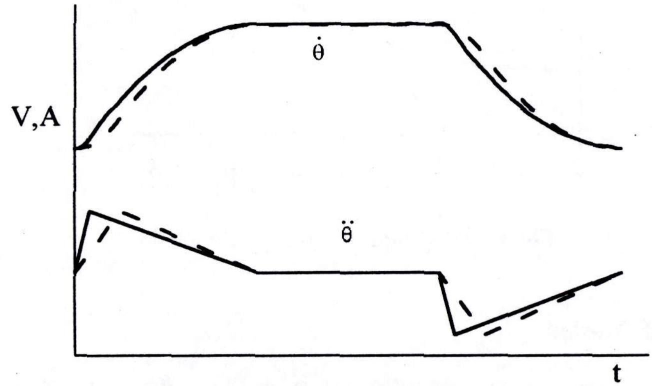


Fig.5 "Parabolic" laws of motions

3 Results of the Simulation.

It was first simulated a path, described by the end effector, that is a straight line 375 mm long, normal to the first link axis as in shown in fig.1. This path was chosen also because, from previous investigations [2,3], it was found that such a planar manipulator describes in this direction a straight line quite perfectly.

As previously said, the natural frequencies of the system depend on θ_2, so they change while the end effector moves along the path; for this reason we refer to "mean" natural frequency and periods T_n that are the ones computed when the end effector is in the middle point of the path.

3.1 Linear Law of Motion

In fig.6 are reported the end effector paths, computed by taking, for the motors, the three linear laws of motion and fixing a time of motion equal to T_n. In this way the fundamental harmonic angular frequency of the law of motion is equal to ω_1.

Have been considered paths that are described both with the end effector that moves away from the first link axis (outwards path) and in the opposite sense (backward paths). The straight line is the path that could be obtained if the transmission was rigid.

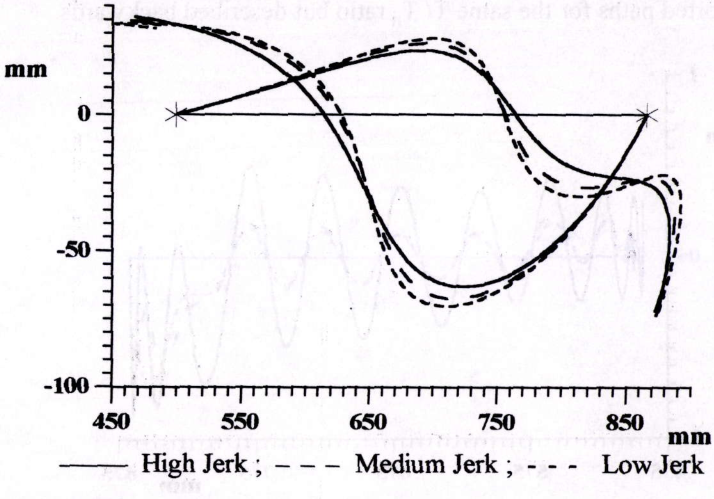


Fig.6: Paths in both directions; $T/T_n = 1$

From this figure we can observe that:

- The end effector describes paths that are very different from the straight line.
- The paths described outwards are different from those backwards.
- The jerk does not seem to have a big influence.

In fig.7 are reported end effector paths computed with the same three linear laws of motion but fixing a time of motion T so that the ratio T/ T_n=10, described outwards.

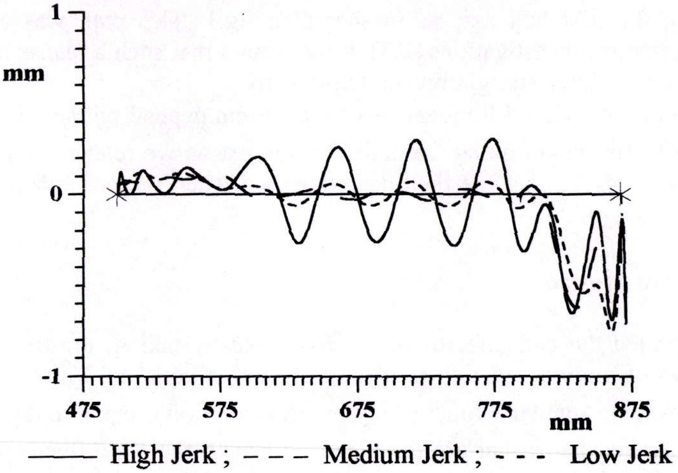


Fig.7 : Outward paths; $T/T_n = 10$

In fig.8 are reported paths for the same T/ T_n ratio but described backwards.

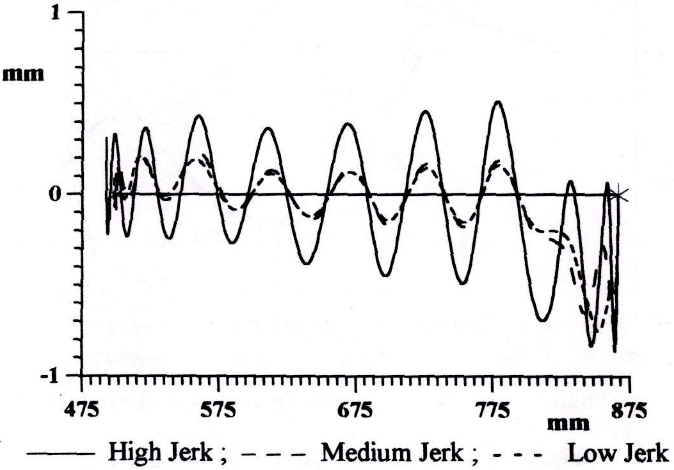


Fig.8 : Backward paths; $T/T_n = 10$

From the above figs it can be observed that:

- The vibration amplitudes are lower than those seen for T/Tn = 1 . This is probably also because, in this case, the 10th harmonic of the law of motion amplitude is lower.

- In this case the higher the jerk is, the higher are the errors of the end effector path.
- The paths descibed outwards show lower errors than those described backwards.

About this last aspect, it can be observed that when ϑ_2 increases (end effector moves forward), the damping ratio σ/σ_{cr2} (see fig.3) increases.

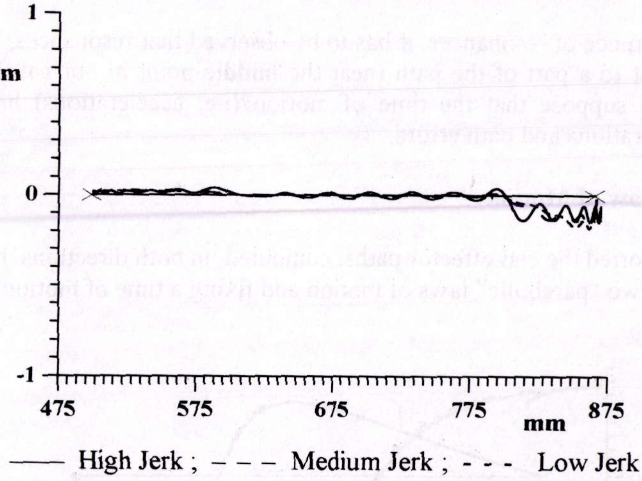


Fig. 9 : Outward paths; $T/T_n = 20$

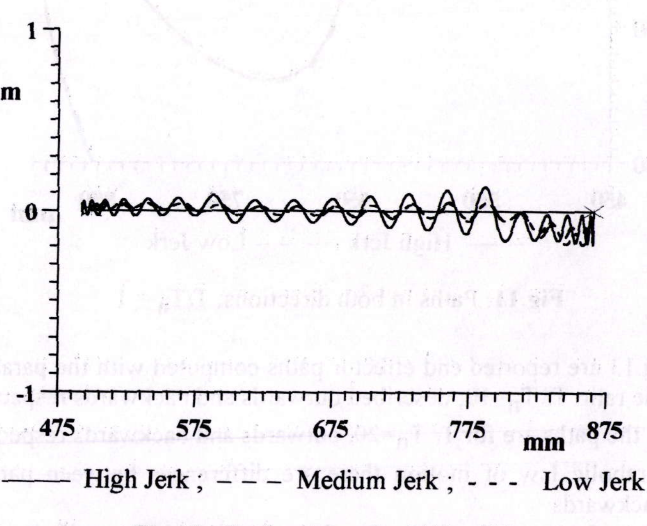


Fig.10 : Backward paths; $T/T_n = 20$

In fig.9 and fig.10 are reported end effector paths computed for T/ T_n=20, describing outwards and backwards respectively.

From these last figs it can be observed that the errors are very lower than those for T/ T_n=10. This is probably also because, in this case, no resonance occurs; in fact each law of motion has been given in eq.(1) by means of 18 harmonic components, as previously said.

As for the occurrence of resonances, it has to be observed that resonances, if they occur, are confined just to a part of the path (near the middle point in our case); so it seems more correct to suppose that the time of motion (i.e. accelerations) has the higher influence on vibrations and path errors.

3.2 Parabolic Law of Motion

In fig.11 are reported the end effector paths, computed, in both directions, by taking, for the motors, the two "parabolic" laws of motion and fixing a time of motion equal to T_n.

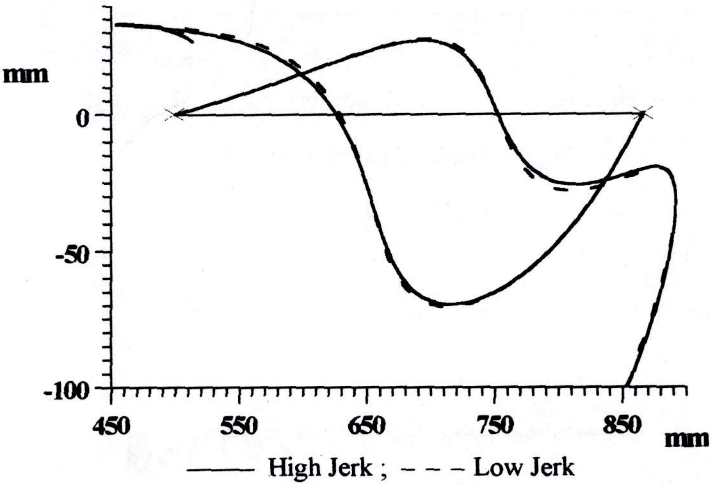


Fig.11: Paths in both directions; T/T_n = 1

In fig.12 and fig.13 are reported end effector paths computed with the parabolic laws of motion fixing the ratio T/ T_n=10, described outwards and backwards respectively.

In fig.14 and 15 the paths are for T/ T_n=20, outwards and backwards respectively.

Also with a parabolic law of motion there are differences between paths described outwards and backwards.

From the last five figures it is also clear that, also in this case, if the ratio T/T_n is increased the errors of the end effector paths are decreased.

If the five last figures are compared to those referred to linear laws of motion, it can be observed that the parabolic law of motion seems to bring to higher vibrations. This is probably because a parabolic law for the velocity has higher jerk values (see fig.5).

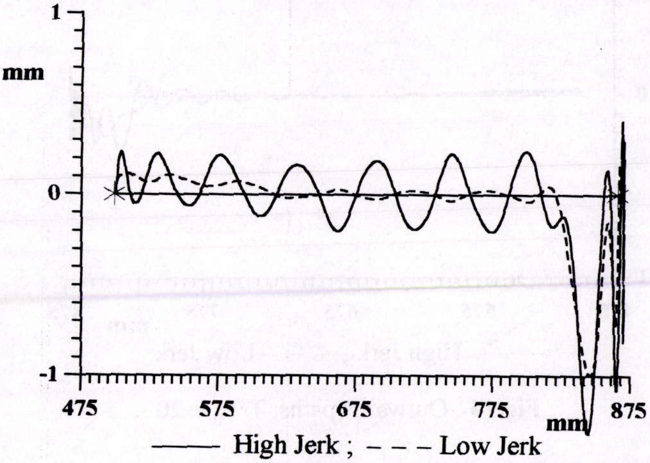


Fig.12 : Outward paths; $T/T_n = 10$

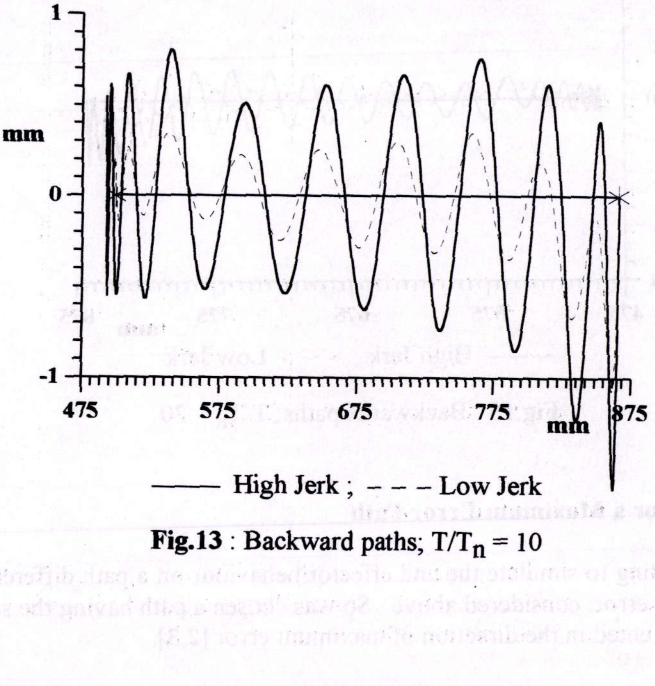


Fig.13 : Backward paths; $T/T_n = 10$

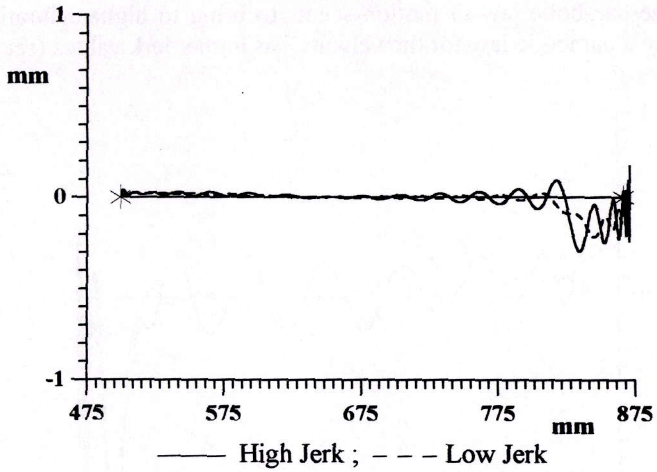


Fig.14 : Outwards paths; $T/T_n = 20$

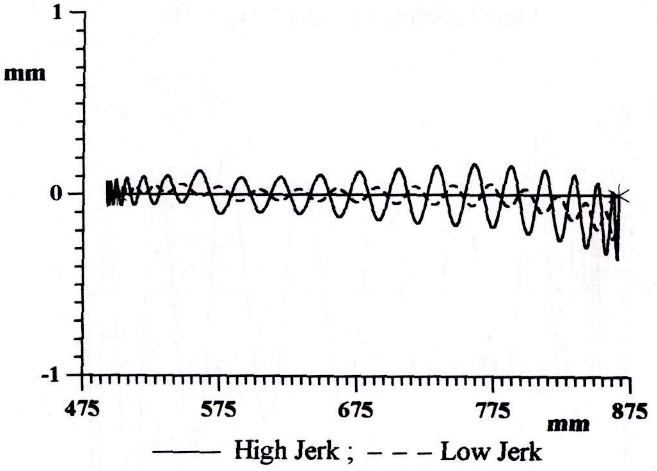
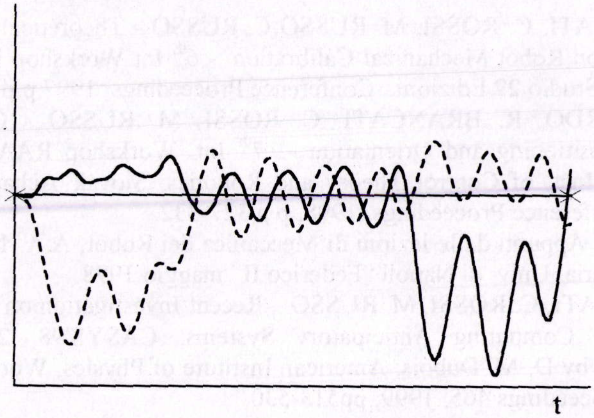


Fig.15 : Backwards paths; $T/T_n = 20$

3.3 Simulation for a Maximum Error Path

It seemed interesting to simulate the end effector behaviour on a path different from the one, of minimum error, considered above. So was chosen a path having the same lenght of 375mm but oriented in the direction of maximum error [2,3].

In this direction, if the transmission was rigid, the end effector describes a path that is rather different fom a straight line but it is almost an arc of a circle [2,3]. For this reason, for a better understanding, in fig.16 are not reported the paths but the differences between the paths obtained with "rigid" transmission and the ones with "elastic" transmission. These last ones have been computed for T/Tn = 10; linear low of motion; medium jerk.



——— Outwards; – – – Backwards

Fig.16 : Differences between paths with rigid and elastic transmission

It can be observed that, in this case, the vibrations due to the transmission elasticity are higher than the ones for the previous path.

4 Conclusions

From this first results of a recently started investigation it is clear that the law of motion can force to vibrate the links even significantly, and this leads to inaccuracies of the end effector motion.

How the law of motion (jerk, acceleration, resonances) mostly influences the end effector motion accuracy is not jet completely clear, so a number of computer simulations and investigations have been planned.

It is evident that, if the ratio T/T_n increased, the vibrations and hence the errors decreased.

Probably this is also due to the amplitudes of the lower order harmonic of the law of motion which are bigger than the amplitudes of the higher order ones. But it seems to be more important that, by increasing the time of motion T , the actuators give lower accelerations so that smaller forces act on the system.

The system behaviour is always a transient; resonances, if they occur, are confined to a small part of the path, as the system natural frequencies change while the end effector runs the path.

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