Surveying an Approximate Solution for Traveling Salesman Problem

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Abstract
One of the interesting problems in 0-1 programming in operation research science is called "Traveling salesman problem". In this paper two inventional methods for calculating the approximate answer have been surveyed. The first inventional method that has been surveyed is simple and its base is selecting a chain of cities that every selected city has the minimum cost with its previous city. The second inventional method is a little harder and has a special algorithm for selecting the first city and next cities. Both inventional methods are surveyed by simulation. Then it seems that the first inventional method has a big error index but the second inventional method has an acceptable error index and it can accept as a heuristic method.
Keywords: Traveling salesman problem, 0-1 programming, Operation research, Simulation, Heuristic method.

1 Introduction

0-1 programming is one of the most important branches of operation research science and in it, two-state variables, one and zero, are used for assigning and not assigning [1]. The order nominating problem between some nodes for obtaining a single closed loop with minimum total cost is famous to traveling salesman problem. One of the applications of this problem is nominating the order of some machines in a single row. In this problem for modelling with 0-1 programming we need main and complement conditions. If the number of nodes increases, modelling of this problem and nominating the complement conditions will be hard. So for solving the problem we will need to enter the model to special 0-1 programming software. In this paper a method for calculating a good answer for the problem has been surveyed. This heuristic method has a simple structure and can develop easily in every software package. A lot of heuristic methods have been invented for the problem. For example, Eeban and Pintea [2] and Dorigi and Gambardella [3] used Ant algorithm for giving a heuristic method. Also Montemanni, Barta, Mastrolilli and Gambardella [4], Achterberg, Berthold, Pfetsch and Wolter [5], Liu, Ng and Ong [6] and Hahsler and Hornik [7] and many others developed heuristic algorithms for the problem.

2 Traveling Salesman Problem

Traveling salesman problem is one of the famous problems in 0-1 programming. In this problem we suppose that there are n cities in a region and the cost (distance) between every two selected cites from all cities is definite. A salesman wants to start his travel from one of those cities and travel to all cities with these conditions that he enters only one time to each city and at last returns to the first city. The cost for traveling from one city to another city can be different from inverse traveling state. In the figure 1 a feasible solution for this problem is shown. The bold circle is the city that travel is started from it.

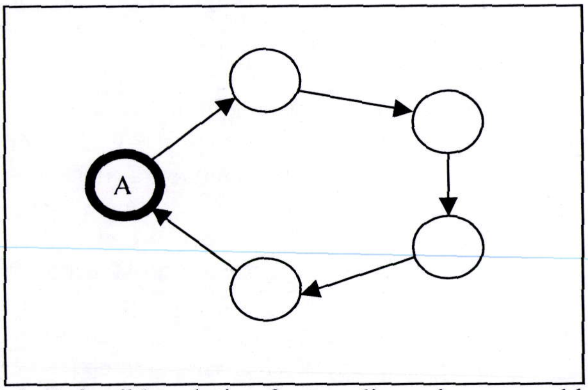

Figure 1: A feasible solution for traveling salesman problem

For solving this problem we use 0-1 programming. Parameters, decision variables, conditions and purpose function are defined as below:

1- Parameter C_{ij} is the travel cost from city i to city j.
2- Decision variable X_{ij} is equal to one if the salesman in his travel goes from city i to city j, otherwise decision variable is equal to zero.

Main conditions defined as below:

$$\sum_{i=1(i\neq j)}^{n} X_{ij} = 1, \quad j = 1,2,\ldots,n \tag{1}$$

The conditions in eq. 1 describe that the salesman enters to each city only for one time.

$$\sum_{j=1(i\neq j)}^{n} X_{ij} = 1, \quad i = 1,2,\ldots,n \tag{2}$$

The conditions in eq. 2 describe that the salesman exits from each city only for one time.

The conditions in eqs. 1,2 ensure that the salesman enters to each city and exits from each city only for one time. These conditions are necessary but they are not enough because it is possible that more than one closed loop is made. For example it is possible that an answer like it is shown in below figure is made, that it is not a feasible answer.

For preventing to make this non-feasible answer we need to use complement conditions. These conditions prevent to make a loop with number of cities less than n. These conditions must be written for all states of cities and prevent to make loops with 2,3,…,(n-2) cities. For example, one of the eqs. 3,4 is necessary to prevent to make the non-feasible answer that has been shown in figure 2.

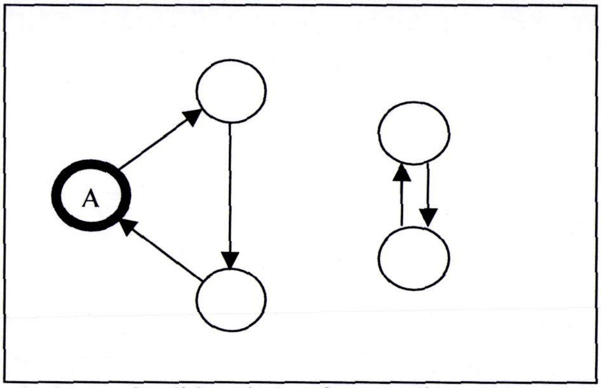

Figure 2: A non-feasible solution for traveling salesman problem

$$X_{AB} + X_{BE} + X_{EA} \leq 2 \quad (3)$$

$$X_{CD} + X_{DC} \leq 1 \quad (4)$$

In the problem that was shown before, the complement conditions must be written for all states for loops with 2 and 3 cities, the number of these complement conditions is 10.

The purpose function of this problem is defined as eq. 5.

$$f(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij} \quad (5)$$

3 Surveying the Methods for Solving the Traveling Salesman Problem

The number of conditions for this problem is very and developing software like professional software for 0-1 programming in software packages is hard and because of it we use inventional methods when an approximate answer is enough for us. For evaluating the power of the inventional methods we must compare their answers with the answer of optimal method.

3.1 The Optimal Method

In this part, for getting the optimal answer without using 0-1 programming, we must survey all states for making a unique loop of cities and compare their costs and finally nominate the best answer. Because of the order of cities is as a loop, if we have n city, the (n-1)! States must be survey. Unfortunately this method has a lot of calculations and if the number of cities increases this method will not be suitable. So using inventional methods is better. The below figure is shown the calculating time for the optimal method depends on the number of cities. Like it seems in figure 3 with increasing the number of cities the calculating time increases exponentially.

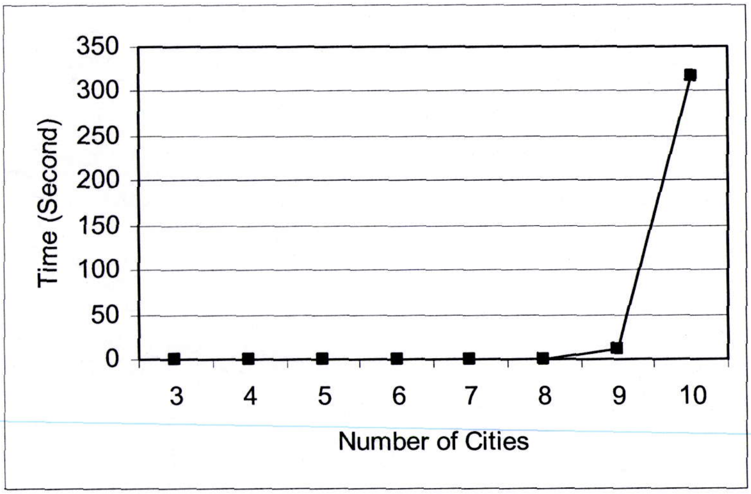

Figure 3: Comparing the optimal method calculating time depend on the number of cities

3.2 The Inventional Methods

In this part two inventional methods have been surveyed.

3.2.1 Inventional Method 1

This method is the simplest method that can be use to solve traveling salesman problem. In this method, a city is selected between all cities randomly and is assigned as the city that the travel is started from it. The next assigned city is selected between unassigned cities and it is the city that has the lowest connection cost from previous city to it. Finally the last city is connected to first city. Like it seems no decision is made for connecting the last city to first city and its connection cost is not surveyed.

3.2.2 Inventional Method 2

This method has two phases. In phase I the first city is selected and is assigned and in the second phase other cities are assigned.

Phase I:

Like it seems in the inventional method 1, no decision is made for connecting the last city to first city and its connection cost is not surveyed. To eliminate this defect we use a special method. We know that in the last step for assigning the cities we will not have any decision on selecting the last city and its connection cost. So at first for selecting the first city we select a city that has the minimum total cost from other cities to it. So we hope that each city that will be select for last city will have a low connection cost from it to first city.

Phase II:

For assigning other cities we use an algorithm that is described in below.

1. Assign the city that was selected in Phase I to city A and as the first city.

2. Select one city between unassigned cities and assign it to city B.
3. Regarding to the city B and other unassigned cities, create a chain of cities that begins from city B and every city in that chain must has the minimum connection cost from its previous city to it.
4. Calculate the cost of the chain and add it with the connection cost from city A to city B.
5. Between before unassigned cities select another city as city B and repeat the steps 3 and 4 for it. Do the loop until all unassigned cities are selected as city B and calculate their cost.
6. Between the calculated costs, select the city B that has the minimum cost and assign it as the next city of A. Then assign the city B as city A and go to step 2 and do the loop until all cities be assigned.
7. Make a connection from last city to first city.

3.3 Evaluating the Methods

For evaluating the inventional methods rather than optimal method we must use simulation. Connection costs must be generated randomly in a distinct region and every order that is resulted from the methods must be evaluated. Generating and evaluating of each order that is resulted by methods must do for a lot of times and the mean of answers be assign as the evaluation criterion. For this purpose we use software that is written with Turbo Pascal programming language. This software can generate all states of order for obtaining the optimal answer and can generate answer by the inventional methods and can calculate their cost. The calculation in this paper is the result of some hours calculating by the software with a PC computer with 2GHz processing speed. To obtaining a good and sure answer the order generations and their cost calculations were done for a lot of times. In the problem the cost of each connection is a random value between 1 and 100. Error percent of each inventional method is calculated with regarding the optimal minimum and optimal maximum cost of each loop by eq. 6. In the eq. 6, e is the error percent, C is the cost of order that is obtained by inventional methods, Min is the optimal minimum cost and Max is the optimal maximum cost.

$$e = \frac{C - Min}{Max - Min} \times 100 \tag{6}$$

In the blow table, the results of calculations and comparing the optimal method with two inventional methods have been shown. Like it seems in the table, calculations to obtaining the optimal answer for number of cities between 3 and 9 have been done for 1000 times but it has been done for 10 cities only 25 times because calculation with 10 cities takes a lot of time.

Like it seems in table 1, the inventional method 2 has a lower and acceptable error percent. So it is selected as a heuristic method for solving traveling salesman problem. Also with surveying the calculation time for this method, that is shown in figure 4, we understand that with increasing the number of cities the calculation time increases exponentially.

Table 1: Results of calculations and comparing the cost of optimal method with inventional methods

Number of cities	Number of calculation repetition	Optimal minimum cost	Optimal maximum cost	Cost of inventional method 1	Cost of inventional method 2	Error percent of inventional method 1	Error percent of inventional method 2
3	1000	121	179	134	128	23.2	12.0
4	1000	136	268	173	145	28.0	7.2
5	1000	146	358	216	158	32.9	5.8
6	1000	151	453	259	171	35.7	6.5
7	1000	155	551	309	177	38.9	5.7
8	1000	158	650	357	186	40.5	5.8
9	1000	159	747	406	193	42.1	5.7
10	25	165	851	481	206	46.0	5.9

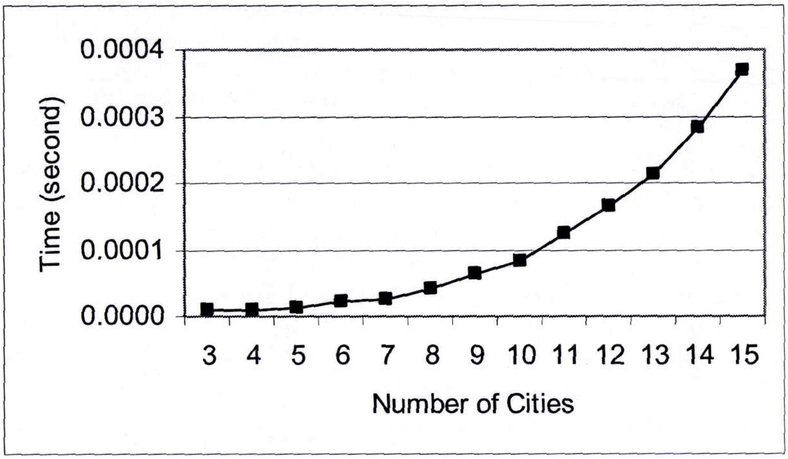

Figure 4: Comparing the calculation time for heuristic method depends on the number of cities for traveling salesman problem

4 Conclusion

Two inventional methods have been surveyed in above for solving the traveling salesman problem and after surveying their validity by simulation it seems that the inventional method 2 has less error percent (approximately 6%) than inventional method 1. So inventional method 2 seems as a simple and good heuristic method for getting an approximately answer of traveling salesman problem.

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